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HEAT TRANSFER DURING STORAGE OF HOT LIQUID IN THE TANK

PRZENOSZENIE CIEPŁA PODCZAS MAGAZYNOWANIA GORĄCEJ CIECZY W ZBIORNIKU

Abstract

In the article, a mathematical model of heat transfer in a storage tank for hot water with a non-uniform initial temperature is presented. The influence of the initial temperature distribution of the liquid in the tank and the influence of thermal resistance of the tank's walls, bottom and cover on temperature profiles of the liquid in the tank and changes of these profiles in time were analysed. A good conformity of the results obtained based on the process model with the results of measurements of water temperatures carried out under laboratory conditions was obtained.

Keywords: thermal stratification, thermal energy storage, renewable energy sources

Streszczenie

W artykule przedstawiono model matematyczny przenoszenia ciepła w zbiorniku magazynującym gorącą ciecz o niejednorodnej temperaturze początkowej. Przeanalizowano wpływ początkowego rozkładu temperatury cieczy w zbiorniku oraz wpływ oporu cieplnego ścian, dna oraz pokrywy zbiornika na profile temperatur cieczy w zbiorniku, a także czasowe zmiany tych profili. Uzyskano dobrą zgodność wyników otrzymanych na podstawie modelu procesu z wynikami pomiarów temperatur wody przeprowadzonych w warunkach laboratoryjnych.

Słowa kluczowe: stratyfikacja termiczna, magazynowanie ciepła, odnawialne źródła energii

Nomenclature

a	– thermal diffusivity,
A	– heat transfer area,
B	– dimensionless constant defined by (17),
Bi	– Biot number,
c	– heat capacity,
D	– tank diameter,
h	– convective heat transfer coefficient,
k	– thermal conductivity,
q_v	– volumetric internal heat source capacity,
\dot{Q}	– rate of heat transfer,
R	– heat resistance,
U	– overall heat transfer coefficient between the liquid and the environment,
s	– height of the tank,
t	– time,
T	– temperature,
T_a	– ambient temperature,
T_m	– average temperature,
x	– position coordinate,
V	– volume,
Δs	– thermocline thickness
ρ	– liquid density,
ϑ	– dimensionless temperature,
τ	– dimensionless time,
η	– dimensionless position coordinate,
$\Delta\eta$	– dimensionless thermocline thickness,
0 (<i>index</i>)	– bottom of the tank,
1 (<i>index</i>)	– top of the tank.

1. Introduction

The tasks of heating equipment consists in generating heat streams for specific energetic needs of persons or technological processes. The amounts of heat generated and received are generally different in heating systems. These differences are particularly important in solar installations serving the purpose of acquiring heat from the solar radiation energy, where both daily and yearly cycles occur. In order to ensure effective operation of such installations, in which periodical production excesses or deficiencies occur, tanks with exchangers for heat storage, so-called storage containers, are used. The heat transfer to a storage container may occur in a spiral-tube exchanger (pipe coil), an external exchanger (plate heat exchanger, JAD type) or a combined storage container [2]. At the heat take-off side in the exchanger, most often water is used as a widely available medium with

a high thermal capacity. Usually, it is hot service water or a thermal buffer, simultaneously serving the purpose of a fluid coupling. Heat take-off by the storage container may take place using the whole volume of the water stored, or by layers. In a tank loaded by layers, the upper layers are heated at first, not the whole volume of the water present in the storage container. It provides the following advantages: an increase in thermal output of the heating system (better performance of the heat source, e.g. solar collectors) and availability of water with a higher temperature with the same total amount of heat stored in the storage container [1].

The phenomenon of temperature stratification has been a subject of numerous experimental papers. Current review of issues connected with temperature stratification in storage containers filled with water is presented in paper [8]. Various types of storage containers for temperature stratification and the research method is described in [3].

In paper [9], the results of a test of a closed tank completely filled with water, with temperature stratification, are reported. Water has played the role of a heat accumulator and there have been no water inflow to or water outflow from the tank. The heat has been delivered to the tank and collected from it via two spatial spiral pipe coils. Profiles of water temperatures storing the heat have been determined. Also, the influence of various configurations, heights and locations of the pipe coils on the water temperature in the tank and at the heat collection side has been taken into account.

In this paper, a mathematical model of heat transfer in a storage tank for hot water with a non-uniform initial temperature is presented. The model takes into account the temperature equalisation of the liquid in the tank and heat losses to the environment through the walls, bottom and cover of the tank. The goal of the paper is to verify the model experimentally and study the effects of various model parameters on the temperature profiles of the liquid in the tank, and their changes in time.

2. Thermocline

A thermocline, as a geographic term, is the middle water layer in lakes of the temperate zone, with temperature decreases rapidly with an increase in depth. A thermocline in tanks storing warm water is defined analogously.

The thermocline precludes convection mixing of the water layers, enabling maximum utilisation of the storage container volume [5].

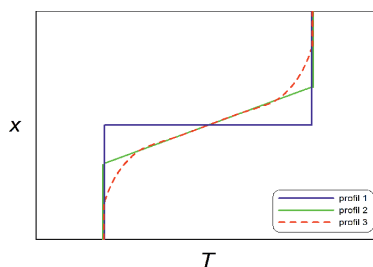


Fig. 1. Temperature profiles of the liquid in the tank with thermal stratification: 1 – profile without an intermediate zone, 2 – profile with a linear intermediate zone, 3 – realistic profile

Fig. 1 shows various temperature profiles of the water in the tank with thermal stratification. Profile 1, without an intermediate zone, does not occur in practice and is of theoretical significance only. Profile 2, with a linear intermediate zone, is closer to reality and easy to describe mathematically. The actual temperature profile in the tank with thermal stratification is curvilinear (profile 3).

3. Model of the process

A mathematical model of the heat transfer process in the tank with thermal stratification is based on the following assumptions:

- ▶ The tank has a cylindrical shape with a vertical axis,
- ▶ Temperature of the liquid in the tank changes only in the axial direction,
- ▶ Thermal properties of the liquid are constant in time and location-independent,
- ▶ Equalisation of temperature of the liquid may be described using the equation of heat conduction,
- ▶ The initial temperature distribution of the liquid in the tank corresponds to profile 1 or 2, shown in Fig. 1.

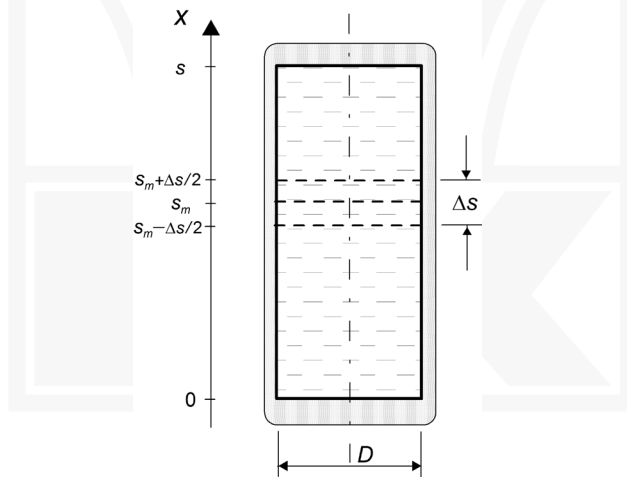


Fig. 2. A tank with thermal stratification

Fig. 2 presents a diagram of the discussed tank having a height s and a diameter D . For the initial temperature distribution without an intermediate zone (profile 1), the tank is filled with colder liquid having a temperature T_0 up to the height s_m ; and there is warmer liquid above this height, having a temperature T_1 .

For the initial temperature distribution with a linear intermediate zone (profile 2), the tank is filled with a colder liquid up to the height $s_m - \Delta s/2$, and there is warmer liquid above the height $s_m + \Delta s/2$. In the intermediate zone with a thickness Δs , the temperature of the liquid changes linearly from T_0 to T_1 .

The heat equation has the following form:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} + \frac{q_v}{\rho c} \quad (1)$$

Thermal diffusivity of the liquid is defined by the following dependence:

$$a = \frac{k}{\rho c} \quad (2)$$

Heat transfer through the sidewall is described by the equation of heat transmission:

$$d\dot{Q} = U(T - T_a) dA \quad (3)$$

Because of the assumed lack of changes in the liquid temperature in the radial direction, the rate of heat transfer per unit volume in the source term of equation (1) amounts to:

$$q_v = -\frac{d\dot{Q}}{dV} = \frac{dA}{dV} U(T_a - T) \quad (4)$$

Geometry of the cylindrical tank leads to a conclusion that:

$$\frac{dA}{dV} = \frac{4}{D} \quad (5)$$

so:

$$q_v = \frac{4U}{D} (T_a - T) \quad (6)$$

The initial condition with a lack of an intermediate zone (profile 1) in the tank has the following form:

$$t = 0 \quad T = \begin{cases} T_0 & \text{for } 0 < x \leq s_m \\ T_1 & \text{for } s_m < x \leq s \end{cases} \quad (7)$$

When the temperature in the intermediate zone is a linear function of location (profile 2), the initial condition is as follows:

$$t = 0 \quad T = \begin{cases} T_0 & \text{for } 0 < x \leq s_m - \frac{\Delta s}{2} \\ T_0 + \left(x - s_m + \frac{\Delta s}{2}\right) \cdot \frac{T_1 - T_0}{\Delta s} & \text{for } s_m - \frac{\Delta s}{2} < x \leq s_m + \frac{\Delta s}{2} \\ T_1 & \text{for } s_m + \frac{\Delta s}{2} < x \leq s \end{cases} \quad (8)$$

In both cases, the average liquid temperature in the beginning of the process amounts to:

$$T_m = \frac{s_m}{s} T_0 + \left(1 - \frac{s_m}{s}\right) T_1 \quad (9)$$

The boundary condition for the tank's bottom has the following form:

$$x=0 \quad k \frac{\partial T}{\partial x} = \frac{T - T_a}{R_0} \quad (10)$$

An analogous condition was assumed for the tank's cover:

$$x=s \quad -k \frac{\partial T}{\partial x} = \frac{T - T_a}{R_1} \quad (11)$$

External thermal resistances R_0 and R_1 result from summation of external convective heat transfer resistance ($1/h$) and conduction through insulation layers (s_i/k_i), respectively:

$$R_0 = \frac{1}{h_0} + \frac{s_{i0}}{k_{i0}} \quad R_1 = \frac{1}{h_1} + \frac{s_{i1}}{k_{i1}} \quad (12a \text{ and } b)$$

In the proposed model, heat transfer through the side walls is described by the source term of the heat equation (1), while transfer through the tank's bottom and cover – by the boundary conditions (10) and (11) [4].

The model equations were transformed into dimensionless forms. Dimensionless temperature:

$$\vartheta = \frac{T - T_a}{T_m - T_a} \quad (13)$$

dimensionless time

$$\tau = \frac{at}{s^2} \quad (14)$$

and dimensionless location coordinate were introduced:

$$\eta = \frac{x}{s} \quad (15)$$

The dimensionless heat conduction equation has the following form:

$$\frac{\partial \vartheta}{\partial \tau} = \frac{\partial^2 \vartheta}{\partial \eta^2} - B\vartheta \quad (16)$$

while the constant B amounts to:

$$B = \frac{4Us^2}{Dk} \quad (17)$$

The initial condition for the profile without an intermediate zone has the following form:

$$\tau=0 \quad \vartheta = \begin{cases} \vartheta_0 & \text{for } 0 < \eta \leq \eta_m \\ \vartheta_1 & \text{for } \eta_m < \eta \leq 1 \end{cases} \quad (18)$$

and the initial condition for the linear temperature profile in the intermediate zone is as follows:

$$\tau=0 \quad \vartheta = \begin{cases} \vartheta_0 & \text{for } 0 < \eta \leq \eta_m - \frac{\Delta\eta}{2} \\ \vartheta_0 + \left(\eta - \eta_m + \frac{\Delta\eta}{2} \right) \cdot \frac{\vartheta_1 - \vartheta_0}{\Delta\eta} & \text{for } \eta_m - \frac{\Delta\eta}{2} < \eta \leq \eta_m + \frac{\Delta\eta}{2} \\ \vartheta_1 & \text{for } \eta_m + \frac{\Delta\eta}{2} < \eta \leq 1 \end{cases} \quad (19)$$

while the constants amount to:

$$\eta_m = \frac{s_m}{s}; \quad \Delta\eta = \frac{\Delta s}{s}; \quad \vartheta_0 = \frac{T_0 - T_a}{T_m - T_a}; \quad \vartheta_1 = \frac{T_1 - T_a}{T_m - T_a} \quad (20 \text{ a, b, c, d})$$

The boundary conditions have the following form:

$$\eta=0 \quad \frac{\partial \vartheta}{\partial \eta} - Bi_0 \vartheta = 0 \quad (21)$$

$$\eta=1 \quad \frac{\partial \vartheta}{\partial \eta} + Bi_1 \vartheta = 0 \quad (22)$$

Biot numbers for the bottom and the cover are defined as follows, respectively:

$$Bi_0 = \frac{s}{R_0 k} \quad Bi_1 = \frac{s}{R_1 k} \quad (23a \text{ and b})$$

Equations of the model were solved numerically using the CrankNicolsonmixed procedure [6].

4. Calculation results

In order to determine the influence of the individual process parameters on the courses of temperature in the tank filled with liquid (water) with a non-uniform initial temperature, parametric analysis of the process was carried out. Temperature profiles for various values of the considered process parameters were determined. In the analysis, dimensional quantities were used, so the graph of the temperature profiles in the tank pertains to the following coordinate system: dimensionless location variable vs. dimensionless temperature. The following parameters were chosen: Biot numbers Bi_0 and Bi_1 , dimensionless thickness of the intermediate layer (thermocline) in the initial condition $\Delta\eta$ and parameter B in equation (16). In all calculations,

$\vartheta_0 = 0.2$ and $\vartheta_1 = 2$ were assumed. For a defined tank height s and thermal conductivity of the liquid k , the value of Biot number is inversely proportional to the external thermal resistance, which in turn depends on the thermal resistance of the external convective heat transfer and the insulation thermal resistance. The higher the thermal resistances, the lower the value of Biot number; with a perfect insulation of the tank's bottom or cover, the respective Biot numbers are equal to zero. It was assumed in the calculations that the Biot numbers for the bottom Bi_0 and the cover Bi_1 are equal. In this calculation series, the other parameters were assumed constant: $B = 300$, $\Delta\eta = 0.2$. The latter value means that the intermediate temperature zone in the beginning of the process constitutes 20% of the tank's height.

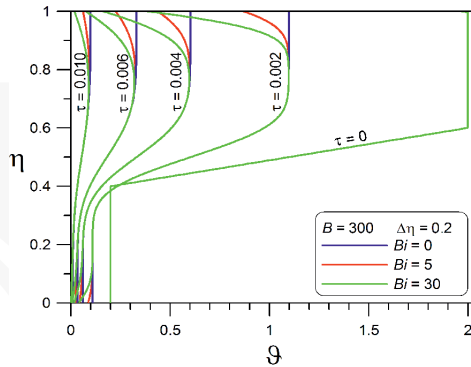


Fig. 3. Dimensionless temperature profiles in the tank for various process durations and various values of Biot number

Results of the calculations are shown in Fig. 3. With time, the profiles shift towards lower temperatures, and the changes in the upper portion of the tank are faster because of a higher driving force of convective heat transfer. Moreover, the upper portion of the tank transmits a part of heat to the lower portion because of temperature equalisation inside the tank. Cooling of the tank content in the lower portion indicates that more heat is transmitted to the walls than transferred from the upper portion in the result of temperature equalisation. At the infinite duration of the process, the system reaches a dimensionless temperature of $\vartheta = 0$, corresponding to equalisation of the liquid temperature and that of the environment. Profiles corresponding to various values of Biot number are designated with various colours. As one can see, various values of Bi give various profiles, but only near the tank ends, particularly the upper end. Heat transfer at the tank ends does not affect the temperature profiles in the central portion of the tank. For perfectly insulated bottom or/and the cover, the temperature profiles are vertical. The worse the thermal insulation, the more deformed the profiles at the ends of the tank. The profile deformation strongly depends on the process duration and the heat transfer location; the biggest deformations occurs for short durations and pertain to the cover. In both cases, it results from a high temperature of the liquid, causing a high driving force of heat transfer.

Fig. 4 shows the influence of the thickness of the intermediate layer in the beginning of the process. The calculations concern constant values of $Bi = 5$ (for the bottom and the cover)

and $B = 300$. The value of dimensionless thickness $\Delta\eta = 0$ means a lack of an intermediate zone (profile 1 in Fig. 1). Such a case is hard to realise in practice; it corresponds to an abrupt border between the layers of the liquid having different temperatures. A larger initial thickness of the intermediate zone results in the liquid temperature more equalised for a significant duration of the process than for smaller $\Delta\eta$ values. However, for long durations of the process, the temperatures in the tank become increasingly more equalised, and the influence of the initial condition becomes increasingly less important.

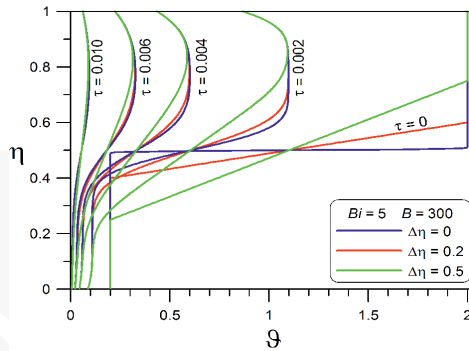


Fig. 4. Dimensionless temperature profiles in the tank for various process durations and various values of $\Delta\eta$

The influence of the parameter B on the dimensionless temperature profiles in the tank is shown in Fig. 5. For defined dimensions of the tank and a defined type of the liquid in the tank, the parameter B is proportional to the overall heat-transfer coefficient through the walls of the tank U . Most often, the strongest influence on the values of U is exerted by the thickness of the thermal insulation of the tank; thus the thicker the insulation layer, the lower the value of B . For a lower B value, the tank cools itself slower. The calculations were carried out for constant values of the other parameters: $Bi = 5$ and $\Delta\eta = 0.2$.

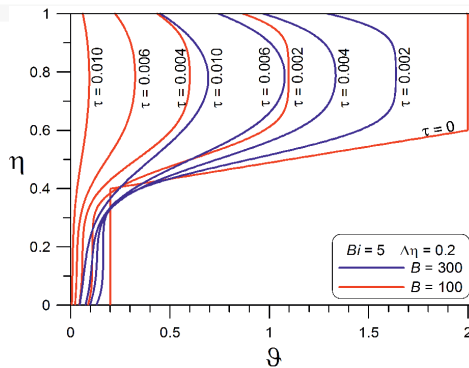


Fig. 5. Dimensionless temperature profiles in the tank for various process durations and various values of parameter B

5. Experimental setup

The experimental studies were carried out in a vertical tube with an inner diameter 44 mm and length 1.8 m, wherein sensors for temperature measurements spaced at 0.1 m were located. The glass tube had an insulation layer. Temporal courses of the water temperature were measured at different heights. The measurement results were recorded automatically. The measurements were performed using two layers of water at different temperatures. At the beginning of the experimental studies, a glass tube was filled with cold water and then through pipe 1 hot water was entered in a controlled amount in order to achieve the position of the two water layers boundary at the desired level. During the supply of hot water to the measuring tube, the cold water from the bottom part of the glass tube was discharged to the outside through pipe 2 and its volume, equal to the volume of the hot water at the inlet, was measured. Measurement details and the installation diagram are presented in paper [4].

6. Comparison of the calculation results with the experimental results

The results of one of the measurement series are shown in Fig. 6 and 7 in the form of symbols. For comparison, a course resulting from the solution of the equations of the presented mathematical model. Physical properties of water at average temperature were assumed: $c = 4190 \text{ J}/(\text{kg}\cdot\text{K})$; $\rho = 1000 \text{ kg}/\text{m}^3$, $k = 0.6 \text{ W}/(\text{m}\cdot\text{K})$; initial temperatures of both layers of water amounted to: $T_0 = 23^\circ\text{C}$, $T_1 = 48^\circ\text{C}$; ambient temperature $T_a = 22^\circ\text{C}$; tube dimensions $s = 1.8 \text{ m}$, $D = 0.04 \text{ m}$. Moreover, as the initial amounts of colder and warmer water were equal, $s_m = 0.9 \text{ m}$ was assumed. Based on these data, the average water temperature in the beginning of the process was calculated: $T_m = 35.5^\circ\text{C}$. Parameters associated with thermal resistances were estimated as: $R_0 = R_1 = 1 \text{ m}^2\text{K}/\text{W}$; $U = 0.6 \text{ W}/(\text{m}^2\text{K})$. The thickness of the intermediate zone was considered a set parameter; $\Delta s = 0.35 \text{ m}$ was assumed. The following values of the dimensionless parameters were determined: $\mathcal{Q}_0 = 0.074$, $\mathcal{Q}_1 = 1.93$, $\eta_m = 0.5$; $\Delta\eta = 0.194$; $B = 324$; $Bi_0 = Bi_1 = 3$.

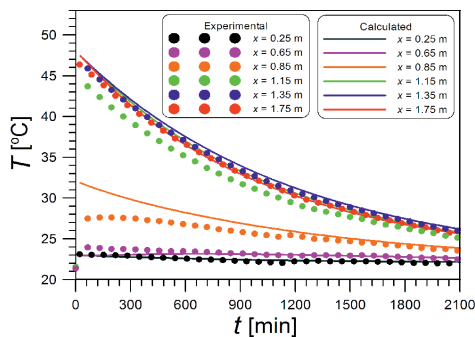


Fig. 6. History of water temperatures at various tube levels – comparison of the results of measurements with the results of model calculations

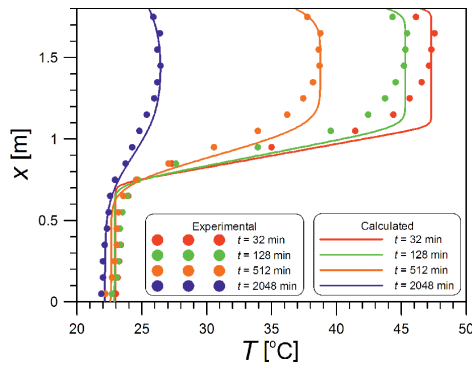


Fig. 7. Profile of water temperatures in the tube for various process durations – comparison of the results of measurements with the results of model calculations

Figure 6 pertains to the course of the water temperature in time at various heights in the tank, presented in a dimension coordinate system. The experimental and calculated courses are quite consistent with one another, apart from the central zone of the tank, where the calculated values are higher than the experimental values.

The temperature profiles are shown in Fig. 7. Also in this case, discrepancies between the experimental and calculated values occur in the central zone. Near the ends of the tank, and in the range of long durations of the process, the accordance between the results is very good.

7. Conclusions

- ▶ The thermocline layer is very important for maintaining thermal stratification of the liquid stored in the tank. The thermocline precludes a convective heat transfer between the layers of the liquid having different temperatures.
- ▶ Heat transfer in the tank with thermal stratification may be described by the heat equation with the heat source term. Thereby, heat transport inside the tank, heat transmission through the side walls (via the source term) and convective heat transfer to the bottom and the cover of the tank are taken under consideration.
- ▶ The numerical calculations confirmed the influence of the thickness of the intermediate zone (thermocline) on temperature distributions in a tank cooling down. However, this influence decays in time.
- ▶ A good accordance between the calculated and the experimental results confirms the correctness of the proposed model. Slight discrepancies were found only in the initial stage of cooling down of the tank and only in its central portion.

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