

DANUTA CIESIELSKA*, STANISŁAW DOMORADZKI**

ON MATHEMATICAL LECTURES AT THE JAGIELLONIAN
UNIVERSITY IN THE YEARS 1860–1918. ESSAY BASED
ON MANUSCRIPTS

O MATEMATYCZNYCH WYKŁADACH
W UNIWERSYTECIE JAGIELLOŃSKIM W LATACH
1860–1918. OPRACOWANIE NA PODSTAWIE RĘKOPISÓW

Abstract

This article is a partial report of research on mathematical education at the Jagiellonian University in the period 1860–1945. We give a description of the selected lectures: *Calculus of Probability* by Michał Karliński, *Analytic Geometry* by Franciszek Mertens, Marian Baraniecki's lectures, *Higher seminar (Weierstrass preparation theorem)* by Kazimierz Żorawski, *Principles of Set theory* by Zaremba and *Analytic function* and *Number theory* by Jan Sleszyński. Moreover, short biographical notes of professors of mathematics of the Jagiellonian University Michał Karliński, Franciszek Mertens, Marian Baraniecki, Stanisław Kępiński, Kazimierz Żorawski, Stanisław Zaremba and Jan Sleszyński – are given.

Keywords: mathematics in Cracow in XIX century and the beginning XX century

Streszczenie

Niniejszy artykuł jest fragmentem badań autorów w zakresie historii matematycznej edukacji w Uniwersytecie Jagiellońskim w latach 1860–1945. Prezentujemy opis wybranych wykładów: Michała Karlińskiego *Rachunek prawdopodobieństwa*, Franciszka Mertensa *Geometrię analityczną*, informacje o wykładach Mariana Baranieckiego, Kazimierza Żorawskiego zajęcia w wyższym seminarium na temat twierdzenia przygotowawczego Weierstrassa, Stanisława Zaremby *Wstęp do teorii mnogości* oraz Jana Sleszyńskiego *Teorię funkcji* (analitycznych) i *Teorię liczb*. Prezentujemy również krótkie biografie profesorów matematyki UJ: Michała Karlińskiego, Franciszka Mertensa, Mariana Baranieckiego, Stanisława Kępińskiego, Kazimierza Żorawskiego, Stanisława Zaremby oraz Jana Sleszyńskiego.

Słowa kluczowe: matematyka w Krakowie w XIX i na początku XX w.

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1. Historical background

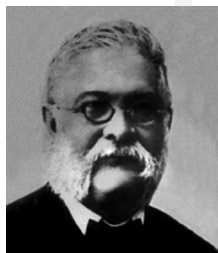
From 1846 to 1918 Kraków was a city in the Austro-Hungarian Empire, situated in the Kingdom of Galicia and Lodomeria. In 1861 Galicia got autonomy; the National Parliament and local government were formed in the capital city Lvov (Lemberg). The December Constitution of 1867 guaranteed the right to instruction in Polish at the universities (see: [5]). In the Kingdom of Galicia and Lodomeria Polish cultural, artistic and scientific life developed. There were four universities: the Jagiellonian University in Kraków, Lvov University, Lvov Polytechnic and for a time the University in Czernowitz (later Bukovina Duchy). For further studies see [14] and [19].

2. From 1860 to 1895

2.1. Mathematical staff in the period 1860–1895

In that period the staff of the Philosophical Faculty at the Jagiellonian University was small and there were only a few professors who published original mathematical papers: Jan Kanty Steczkowski¹, Franciszek Mertens and Marian Baraniecki. As a result, the number and scope of mathematical lectures were very limited. Michał Karliński lectured on classical calculus, calculus of variation, probability and mathematical geography, Franciszek Mertens on analytic geometry, trigonometry and algebraic equations), Marian Baraniecki on theory of determinants, algebraic equations, geometry and Number Theory), Ludwik Birkenmajer² gave lectures on the history of mathematics. For detailed studies see: [19] and [20].

2.2. *Calculus of Probability* by Michał Karliński



Franciszek Michał Karliński (1830–1906) was appointed to the Chair of astronomy and mathematics at the Jagiellonian University in 1862. Karliński worked primarily on observational astronomy, and – when the eye disease made celestial observations impossible – on meteorology. To Karliński's lecture duties belonged both astronomy lectures, as well as higher mathematics. After that lectures were separated between specialists. In addition, in 1874–1877 (untill the foundation of the Chair of Geography at the University).

Karliński taught mathematical geography and theory of geographical maps. Karliński's surviving notes show, among others, that in the nineteenth century Poland was familiar with world literature in probability. In his notes Karliński mentioned the works of such masters as Jacob Bernoulli (*Ars conjectandi* 1713), A. de Moivre, Nicholas, Daniel and Johann Bernoulli, J. d'Alembert, J. Lagrange, P.S. Laplace (*Théorie*

¹ Jan Kanty Steczkowski (1807–1872), a mathematician, professor of the Jagiellonian University, the author of a series of mathematical books for students published by Scientific Society of Kraków (Towarzystwo Naukowe Krakowskie: 1815–1872).

² Ludwik Antoni Birkenmajer (1855–1929), a historian of sciences, mathematician, physicist, astronomer and professor of the Jagiellonian University, a member of Polish Academy of Arts and Sciences.

analytique des probabilités 1812). In the preliminary remarks to the lecture Karliński wrote: “When, in studying the truth, gaps or breaks occur as a result of our ignorance so that it is impossible to find out what is in contradiction to the acknowledged truth, whereas we cannot prove the total compatibility of what we already know with what we want to know or investigate, then, instead of essential truth we have probability”.

Karliński classified probability as:

- *Philosophical* – “it occurs when we conclude the uniformity of rule from a multitude of cases”,
- *Aesthetic* – “in the fine arts, it consists in our ability to consider as true and real the thing which is presented by the artist, either as complete, or as happening in accordance to the assumptions made by him and to the fundamental conditions of art, or more succinctly, in comparison of what artist tells us with what experience has taught us”.
- *Mathematical* – “it is the relationship that exists between the number of cases favorable to some event, or, as it is commonly said, of chances, and the number of all probable cases, however under the assumption that all cases are equally possible”.

In Karliński’s lecture one can find thoughts about axiomatic definition of probability terms: “probability of the favorable effect is $v = \frac{a}{a+b}$, while the probability of unfavorable

effect is $v_1 = \frac{b}{a+b}$. Both these probabilities are fractions whose sum $v + v_1 = \frac{a+b}{a+b} = 1$.

Unity is the symbol of certainty ...” (a is the number of favorable effects, b – unfavorable).

Karliński gave a lot of historical information, solved problems about playing cards and dice, and about drawing lottery balls from an urn. His manuscript deserves attention because of the mathematical terminology. For each term Karliński gave a Polish, Latin, French and German name, and noted some historical facts, such as the name of an inventor, the time of introduction and the origin of the introduced term. The second part of the lecture was devoted to the applications of probability theory. “According to already presented rules for calculating the direct probability – writes Karliński – we are able to calculate in advance the benefits and losses connected with uncertain accidents of any kind”. He discussed issues concerning lottery, and devoted much attention to the mathematical concept of expectation.

2.3. *Analytic Geometry* by Franciszek Mertens



Franciszek Mertens (1840–1927) was born in Środa (Prussian Emporium, now Poland) and died in Vienna. Mertens studied at the University of Berlin where he attended lectures by Weierstrass, Kronecker and Kummer. In 1865 he obtained his doctorate with a dissertation *De functione potentiali duarum ellipsoidium homogenearum*; his advisors were Kummer and Kronecker. He worked first at the Jagiellonian University in Kraków from 1865 to 1884. He moved to the Polytechnic in Graz, and in 1894 he became an ordinary professor of mathematics at the University of Vienna.

Mertens worked on a number of different topics including potential theory, geometrical applications of determinants, algebra and analytic number theory. He published 126 papers

but is probably best known for the conjecture on the Möbius function (so-called Mertens' conjecture). Since a proof of it would have implied the truth of the Riemann hypothesis, many mathematicians attempted to prove Mertens' conjecture, but in 1985 Andrew Odlyzko and Herman te'Rielee disproved it.

In 1880's Franciszek Mertens lectured on the theory of determinants and projective geometry at the University, though the lecture was entitled *Analytic geometry*. Some copies of the lithographed notes from this lecture survived. One copy is a property of the Library of the Faculty of Mathematics and Computer Sciences of the Jagiellonian University (Bibl. Inst. Matem. U.J. sygn. 216, L. inw. 234). In 1880's this copy was a property of the Mathematical Seminar in Kraków. Another copy of the book is available at the Jagiellonian Library (see: [16]). The notes were taken by Leon Wątorski, a member of the Mathematical and Physic Society of Students' of the Jagiellonian University and the lithographed copies were produced by J. Pacanowski. The book has 462 pages

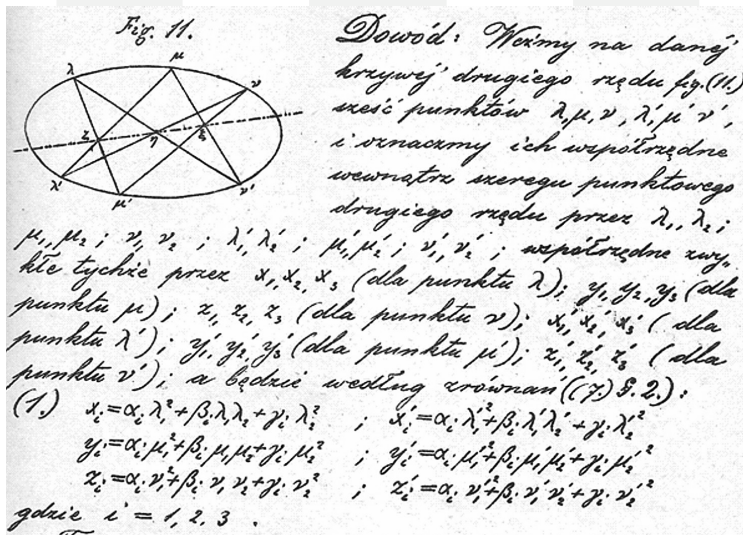


Fig. 1. Proof of Pascal Theorem by Mertens

and consists of 12 chapters. The notes start with foundations of the theory of determinants and its application to solving algebraic equations in many variables. Mertens gave a definition of determinant and its analytic (Laplace) form, basic applications (e.g. Cramer's Rule and the Kronecker-Capelli-(Rouché-Fontené-Frobenius) Rule). In the second part of the notes the theory of algebraic curves is discussed. Mertens introduced projective coordinates in both forms: Plücker's and Möbius's homogeneous coordinates³. He also

³ Homogeneous coordinates were introduced by August Ferdinand Möbius in his 1827 work *Der barycentrische Calcul*. Julius Plücker in 1832 suggested to assign six homogeneous coordinates, for the homogeneous coordinates of two points in projective 3-space, as determinants. A generalization of this methods is called Plücker embedding. Homogeneous coordinates can be used to locate the point of intersection of two algebraic curves. Władysław Zajaczkowski in early

gave fundamentals of the theory of quadratic forms and classification of curves of degree 2. Mertens presented the fundamental theorem of projective geometry and the theory ended with: Desargues theorem, Pascal hexagon theorem and Brianchon's theorem. All theorems were completely discussed and proved.

In 1880's Mertens introduced in Kraków the basics of Felix Klein's Erlangen Program. In 1872 Klein proposed to focus on projective geometry and group theory to produce completely new characterization of geometry. Klein in his consideration, emphasized projective geometry as a unified way of looking at various geometries and Mertens was following his idea, presenting the projective space as fundamental for any geometry. For the detailed studies on the perception of analytic geometry⁴ in Poland see [13] and for the presentation of the most comprehensive Polish lecture on projective space in XIX century see [8].

2.4. Marian Baraniecki and his lectures



Marian Alexander Baraniecki (1848–1895) attended a gymnasium in Warsaw and studied mathematics at the University in Warsaw (Szkola Główna Warszawska) since 1865. He graduated from the Imperial University of Warsaw⁵ in 1870 and he obtained a master's degree in mathematical and physical sciences. Then he studied at the Jagiellonian University in Kraków and in the Leipzig University, where in 1871 he obtained a PhD on the basis of the dissertation *Über die gegeneinander permutable Substitutionen*. Three years later, after further studies in St. Petersburg and Moscow, he obtained a master's degree in pure mathematics on the basis of a dissertation about hypergeometric functions (for a career, this degree meant more than PhD but less than habilitation). Baraniecki was a teacher in high schools in Warsaw and between 1876 and 1885 he was a reader at the Imperial University of Warsaw. In 1885, he was offered a job at the Jagiellonian University and at the Polytechnic School in Lvov. He chose Jagiellonian University, where he headed the Mertens' Chair. Baraniecki published several papers on algebra and the theory of functions in *Memoir of the Society of Sciences in Paris (Pamiętniki Towarzystwa Nauk Ścisłych w Paryżu)* and publications of Academy of Arts and Sciences. His book on theory of determinants, published in 1879 in Paris by the Kórnik Library, and the lithographed *Course of algebraic analysis (1879–1880)* and *The initial synthetic lecture of properties of conic*, were a basis for major Polish university-level courses on the theory of determinants. Baraniecki distinguished himself as one of the founders of the Mathematical and Physical Library – a publishing series founded in Warsaw in 1884 with support of Mianowski Fund. The series was directed at readers at various levels of education, up to university level.

1880's presented at the Lvov University in his lecture on analytic geometry a proof of Bézout's theorem on algebraic curves involving Plucker coordinates.

⁴ Unfortunately J. Dianni, the author of the paper on perception on analytic geometry in Poland, did not distinguish between Cartesian and projective methods in geometry. She discussed them jointly, to some disadvantage for the latter.

⁵ The Warsaw University (Szkola Główna Warszawska) was closed in 1869 by the tsarist authorities as an act of repression after January Uprising. In 1870 the Imperial Warsaw University was founded.

In Kraków Baraniecki taught algebra and geometry and gave courses in mathematics for naturalists. He also had classes in the lower seminar for mathematician. The scope of his lectures and classes was related with the theory of determinants, set theory, algebraic equations and numerical theory of functions of a complex variable and periodic functions analytic geometry and synthetic theory of surfaces and lines of double curvature, the study of invariants and coefficients of double and triple forms. He died at the age of 46 years. His scientific and teaching activity and efforts to elevate the mathematical culture are not adequately appreciated.

The successor of Baraniecki at the Chair was Stanisław Kępiński (1867–1908), who studied mathematics and physics at the Faculty of Philosophy of the Jagiellonian University in the years 1885–1889. In 1896 he was appointed as an extraordinary professor of mathematics at the Jagiellonian University, and in 1899 he became the Dean of the Faculty of Mathematics in Lvov Polytechnic School.

3. From 1895 to 1918

3.1. Staff and regular lectures

In 1895 a new era of mathematics at the Jagiellonian University began. Prominent Polish mathematicians started their academic career and brought to the university modern mathematics. Michał Karliński continued lecturing on *Calculus* and *Spherical trigonometry*. Kazimierz Żorawski, who started his career at the Jagiellonian University in 1895, lectured on: *Higher mathematics*, *Differential geometry*, *Analytic geometry*, *Elementary geometry*, *Curves and surfaces*. He had also a special class for students, called a higher seminar. Stanisław Zaremba, who arrived to Kraków in 1900, lectured on: *Higher algebra*, *Projective geometry*, *Calculus*, *Analytic function*, *Differential equations* and *Analytic geometry*, and held some seminars for students. Maurycy Pius Rudzki⁶ lectured on *Mechanics*, Cezary Russjan⁷ on *Ordinary differential equations*, *Partial differential equations*, *Calculus of variations* and *Projective geometry*. At the university there were some special lectures founded by a private foundation: Ludwik Birkenmajer's *History of mathematics*, Antoni Hoborski's⁸ *Calculus* and Alfred Rosenblatt's⁹ *Curves and Surfaces of Second Degree*. Also Jan Sleszyński, who had

⁶ Maurycy Pius Rudzki (1862 Uhryńkowce, now Ukraine – 1916 Kraków), an astronomer and geophysicist. He studied in Lvov, Vienna and Odessa. He obtained PhD at the Vienna University and Odessa University. A professor of the Jagiellonian University, member of Polish Arts and Sciences in Kraków.

⁷ Cezary Russjan (1867 Makieievo, now Ukraine – 1937 Charkov), a mathematician. He studied in Kiev and Odessa, where he graduated in mathematics. He also studied in Berlin, Paris and Leipzig. He was a docent of the Odessa University, professor of the Jagiellonian University, Lvov Polytechnic and Charkov University.

⁸ Antoni Hoborski (1879 Tarnów – 1941 Sachsenhausen), a professor of mathematics at the Mining Academy in Kraków and Jagiellonian University. Rector of the Mining Academy in Kraków.

⁹ Alfred Rosenblatt (1882 Kraków – 1946 Lima), a professor of mathematics at the Jagiellonian University and St. Marc University in Lima, a member of *Academia Nacional de Ciencias Exactas, Físicas y Naturales del Perú*.

many lectures: *Number theory*, *Analytic function*, *Probability*, *High algebra*, *Methodology of mathematics*, *Determinants*, was a beneficiary of the foundation (see [9]). For detailed study see: [19] and [20].

3.2. Special lectures

In 1900 Stanisław Zaremba joined the university and started his academic career presenting the modern methods of mathematics in a special lecture: *On the Dirichlet Boundary Condition and Related Problems*. The idea of the presentation of modern mathematics was continued and Kazimierz Żorawski, Jan Sleszyński, Alfred Rosenblatt and Antoni Hoborski contributed to this project. Many of these lectures were sponsored by a private foundation. Each professor presented two lectures: Kazimierz Żorawski – *Curves and Surfaces* (1909, 1910); *Kinematics*¹⁰ (1911), Stanisław Zaremba – a very important lecture *Principles of the Sets Theory* (1912) and *Theoretical Physics* (1915) and Jan Sleszyński – *Mathematical Logic* (1913) and *Probability* (1912). Alfred Rosenblatt, a private docent with no chair, presented a lecture entitled *Algebraic curves*¹¹ (1913).

In the Jagiellonian Library there is the legacy of Aleksander Birkenmajer¹². In this collection there are many handwritten notes of mathematical, physical and other lectures, and among them there are notebooks entitled: *Principles of Set Theory* by Stanisław Zaremba (see: [1]), *Number theory* by Jan Sleszyński (see: [2]), *Analytic Function* by Jan Sleszyński (see: [3]) and *Higher Seminar in mathematics* by Kazimierz Żorawski (see: [4]).

3.3. Weierstrass preparation theorem by Kazimierz Żorawski



Kazimierz Żorawski (1866–1953) was born in Szczurzyn near Ciechanów. After graduation from a classical gymnasium in Warsaw and four years of study at the Imperial University of Warsaw, Żorawski graduated in 1888 with a first degree in mathematics. He continued his study in Leipzig, where he was a student of Lie, and in Göttingen. Żorawski obtained a PhD in mathematics from the Leipzig University. In 1892 he moved to Galicia and was appointed as a lecturer at the Lvov Polytechnic. A year later he got habilitation at the Jagiellonian University. In 1895 he obtained the I Chair of mathematics at the Jagiellonian University. In 1917 Żorawski was a Rector of the Jagiellonian University. Two years later he moved to Warsaw, to work in the Ministry of Education, Warsaw Polytechnic and Warsaw University. He was a member of the Polish Academy of Arts and Sciences. Żorawski wrote more than 70 papers, mostly

¹⁰ This was a special lecture meant for broad audience.

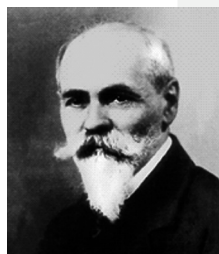
¹¹ In that period, Rosenblatt was interested in algebraic geometry. He published about 200-page monograph on algebraic surfaces in 1912 and 28 papers on algebraic geometry before 1929.

¹² Aleksander Ludwik Birkenmajer (1890–1976), a historian of exact sciences and philosophy, bibliologist, professor of the Jagiellonian University and Warsaw University, an expert in the field of research of Copernicus. A son of Ludwik Birkenmajer and a grandson of Franciszek Michał Karliński. He was a member of Polish Academy of Arts and Sciences in Kraków and of Royal Historical Society in London.

in real and complex analytic and differential geometry, differential equations, kinematics and the theory of continuous groups of transformations.

The Weierstrass preparation theorem states that an analytic function of several complex variables in an open neighborhood of a point factors into a monic polynomial in one (fixed) variable whose coefficients are analytic function of the remaining variables, and a function not vanishing in the neighborhood. This theorem, until mid-1930s, was a fundamental tool for investigating a function at singular points. About 1912, Żorawski presented the Weierstrass preparation theorem¹³ to the students of the Faculty in the classes of higher mathematical seminar. He also sketched a proof of the theorem and discussed the type of singularity of selected points and functions using the theorem.

3.4. Principles of Set theory by Zaremba



Stanisław Zaremba (1863–1942) was born in Romanówka (Austro-Hungarian Empire, now Ukraine) and died in Kraków. He attended a German gymnasium in St. Petersburg. In 1886 he graduated from the Institute of Technology in St. Petersburg. In 1887 he went to Paris, where he studied mathematics. He obtained a doctorate from Sorbonne in 1889 on the basis of the thesis *Sur un problème concernant l'état calorifique d'un corps solide homogène indéfini*, advised by Darboux and Picard. Zaremba stayed in France until 1900, making many contacts with mathematicians of the French school at that time;

publishing his results in French mathematical journals made him well known and highly respected by leading French mathematicians such as Poincaré and Hadamard. In 1900 Zaremba returned to Poland, where he was appointed to the Chair in the Jagiellonian University. Much of Zaremba's research was in partial differential equations and potential theory. He also made major contributions to mathematical physics and crystallography. He studied elliptic equations and he contributed mostly to the Dirichlet principle. Zaremba wrote more than 100 papers and 7 books¹⁴.

Principles of Set Theory (see: [1]). The lecture took place in the summer semester of the academic year 1910/1911, on Saturdays from 8 to 9 am (see: [18]). Let us recall that

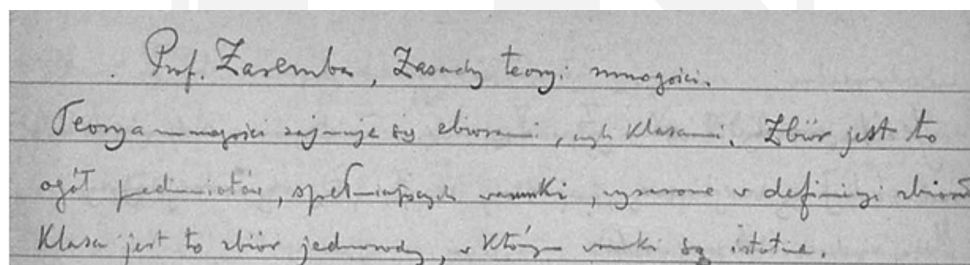


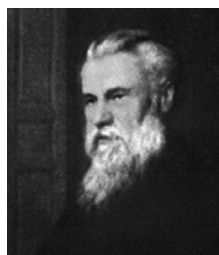
Fig. 2. An extract from the *Principles of the Sets Theory* by Zaremba

¹³ The notes of the student A. Birkenmajer entitled *Higher seminar with prof. Żorawski*.

¹⁴ There is no complete bibliography of Zaremba.

in the academic year 1909/1910 in Lvov Waclaw Sierpiński lectured, first time in Polish lands, on set theory (see: [14]). Unfortunately, no copy of notes from this lecture survived. However, fortunately, in the Jagiellonian Library there is the notebook entitled *Principles of Set Theory*. The notes were taken during Zaremba's lecture. Zaremba presented 'naive set theory', and in fact he gave a study of the structure of the real number line. Let us list some problems selected from the book: On the matter of the set theory, decimal representation of a real number and unique decimal representation, on the continuum nature of the interval $(0,1)$, on the continuum nature of any interval. Zaremba's original argument on equicardinality of the square and the interval is finally given.

3.5. *Analytic function and Number theory* by Jan Sleszyński



Jan Sleszyński (1854–1931) was born in Łysianka (Lisianka), now Ukraine. In 1871 he graduated from a secondary school in Odessa and studied mathematics at the Odessa University, from which he graduated in pure mathematics in 1875. He studied at the University of Berlin under Karl Weierstrass and obtained his doctorate in 1882. From 1883 to 1909 he was a professor of mathematics at the Odessa University. In 1911 Sleszyński was appointed to the III Chair of mathematics at the Jagiellonian University. Sleszyński worked at the Jagiellonian University until his retirement in 1924. In 1921 he became a member of Polish Academy of Arts and Sciences. Sleszyński was probably the very first to state: "The point of civilization is the exchange of ideas. And where is this exchange, if everybody writes and nobody reads?"

Sleszyński, recognized in Poland as a logician, was in fact a prolific mathematician. His main work was on continued fractions, least squares and axiomatic proof theory based on mathematical logic. Sleszyński is an author of the first rigorous proof of a restricted form of the Central Limit Theorem and the Sleszyński–Pringsheim theorem on convergence of certain continued fractions. In 1910 Kazimierz Żorawski, the president of the Foundation of Dr. Kretkowski¹⁵, was looking for a candidate for the III Chair of Mathematics. The candidate should be a well-known Polish-speaking professor of pure or applied mathematics. Sleszyński, who just retired from the Odessa University, accepted the offer. From 1911 to 1918 Sleszyński lectured at the University on *Number theory* (1911, 1915), *Analytic functions* (1911, 1913, 1915–1918), *Theory of determinants* (1913, 1916–1918), *Differential calculus* (1918), *Calculus of Probability*¹⁶ (1912, 1916) and *Mathematical logic or Methodology of mathematics* (1913, 1915, 1916, 1918).

The lecture *Analytic functions* (see: [3]) by Sleszyński was arranged in a modern style. Sleszyński suggested to students not only classical monograph by Jordan and Goursat but

¹⁵ Władysław Kretkowski (1840–1910), a mathematician and engineer, graduated from Sorbonne and Imperiale École des Ponts et Chaussées in Paris. Kretkowski obtained a PhD in mathematics from the Jagiellonian University in 1882. In his last will he bequeathed all his property for the development of mathematics in Kraków, especially suggesting endowment of the III Chair of Mathematics at the University.

¹⁶ It was classical probability.

also quite new Stolz's *Grundzüge der Differential und Integralrechnung*. The notes consist of 151 pages and of 25 separate pages for the table of contents. Pages from 2 to 25 are devoted to metrical (topological) properties of real line and complex plane, pages from 25 to 58 to the theory of real function, and 59 to 151 to analytic function of a one complex variable. Probably Sleszyński was the very first person who lectured on topology (sic!) at the Polish university. Starting with metrical properties of the real line he defined: an accumulation point of a set and a closed set. Next, he defined a set dense in itself, a perfect set and proved a theorem on such sets. He also discussed Weierstrass theorem on boundary classes, fundamental definitions and theorems on real line; supremum and infimum and the Bolzano theorem, convex set. etc. His treatment of the theory of function of real variable was classical, so we will move here to a discussion of analytic functions of one complex variable. Sleszyński first gave a foundation of a 'class theory' in complex plane. By a set or class of complex numbers he meant all complex numbers which satisfy a set of assumptions. In the theory of classes Sleszyński gave only the 'definition of inclusion' and did not discuss any problem of the (naïve) sets theory. He gave many definitions and theorems of classical topology, in particular he discussed limit point and accumulation point, the class (set) of all accumulation points of any given class. He introduced a (topological) classification¹⁷ of classes: closed (zamknięta), open (otwarta), everywhere dense (wszędzie gęsta), perfect

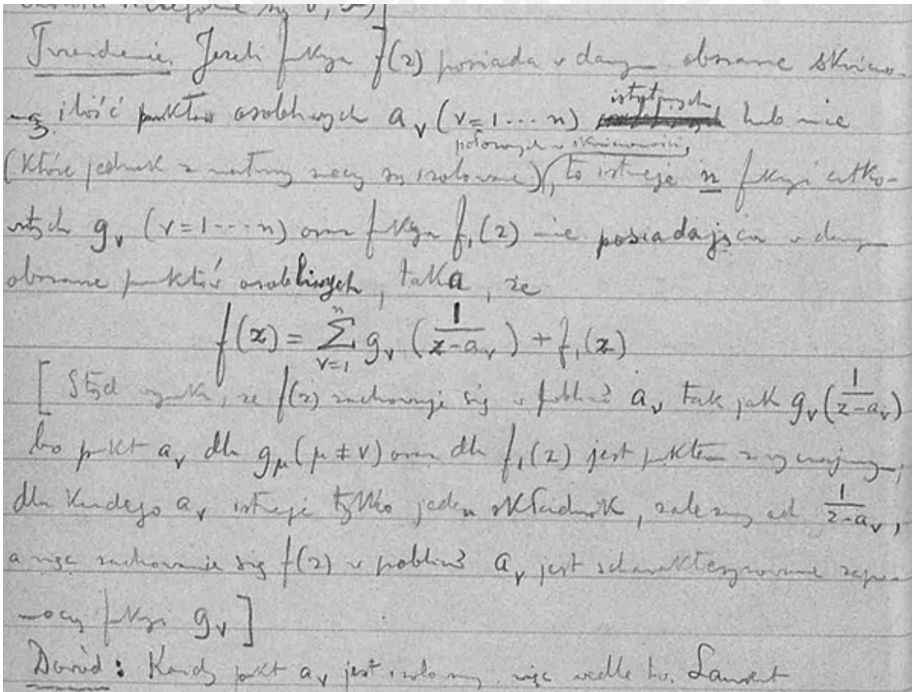


Fig. 3. An extract from the *Analytic functions* by Sleszyński

¹⁷ In parentheses there are Polish name proposed by Sleszyński.

(doskonała), connected (ciągła¹⁸), simply connected (związła¹⁹), convex (wypukła) and some fundamental properties of these classes, for example the fact that a convex set is simply connected.

After these metrical (topological) preparations Sleszyński introduced the theory of analytic function with a definition of derivative of a function at a point and Cauchy-Riemann equations (a necessary condition for differentiability). Next he defined the derivative of a function of complex parameter, a curvilinear integral, and concluded with Weierstrass mean value theorem for curvilinear integral. Next Sleszyński switched his consideration to series. He started with the classical definition of convergent and absolutely convergent series and Cauchy-Mertens theorem. Then he discussed differentiability, uniqueness of power series and Weierstrass theorem, power series, Cauchy-Hadamard theorem, differentiability and uniqueness and integrability of power series. He finished this part with Euler Formulae. The Cauchy integral formula and Taylor series are given. Next he introduced Liouville²⁰ series and defined isolated singularities, gave their classification and finally presented classical theorems on singularities and their application – Riemann principle, Casorati-Weierstrass²¹, Cauchy residue and the definition of the winding number. Fundamental facts on entire functions – Liouville theorem and Mittag-Leffler theorem – and uniqueness theorem

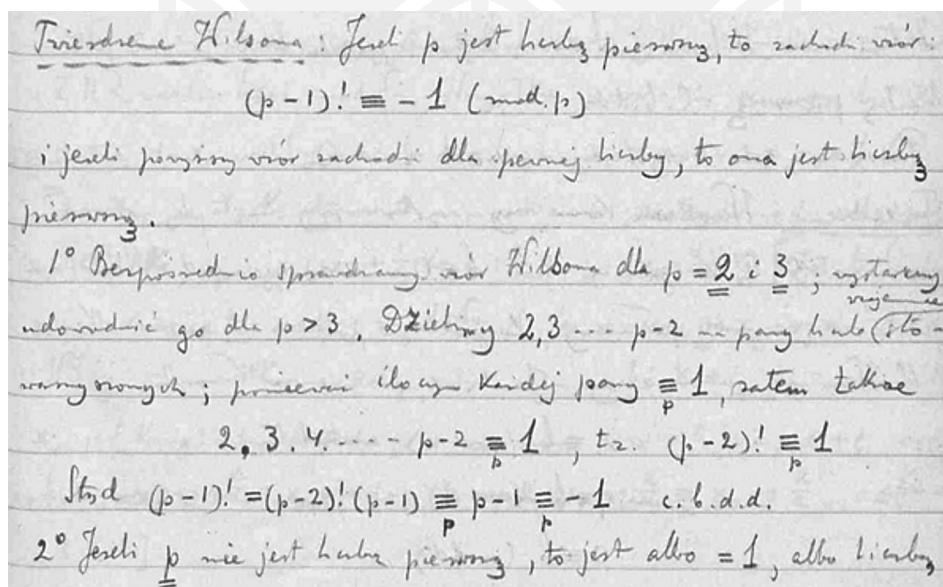


Fig. 4. An extract from the *Number theory* by Sleszyński

¹⁸ Now 'spójna'.

¹⁹ Now 'jednospójna'.

²⁰ Sleszyński meant Laurent series.

²¹ Sleszyński did not mention the name of Sochocki, who was first to prove this theorem. For a detailed study see e.g.: St. Domoradzki, *Julian Karol Sochocki (1842–1927)*, Opuscula Mathematica (1522), Kraków 1993, pp. 137-142.

completed the theory of analytic function. In the last part of the lecture Sleszyński made some remarks about multi-valued functions and Riemann surface.

Number theory (see: [2]) by Sleszyński. The notebook has 83 pages and there is no date of the lecture, but it was probably in the summer semester of the academic year 1912/13. There is no table of contents. Probably the notebook is not complete.

Sleszyński presented basic facts on the numbers theory: divisibility and Euclid's algorithm, definition of prime and composite numbers and the theorem on the infinite number of prime numbers. He gave some facts about divisibility of the integers: the divisibility rules, definitions of greatest common divisor (GCD) and least common multiple (LCM) for two or a finite number of integers and some fundamental theorems, Gauss algorithms for GCD and LCM of two integers. Next Sleszyński switched to the theory of congruences starting with definition of congruence relation for two numbers and moving to the Gaussian function²², Euler theorem for prime numbers and Fermat little theorem. He gave the explicit form of the Euler totient function. Then he considered polynomials with integer coefficients and problem of solving linear congruencies, and presented an algorithm for solving a reduced congruence. Sleszyński also discussed Diophantine equations and gave some classical facts: Wilson theorem, Chinese remainder theorem and general Euclid algorithm. He introduced Gauss' function and symbol, Legendre's symbol and Jacobi's symbol. Continued fractions, one of his main areas of research, are also broadly discussed. Finally, the theory of quadratic residues and Eisenstein theorem are presented.

This work was partially supported by the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge, University of Rzeszów.

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²² Sleszyński meant Euler totient function, for which Gauss gave an explicit presentation.

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