TECHNICAL TRANSACTIONS 3/2018
CZASOPISMO TECHNICZNE 3/2018

ELECTRICAL ENGINEERING

DOI: 10.4467/2353737XCT.18.040.8302
SUBMISSION OF THE FINAL VERSION: 5/3/2018

Ilona Lagun
Adrian Nakonechnyy
Department of Computer Systems and Automatic, Lviv Polytechnic National University, Ukraine

Volodymyr Samotyy (vsamotyy@pk.edu.pl)
Department of Automatic Control and Information Technology, Cracow University of Technology

Using fuzzy logic to optimise the selection of mother wavelets

Wykorzystanie logiki rozmytej dla optymalizacji wyboru falek bazowych

Abstract

The effectiveness of signal processing using discrete wavelet transformation depends on the correct choice of basic wavelet function. A model of multi-criteria optimisation of selection of wavelet function was developed based on fuzzy logic. An experimental study of the model was carried out for the most common types of signals selected from the MATLAB database package.

Keywords: selection of criteria, wavelet function, multi-criteria optimisation, fuzzy set, fuzzy criteria

Streszczenie

Skuteczność przetwarzania sygnału przy użyciu dyskretnej metody falkowej zależy bezpośrednio od prawidłowego wyboru falki bazowej. Opracowano model optymalizacji multicriterialnej doboru falki bazowej na podstawie logiki rozmytej. Wykonano badania eksperymentalne opisanego modelu dla najbardziej popularnych typów sygnałów wybranych z środowiska MATLAB.

Słowa kluczowe: wybór kryteriów, funkcja falkowa, optymalizacja wielokryterialna, zbiór rozmyty, kryteria rozmyte

1. Introduction

The efficiency of processing signals using a discrete wavelet transform is directly dependent upon the selection of the corresponding basic wavelet functions [12].

When choosing a mother wavelet, mainly, properties such as carrier size, the number of zero moments, and the smoothness of the basic functions need to be taken into account. However, such characteristics only have mathematical descriptions of the basic functions and do not enable the obtaining of explicit recommendations for their practical use for the analysis and processing of certain types of signals. Thus, it is expedient to form a certain set of wavelet functions, the optimal choice of which will be determined by certain criteria.

2. Formulation of the problem

In work [1], the properties of wavelets with final and infinite support are generalised. This approach facilitates the basic wavelet function selection process which is dependent upon usage. The amount recommendations reduce the error possibility during the process of basic wavelet selection. Sometimes, standard wavelet plural functions are used [2] – these include Haar (db1), Daubechies (db), Coiflets (coif) and Symlet (sym). Unfortunately, the rules of basic wavelet function selection have not been established in [2]. The basic wavelet function choice is executed using the previous experience. The basic wavelet function selection remains a problem. The best basic wavelet function selection has been executed for Electroencephalogram (EEG) data plural. Several standard wavelet plural including Daubechies, Symlet, Coiflets, Morlet Mexicanhat and Meyer have been reviewed. The most widespread wavelet functions for biomedical signal processing have also been reviewed. The optimal choice for the basic function is absent. In work [3], the choice of basic wavelet could be used for increasing accuracy of approximation and automation speed increasing To improve electrocardiogram (ECG) approximation accuracy the analyses of the basic wavelet functions Cubic Spline Wavelet, Haar Wavelet, Db4 Wavelet and Db6 Wavelet have been conducted. The comparative analysis have been performed as a table. The analysis of different wavelet functions for discrete wavelet transformation is shown in [9]. It is used for watermarks drawing. In [10], the automation of wavelet function attempt have been occurred during gear wheels and bearings diagnostics. The 324 wavelet function has been investigated. The (Daubechies) (db44) function have the most suitable form for this case. The algorithm of automatic basic function choice have been performed. The problem of the silence of EEG signals using wavelet transformation has been reviewed in [13]. The investigation has been performed using the standard EEG data set. To improve the characteristics, the wavelet functions choice db4, sym7, coif3, bior3.9. have been executed. The wavelet function choice was made on the basis of five criteria: MSE, RMSE, SNR, the improved SNR and PRD. In this article, it is emphasised that using this criteria, the unambiguous choice of required function could not be achieved. In this work for unambiguous choice of wavelet functions optimization the fuzzy logic have been used.

Currently, in the technique of signal processing, criteria have been developed based on energy, entropy and correlation dependences [7]: Energy to Shannon Entropy Relations (EER)); entropy (Mutual Information to Relative Entropy Relations (IER)) and correlation (correlation coefficient (Cr)). The energy criterion foresees the calculation of the following ratio:

$$\mathrm{EER} = \frac{E_C}{En} \tag{1}$$

where:

E_C – signal energy in the wavelet space;

En – Shannon entropy for the wavelet coefficients of the signal decomposition.

The entropy criterion is based on the calculation of the next dependence

$$Cr = \frac{\mathrm{COV}_{S\psi}}{\sigma_S \cdot \sigma_\psi} \tag{2}$$

where:

$\mathrm{COV}_{S\psi}$ – the mutual covariance of discrete signal sequences and the basic wavelet function;

σ_s, σ_ψ – the standard deviations of these sequences.

The correlation criterion involves calculating the relationship:

$$\mathrm{IER} = \frac{I(S,C)}{D(S||C)} \tag{3}$$

where:

$I(S;C)$, $D\left(S||C\right)$ – mutual information and relative entropy between the signal and its wavelet coefficients, respectively.

The analysis showed that it is not always possible to reach the uniqueness of the choice of the basic wavelet functions by the above-mentioned criteria [7]. Therefore, a more generalised criterion is required. At present, the solution to the multi-criteria optimisation problem, in which it is impossible to totally optimize all conflicting criteria by 100%, but only each of them to some extent, allows the use of fuzzy logic. The purpose of this work is to analyse basic methods of optimisation and construct a multi-criterial optimisation model for choosing basic wavelet functions under uncertainty.

3. Multi-criteria optimisation choice of basic wavelet functions

In general, the fuzzy solution $\tilde{D}$ is the result of the intersection of local criteria $\tilde{G}_1 \div \tilde{G}_3$. In the case of non-equilibrium criteria, we have:

$$\tilde{D}=\left\{\frac{\underset{i=1,n}{\min}\left(\mu_{G_i}\left(x_1\right)\right)^{w_i}}{x_1},\frac{\underset{i=1,n}{\min}\left(\mu_{G_i}\left(x_2\right)\right)^{w_i}}{x_2},\ldots,\frac{\underset{i=1,n}{\min}\left(\mu_{G_i}\left(x_k\right)\right)^{w_i}}{x_k}\right\} \quad (4)$$

where:

w_i – the coefficient of relative importance of the criterion G_i.

Weights are normalised with the following condition:

$$w_1+w_2+w_3+\ldots+w_n=1. \quad (5)$$

To determine the coefficients w_i, comparative criteria matrices are formed. As a result, the best basic wavelet function is the function with the highest degree of membership:

$$\mu_D\left(x^*\right)=\max_{i=1,2,3,\ldots,n}\mu_D\left(x_i\right) \quad (6)$$

Since the dimensions of the criteria and their scale are different, it is impossible to compare their quality. In view of this, it is necessary to make all scales the same and dimensionless by the following normalisation [5]:

$$G_i=\frac{G_i-G_{i,\min}}{G_{i,\max}-G_{i,\min}}. \quad (7)$$

The next step is to present the criteria in the form of a fuzzy set on the universal set of basic wavelet functions X [4].

$$\tilde{G}_i=\left(\frac{\mu_{G_i}\left(x_1\right)}{x_1},\frac{\mu_{G_i}\left(x_2\right)}{x_2},\ldots,\frac{\mu_{G_i}\left(x_k\right)}{x_k}\right) \quad (8)$$

where:

$\mu_{G_i}\left(x_j\right)$ – a number in the range '0-1', the degree of membership of the basic low-wavelength function fuzzy set $\tilde{G}_i$. The higher the value $\mu_{G_i}\left(x_j\right)$, the higher the estimation of the base wavelet function x_j by the criterion G_i.

Each of the criteria can be represented as a simple fuzzy value of G_i and a linguistic variable $G_i=\left\{G_{iL},G_{iM},G_{iH}\right\}$, where *L*, *M* and *H* denote the concept of 'low', 'middle' and 'high' for the corresponding *i*-th criterion.

On the basis of the analysis of the effectiveness of the criteria performed on the results of the processing of different types of signals, the following expert pair comparisons are formed:

1. Weak advantage of G_2 over G_1;
2. Significant advantage of G_2 over G_3;
3. Significant advantage of G_1 over G_3.

The practical realisation of the multi-criterial optimisation problem of choosing a base wavelet function is performed on the basis of the FIS-editor of systems of fuzzy output from the Fuzzy Logic Toolbox, which is part of the package of applied mathematical modelling software Matlab R2011b.

It is known that in the process of constructing fuzzy-out systems, the methods of Mamdani and Sugeno have become the most commonly used. An analysis of both methods showed the feasibility of using a fuzzy model based on the Mamdani method.

The level of effectiveness of the criterion for choosing basic wavelet functions can be described as follows: low (L), middle (M), high (H). At the phasing stage, membership functions for the sets of input and output linguistic variables are given. The set of criteria $G = \{G_1, G_2, G_3\}$ is represented by three linguistic variables:

$$EER=\{EER\_L, EER\_M, EER\_H\};$$

$$IER=\{IER\_L, IER\_M, IER\_H\};\ Cr=\{Cr\_L, Cr\_M, Cr\_H\};$$

Subsequently, the sigmoid function is selected as a membership function for each input linguistic variable and the trapezoidal function is selected as a membership function for the output linguistic variable. In the next stage, a rule base is formed in the form of a structure with three inputs and one output based on expert pair comparisons (Table 1).

Table 1. Rules for forming a multi-criterial optimisation model for choosing a basic wavelet function

No.	Entrance			Exit
	G_1(EER)	G_2(IER)	G_3(Cr)	Y(WV)
1	2	3	4	5
1	low	low	low	low
2	low	low	middle	low
3	low	low	high	low
4	low	middle	low	low
5	low	middle	middle	middle
6	low	middle	high	middle
7	low	high	low	middle
8	low	high	middle	middle
9	low	high	high	middle
10	middle	low	low	low
11	middle	low	middle	low
12	middle	low	high	middle
13	middle	middle	low	middle
14	middle	middle	middle	middle

tab. 1 (cont.)

1	2	3	4	5
15	middle	middle	high	middle
16	middle	high	low	middle
17	middle	high	middle	middle
18	middle	high	high	high
19	high	low	low	low
20	high	low	middle	middle
21	high	low	high	middle
22	high	middle	low	middle
23	high	middle	middle	middle
24	high	middle	high	middle
25	high	high	low	high
26	high	high	middle	high
27	high	high	high	high

Finally, in the process of dephasing is used the centre of weight method for a discrete set of values of the membership function. At the dephasing stage, the implementation of a fuzzy output system makes it possible to obtain an estimate of the efficiency of the basic wavelet function.

4. Research results

As a result of the construction of the model, the dependence of the level of efficiency of the basic wavelet functions on the two criteria in the form of surfaces was obtained (Fig. 1) wavelet functions, which are defined as optimal for processing each test signal. For the evaluation of the developed models, families of orthogonal functions were selected with a compact Daubechies carrier (db1 ... db20), Coiflets (coif1 ... coif5), Symlets (sym1 ... sym20) and test signals from the Matlab package: blocks, bumps, doppler, heavy sine, trsin, wcantor [5].

For each of the test signals and the selected basic functions, the Matlab package evaluates the values for the energy, entropy and correlation criteria. As a result, arrays of values for each of the criteria are formed. For each array of values, the developed multi-criteria optimisation model was applied and appropriate dependencies were obtained.

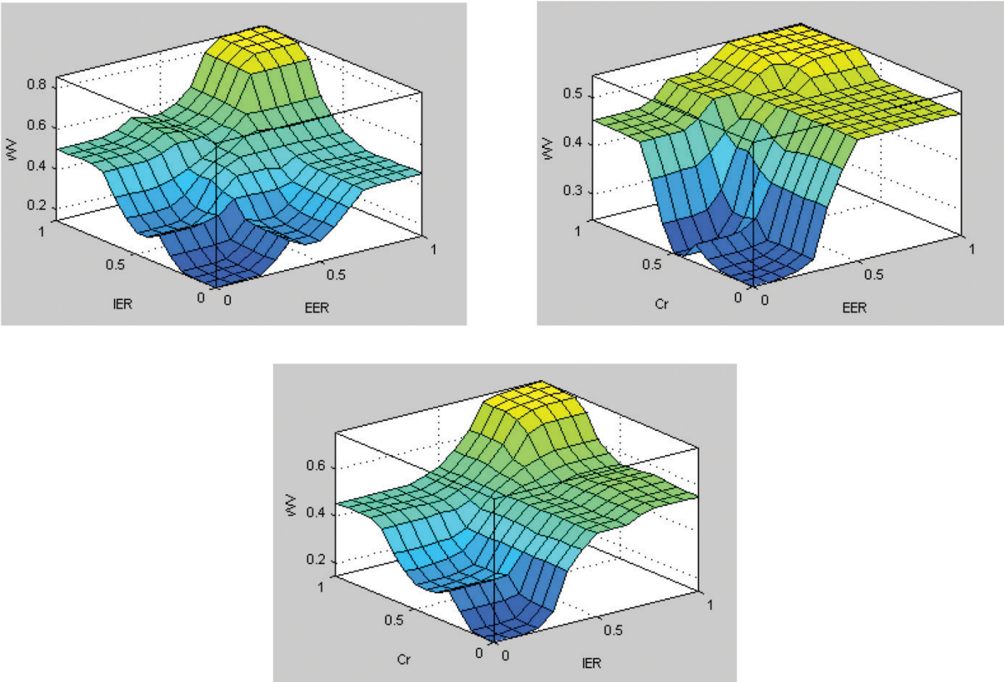

Fig. 1. The surface of a fuzzy model system relative to: a) input variables EER, IER; b) the input variables EER, Cr; c) the input variables Cr, IER

Table 2. Effective base wavelet functions, selected on the basis of multi-criteria optimisation for processing test signals

Test signals	**Base wavelet functions**
'blocks'	db1, sym1
'bumps'	sym15, sym19
'doppler'	sym9, sym13
'heavy sine'	db2, db3
'trsin'	sym7
'wcantor'	db15, db17, db18, db19, db20

5. Conclusions

In this paper, the solution of the ambiguity problem of the choice of basic wavelet functions is proposed using various criteria by means of multi-criteria optimisation with the use of fuzzy logic.

The paper deals with the analysis of algorithms for solving the multi-criterial optimisation problem on the basis of the theory of fuzzy sets. The method of constructing a multi-criterial optimisation model using linguistic variables based on the Mamdani algorithm. As a function of the membership of input variables, a sigmoid function is selected; as a function of the membership of the output variable, the trapezoidal function is selected. Rules for the model of the multi-criteria optimisation of the choice of the basic wavelet function were constructed on the basis of expert information according to the results of previous studies. The practical implementation of the model is based on the FIS-editor of Fuzzy Logic Toolbox fuzzy output systems, which is part of the Matlab R2011b software package of mathematical modelling. The developed model was used to determine the efficiency of the basic wavelet functions for six test signals of the Matlab package – this allowed the selection of optimal functions for the further elaboration of each test signal.

References

[1] Ahuja N., Lertrattanapanich S., Bose N.K., *Properties determining choice of mother wavelet*, IEE Proceedings – Vision, Image and Signal Processing, Vol. 152, 2005, 659–664 .

[2] Al-Qazzaz N.K., Bin Mohd Ali S.H., Ahmad S.A., Islam M.S., Escudero J., *Selection of Mother Wavelet Functions for Multi-Channel EEG Signal Analysis during a Working Memory Task*, Sensors, 2015, 15, 29015–29035.

[3] De Moortel I., Munday S.A., Hood A.W., *Wavelet Analysis: the effect of varying basic wavelet parameters*, Solar Physics, Vol. 222, 2004, 203–228.

[4] Chong E.K.P., Zak S.H., *An Introduction to Optimization*, 4th Edition, John Wiley & Sons, 2013.

[5] Hao Ying, *Fuzzy Control and Modeling: Analytical Foundations and Applications*, Wiley-IEEE Press, 2000.

[6] Kapil Tajane, Rahul Pitale, Jayant Umale, *Comparative Analysis of Mother Wavelet Functions with the ECG Signals*, Journal of Engineering Research and Applications, Vol. 4, 2014, 38–41.

[7] Lagun I., Nakonechny A. *Selection of wavelet basis for the effectiveness processing of signals,* Vestnik Brestskogo gosudarstvennogo tekhnicheskogo universiteta, No. 5, 2016, 69–73.

[8] Mallat S., *Wavelet tour of signal processing*, Third edition: The Sparse Way, 2008.

[9] Navdeep Goel, Gurwinder Singh, *Study of Wavelet Functions of Discrete Wavelet Transformation in Image Watermarking*, An International Journal of Engineering Sciences, Vol. 17, 2016, 154–160.

[10] Rafiee J., Rafiee M.A., Tse P.W., *Application of mother wavelet functions for automatic gear and bearing fault diagnosis*, Expert Systems with Applications, Vol. 37, 2010, 4568–4579.

[11] Reiner Horst, Tuy Hoang, *Global Optimization: Deterministic Approaches*, 3rd edition, Springer-Verlag GmbH, 1996.

[12] Thomas Weise, *Global optimization algorithms: theory and application*, 3nd Edition, Thomas Weise, 2011.

[13] Zaid Abdi Alkareem Alyasseri, Ahamad Tajudin Khader, Mohammed Azmi Al-Betar, *Electroencephalogram Signals Denoising using Various Mother Wavelet Functions: A Comparative Analysis*, Proceedings of the International Conference on Imaging, Signal Processing and Communication, At Penang, Malaysia, October 27, 2017, 100–105.