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MODELING OF STEADY AND TRANSIENT TEMPERATURE DISTRIBUTION IN THE DEVICE FOR MEASURING THE THERMAL CONDUCTIVITY

MODELOWANIE USTALONYCH I NIEUSTALONYCH ROZKŁADÓW TEMPERATURY W APARACIE DO POMIARU WSPÓŁCZYNNIKA PRZEWODZENIA CIEPŁA

Abstract

The aim of this paper is the presentation of the method for thermal conductivity measurement and numerical modeling of temperature distribution in apparatus for measuring the thermal conductivity. Experimental studies will be carried out in transient state until the steady state heat conduction in apparatus is achieved. The calculated and measured temperature distribution will be compared. The time to steady state in apparatus for different samples will be estimated based on numerical and experimental results. The influence of the contact resistance between the sample and the measuring device will be analyzed.

Keywords: *heat conduction*, *CFD modeling*

Streszczenie

Celem artykułu jest pokazanie sposobu pomiaru współczynnika przewodzenia ciepła w ciałach stałych oraz przeprowadzenie modelowania rozkładu temperatury w przyrządzie pomiarowym. Przeprowadzone zostaną badania doświadczalne w stanach nieustalonych aż do momentu ustalenia się zjawiska przewodzenia ciepła. Przedstawione zostaną wyniki modelowania numerycznego w stanie nieustalonym. Porównane zostaną zmierzone i obliczone przebiegi temperatury. Zweryfikowany zostanie czas ustalenia się zjawiska przewodzenia ciepła w przyrządzie pomiarowym. Dodatkowo analizowany będzie wpływ oporów kontaktowych pomiędzy próbką i urządzeniem pomiarowym zarówno doświadczalnie jak i numerycznie.

Słowa kluczowe: *przewodzenie ciepła*, *modelowanie CFD*

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1. Measurements of thermal conductivity in solids

Heat conduction is the transfer of internal energy between directly adjoining parts of one body or different bodies [1]. In the liquids transferred is kinetic energy of atoms and molecules and in solids transferred is vibration energy of atoms in the crystal lattice and the free movement of electrons. Heat conduction takes place generally in accordance with the law of the Fourier that is saying that the transferred heat flux density is directly proportional to the temperature

$$\dot{q} = -k\frac{\partial T}{\partial x} \tag{1}$$

where:

$\dot{q}$ – the heat flux expressed in W/m^2,
k – a heat conduction coefficient in W/(m·K),
T – a temperature in °C or K,
x – a coordinate in m.

The minus sign in equation testifies to the fact that heat flows in the direction of the decreasing temperature.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator. The thermal conductivity varies within a wide range, from 0.026 [W/(mK)] for the air to about 5000 [W/(mK)] for the graphene.

Two classes of methods exist to measure the thermal conductivity of a sample: steady-state and non-steady-state (or transient) methods.

Steady-state techniques perform a measurement when the temperature of the material measured does not change with time [2–3]. In these techniques a flat, cylindrical or spherical layer is located between the heat source of a higher temperature and the heat receiver at a lower temperature. The advantage of these methods is simple mathematics.

The disadvantages of these methods include a complex array of regulatory control and the test bench for a long time, set the heat exchange of the sample, the need to ensure perfect contact with the surface of the sample surfaces and the radiator grille and the need to ensure good thermal insulation on the other surfaces of the sample.

The transient techniques perform a measurement during the process of heating up. The advantage is that measurements can be made relatively quickly. Transient methods are usually carried out by needle probes. The disadvantage is that the mathematical analysis of the data is in general more difficult.

2. Experimental research

Apparatus used for the tests is presented in Figure 1. It consists of two sections: hot and cold. In the hot, an electric heater is mounted with a maximum power of 65 [W] (at max. voltage of 240 [V]). Heating power can be continuously reduced. Hot section is at the top of the apparatus. It has a cylindrical shape with a diameter of 25 [mm] and is made of brass with a heat conductivity 121 [W/(mK)].

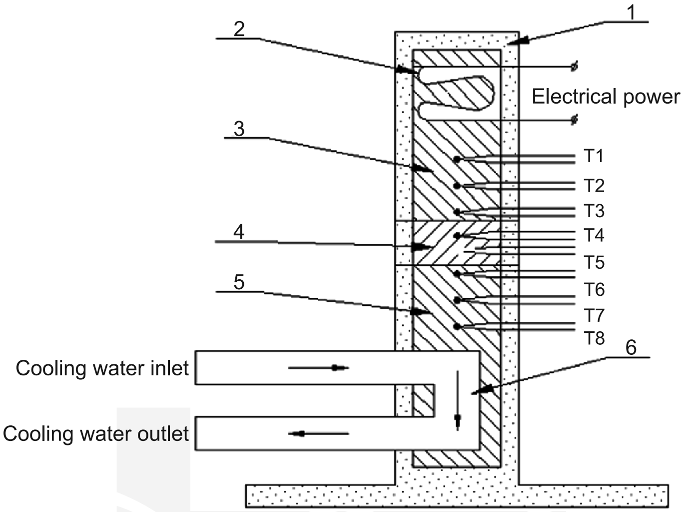

Fig. 1. Apparatus for measurement of thermal conductivity, 1 - insulation, 2 – electric heater, 3 – heating section, 4 – sample test material, 5 – cooling section, 6 – coil, T1, T2, T3, T4, T5, T6, T7, T8 – thermocouples

Rys. 1. Aparat do badań współczynnika przewodzenia ciepła; 1 – izolacja, 2 – grzałka elektryczna, 3 – sekcja grzejna, 4 – próbka materiału badanego, 5 – sekcja chłodząca, 6 – wężownica, T1, T2, T3, T4, T5, T6, T7, T8 – termopary

K type thermocouples are mounted every 15 [mm] on the surface of the heater. They are labeled T1, T2, T3. The cooling section has a cylindrical shape with a diameter of 25 [mm] and is made of the same kind of brass. This section is situated at the bottom of the apparatus. Three thermocouples are mounted with markings T6, T7, T8 on it’s surface. This thermocouples are also located at 15 [mm] distance. In the cooling section, water flows through coil, where receives heat from metal area. Volume flow rate is about 1.5 [l/min]. Both sections are thermally insulated on the outer surface by plastic covers.

A test sample is mounted in the apparatus between the heating and cooling section. The samples are cylindrical with a diameter of 25 [mm] and a height of 30 [mm]. Samples are made of brass, steel and aluminum, respectively with heat conduction coefficients 121, 25, 180 [W/(mK)] according to manufacturer. A sample of brass is equipped with a two thermocouples with labels T4 T5, allowing additional verification of the temperature distribution. The surfaces of the samples that are in contact with heating and cooling sections are lubricated with grease paste to minimize contact resistance.

At the laboratory, temperature are measured in each section. The measurement is carried out since the launch of an electric heater and cooling system until the steady state heat conduction in apparatus is achieved. The contact surfaces are lubricated with grease paste and pressed. First experiment is conducted for a sample made of aluminum.

The measurement results are shown in Figure 2. For the time of 3000 [s] steady state is achieved. The temperature distribution along the height of the apparatus for the time 4000 [s] is shown in Figure 3.

The temperature distribution curve in aluminum sample is at a different angle than temperature distribution curves in heating and cooling sections. This is due to a higher

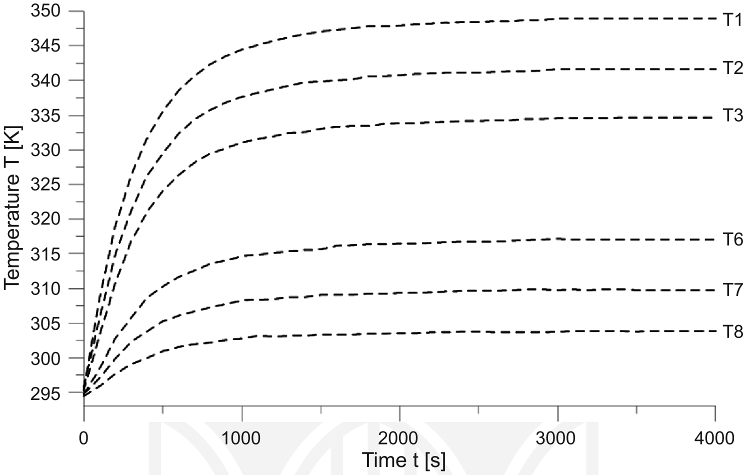

Fig. 2. Measured transient temperature at selected points in the device with inserted aluminum sample

Rys. 2. Zmierzone przebiegi temperatury w wybranych punktach w aparacie z próbką aluminiową

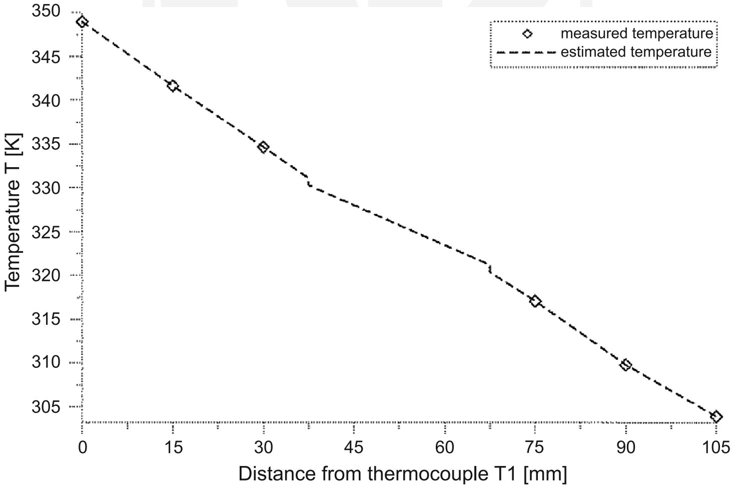

Fig. 3. Measured steady state temperature distribution in the device with inserted aluminum sample

Rys. 3. Rozkład zmierzonej ustalonej temperatury w aparacie z założoną próbką aluminiową conductivity in aluminum than in brass. Temperature discontinuities are caused by contact resistance at the contact surfaces between the sample and cooling or heating sections. They are not higher than 0.8 [K].

Next, thermal conductivity of aluminum sample is estimated. For this purpose, the temperature of the upper and lower surfaces of the sample must be calculated. Finally, thermal conductivity can be determined from the equation (2).

$$k_{a-b} = \frac{4U\,I}{\pi D^2 \left(\dfrac{\Delta T_{a-b}}{\Delta x} \right)} \tag{2}$$

where:

U – the average value of the voltage drop on the heating element [V],
I – the average value of the heating current [A],
D – diameter of samples and heating and cooling part of device,
ΔT_{a-b} – the temperature difference between the upper and lower surface of the sample [K],
Δx – thickness of the sample [m].

During the measurement the following data are observed: the current is 0.186 [A] and the voltage 160 [V] sample diameter is 25 [mm] and height 30 [mm]. Substituting these data into equation (3) the following results is obtained.

$$k_{a-b} = \frac{4 \cdot 160\,[\mathrm{V}] \cdot 0.186\,[\mathrm{A}]}{\pi \cdot (25 \cdot 10^{-3}\ [\mathrm{m}])^2 \cdot \left(\dfrac{10\,[\mathrm{K}]}{30 \cdot 10^{-3}\ [\mathrm{m}]} \right)} = 181.9 \left[\frac{\mathrm{W}}{\mathrm{mK}} \right] \tag{3}$$

Determined value differs from the real value of 180 [W/(mK)] for 1.04% because of temperature measurement errors and contact resistance between the sample and cooling or heating section. A much larger measurement errors of heat conductivity can occur if the measurement is carried out in transient state.

3. CFD modeling

The following part describes the numerical modeling of temperature distribution in the transient state, and steady state in the apparatus for measuring thermal conductivity [4–8].

Geometric model of the apparatus is created basing on the measured dimensions, considering its basic elements such as heating section with the resistance heater and the cooling section with water coil where the cooling water flows. This parts are made of brass with a diameter of 25 [mm] and a height of 100 [mm] each. The samples are installed between them. They are made of aluminum, stell and having the same diameter as upper and lower parts and height of 30 [mm]. Geometric model of this apparatus is shown in Figure 4. The cooling coil interferes with simplification of 3-D model into an axisymmetrical model.

The three-dimensional geometry is divided into 907467 cells, 2242059 faces, 343969 nodes. Finite elements are built from quad pave type of elements. This grid is shown in

Figure 5. Modeling is performed using the equations of balance of mass, momentum and energy. It is assumed that the flow is turbulent and the k-ε model (Viscous Standard *k*–ε, Scalable Wall Functions) is used.

After that material properties for solid and water are defined. Next flow and thermal boundary conditions are defined. At the inlet to the water coil, velocity-inlet type boundary condition is set (speed 1 [m/s], 294.65 Temperature [K]). At the water outlet of the coil, outflow type boundary condition is defined. On the surface of the heater, heat flux density (Heat Flux) is set (28000 [W/m^2]).

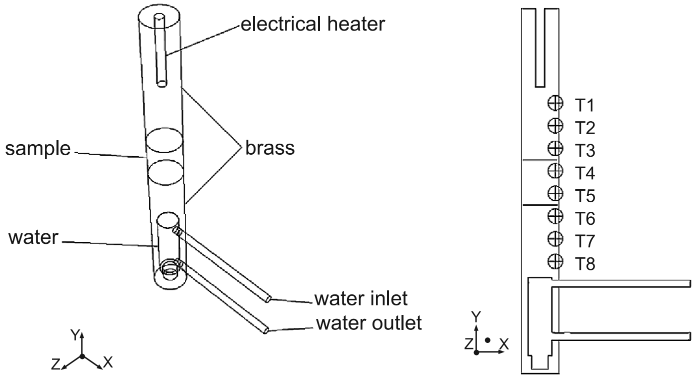

Fig. 4. Geometry of apparatus model

Rys. 4. Geometria modelu aparatu

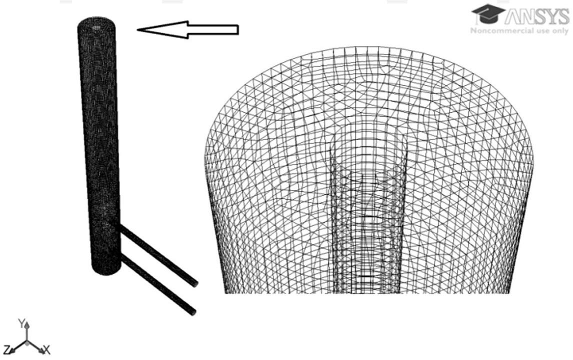

Fig. 5. Division for finite elements

Rys. 5. Podział na elementy skończone

The constant time step is set (1 [s]). Iteration starts taking the whole area parameters as at water inlet to the coil. ANSYS Fluent is used [4]. Steady state is obtained after about 3000 [s].

Figure 6 shows the calculated temperature transient at the points in the apparatus where thermocouples(T1-T8) are installed. Numerically obtained temperatures are compared with values measured at the device. The maximum error does not exceed 1.2%.

Temperature distribution along the height of the apparatus with installed aluminum sample is presented in Figure 7. Well developed and validated numerical model can be used to determine the time when steady state is achieved. This is the time at which measurements of thermal conductivity for samples can be done.

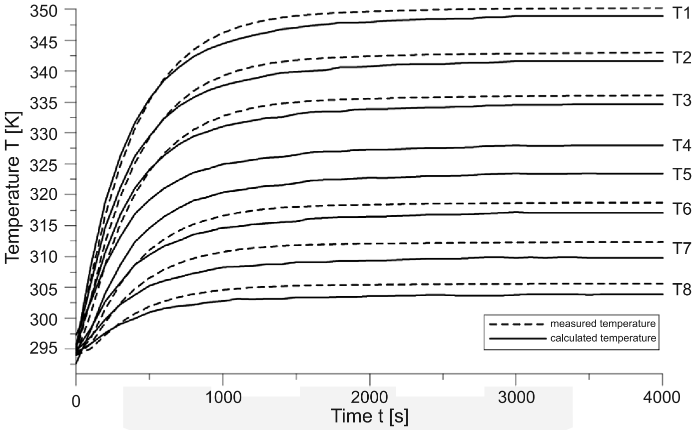

Fig. 6. Measured and calculated temperature transient at selected points in the device with inserted aluminum sample

Rys. 6. Zmierzone i obliczone przebiegi temperatury w wybranych punktach w aparacie z próbką aluminiową

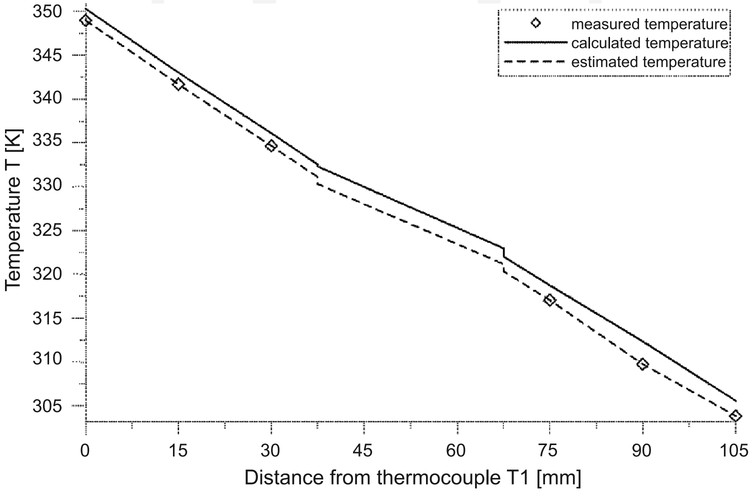

Fig. 7. Measured and calculated steady state temperature distribution in the device with inserted aluminum sample

Rys. 7. Rozkład zmierzonej i obliczonej ustalonej temperatury w aparacie z założoną próbką aluminiową

Finally numerical simulation is performed for the sample made of steel. Time when the steady state is obtained is about 5500 [s].

4. Conclusions

The apparatus for measuring thermal conductivity in solids and the method to calculate thermal conductivity were presented. Experiment was carried out and measured temperature transient at selected points in the device with inserted aluminum sample were presented. Coefficient of thermal conductivity was calculated. Determined value of conductivity differs from the real value of 180 [W/(mK)] for 1.04% because of temperature measurement errors and contact resistance between the sample and cooling or heating section.

Next, numerical model of the apparatus for measuring thermal conductivity was presented. The comparison of measured and calculated temperature transient at selected points in the device with inserted aluminum sample were presented. The maximum error between the measured and calculated values did not exceed 1.2%.

The presented numerical model calculated the time when steady state was achieved in apparatus with aluminum sample. The calculated value of 3000 [s] was validated by measurement. Additionally, the steady state time for apparatus with steel sample was determined (5500 [s]).

The knowledge of steady state time allows users for creation of an experimental plan. The measurements will not be done until the steady state heat conduction in apparatus is achieved.

Additionally, the presented numerical model allows users for contact resistance analysis. Furthermore, knowing that the temperature at any point of apparatus can not exceed 90^{o}C, the numerical model can calculate the maximum power supply.

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