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EMERGENCE OF COOPERATION AS A RESTULT OF MUTATION AND INHERITANCE IN PD/PG-LIKE GAME

WSPÓŁPRACA JAKO REZULTAT MUTACJI I DZIEDZICZENIA W GRACH TYPU PD/PG

Abstract

The approach of evolutionary games like prisoner's dilemma, among others, is based on a consistent strategy. We propose an experimentally verified system where cooperation can arise out of two simple factors: mutation and inheritance. This system has a social dilemma property and allows each agent to set its own desired number of participants. There are two essential mechanisms: fluctuation through which mutation leads to cooperation, and natural selection which tends to promote cheaters and therefore disrupts the cooperation. It is shown in numerical simulations that the interplay between both mechanisms leads to an equilibrium and that no intentional strategies are necessary to establish and sustain cooperation. Thus, starting from a population of non-cooperating agents, natural evolution can end with a population composed of cooperating groups with the mean group size determined by the fluctuation rate and the pay-off function. A thorough analytical explanation of numerical results is provided.

Keywords: social dilemma, cooperation, prisoner's dilemma, evolutionary games, public goods game

Streszczenie

Podejście w grach ewolucyjnych, takich jak Dylemat więźnia, jest oparte przede wszystkim na spójnej strategii. W prezentowanym artykule zaproponowane zostało podejście, w którym współpraca dwóch więźniów może wynikać z mutacji oraz dziedziczenia. System ten ma charakter dylematu społecznego, w którym możliwe jest ustawienie przez każdego z agentów pożądanej dla niego liczby uczestników gry. Ponadto zostały zdefiniowane dwa podstawowe mechanizmy: wahania, które w wyniku mutacji prowadzi do współpracy, a także doboru naturalnego, który ma tendencję do promowania oszustów, w związku z czym zakłóca proces współpracy. W symulacjach numerycznych zostało wykazane, że wzajemne oddziaływanie między dwoma mechanizmami prowadzi do równowagi, a niecelowe strategie są konieczne do nawiązania i utrzymania współpracy między agentami. Tak więc, zaczynając od populacji nie współpracujących agentów, w wyniku procesu ewolucji, może skończyć się na populacji złożonej ze współpracujących grup ze średniej wielkości grupami określonymi przez stopień wahania oraz funkcję spłacającą. Artykuł dokładnie wyjaśnia wyniki numerycznych testów.

Słowa kluczowe: dylemat społeczny, współpraca, dylemat więźnia, gry ewolucyjne, gra dobra publicznego

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1. Introduction

Cooperation is fundamental to numerous biological and social systems. Multicellular organisms, social insects, herds of animals and human societies are all examples of systems dependent on cooperative interactions. Explaining the emergence of cooperation has been a challenge since the time of Darwin. Cooperators are defined as those who help others at a cost to themselves, while the term 'defectors' is used to describe those who receive the benefits of altruism without providing any help in return.

With regard to cooperation, the players are usually given the options of working alone, trusting another in cooperation, or cheating another who has trusted them The choice is usually done by a consistent strategy (an algorithm). Certain pieces of research demonstrate 'wars' of played by actors utilising different strategies. The aim of finding the best strategy, i.e. the one which defeats the others, is achieved through evolutionary selection.

Cheating requires individuals who are being cheated; therefore, not all actors can enter into a cheating strategy at any one time. Too many cheaters also undermine the fraction of cooperators that may lead to the extinction of the cooperators. An equilibrium is therefore desirable, where a specified fraction of cooperators is able to displace another fraction of cheaters.

In contrast to concepts where the cooperation is a result of consistent strategy exercised in a game of skill, our system shows that the cooperation can be a product of two simple factors: mutation and inheritance, both of which are widely observed in nature. For instance, micro-organisms are not expected to make rational decisions. Consequently, an important question immediately arises of whether or not it is possible to propose a social dilemma model capable of evolving towards cooperation when driven only by these two basic natural phenomena. Discovering such models is as important for our understanding of the emergence of cooperation, as models based on rational strategies. In the present paper, we propose a social dilemma game which extends the public goods game [1] and leads to cooperation supported by mutation and inheritance only.

In the public goods game [1], the players contribute to a common pool which is then enhanced by a linear function and distributed evenly among the contributors. Our system investigates a comparable approach, with the main difference being that the enhancement is provided by a power function. With specific restrictions over the exponent value, we are able to introduce social dilemma to this problem. Thus, by contributing, an agent exposes themselves to a potential risk of being cheated, over the prospect of receiving profitable return. The question regarding such an arrangement is how much the agents become willing to contribute.

Another major difference is that there is no common pool in our system. Each agent establishes their own pool with a number of agents from their neighbourhood. The outcome of participation is shared evenly. The problem of reciprocity arises – if an agent A has included B, will B return the favour? By the fact that the agent A is in neighbourhood of a few other agents, another agent C, as a replacement for B, may include A in its group, that will make A treated fairly. This may enable a society-wide indirect reciprocity where each agent is cheated by another but cheats another themselves. Whether such an arrangement would work and under which conditions is another question that is answered in this paper. A certain

biological analogy here is a multicellular organism where a single unit (a cell, an organ, a system) provides a service for other units whilst simultaneously receiving services of other units, not necessarily the same units to which they are providing a service.

2. Related Works

The emergence of cooperation has been studied based on tools developed within the framework of evolutionary games theory [2] to emphasise the problem of cooperation and defection for the evolution of social behaviour [3]. The prisoner's dilemma [4] game is a model game used to study aspects related to corruption [5], incorrect social information [6], shared rewards [7], multiple players [8], fraternity and egoism [9]. Similarly, the snowdrift game [10] has been used to study the problems of structured populations [11], random rematching [12], invasion and expansion of cooperators [13], and other problems. In the stag-hunt [14] game, individual fitness and past group decisions are taken into consideration [15]. The problems of shared rewards [7], sanctions as honest signals [16] and the optimal sizes of groups [4] have been studied within the framework of the volunteer's Dilemma game [17]. Future research should be directed into the area of automatic crawling systems [18], where cooperation can improve crawling performance. The cooperation modelling could be based on Petri net modelling [19] so that the formal and informal presentation could be clarified. The public goods game seems to be a basis for tender behaviour; therefore, results of such research could have a practical application [20, 21]. Additionally, research in the area of emotions and reactions [22] should be performed because they play an equally essential role as rational reasoning when it comes to judging the intelligence of a partner. Another important aspect of the research field is the species formation process [24] in agent--based algorithms, where the basis of this process is sexual selection and the persistent genetic diversity of the population [23].

3. Aim of the study

The aim of the study was to investigate the effect of the mutation and the process of succession to evolutionary games such as the prisoner's dilemma game. In the present method, a higher degree of freedom was introduced which allows the player to determine the number of participants in the game. In this system, the process of natural selection tends to favour cheats, while the second main mechanism, fluctuation, leads to cooperation as a result of mutation. As mentioned, natural selection favours cheats, this means that this process disrupts cooperation.

It is shown at a later stage of testing the system, that the interaction between these two mechanisms leads to balance, and the mutation is an integral factor in maintaining the already established cooperation between agents.

Consequently, even if the initial population consists of non-cooperating groups of agents, as a result of an evolutionary process, it may become a population that consists of the cooperating group of agents.

4. Methodology

In this section, we propose methodology which is experimentally verified in section 4. There are M agents in the system. During the game, each agent can establish cooperation with its neighbours. For simplicity of analytical derivation, we assume that:
1. the number of neighbours is fixed for all agents and equal to system parameter N
2. reciprocity: if A is a neighbour of B, then B is a neighbour of A

We assume that the game is played in ring topology and each agent has a neighbourhood of $\frac{N}{2}$ agents in both directions. Our numerical simulations show that populations defined over random networks behave identically to ring topology, on average, only if N is small enough to keep the above assumptions correct.

The lifetime of an agent is divided into the following steps:
1. Within the neighbourhood, establish a group with other agents
2. Receive a profit from participation in own and other groups
3. Produce an offspring if possible

The establishment of a group of size $0 \leq k \leq N$ is governed by the agent's genetic trait p and population-wide fluctuation rate q. The genetic trait $p = \frac{k}{N}$, with a willingness to cooperate, is established in the agent's direct predecessor as fraction of the size of its group. It should be expected that an agent bearing such a trait establishes a group of the same size as its predecessor; however, we want the fluctuation to interfere in this process. The decision to include a particular neighbour is made by a stochastic process accommodating both p and q in following formula:

$$B(p)xorB(q) \tag{1}$$

repeated N times for each one *agent* in the neighbourhood, where $B(x)$ stands for a single Bernoulli trial, returning *true* with the probability of x. The exclusive-or (*xor*) is the only logical operator that provides deserved symmetry – it allows fluctuation to affect the genetic trait both ways: if $B(p)$ produces a negative decision, it still can be changed with the probability of q; additionally, if $B(p)$ gives a positive decision, it may be revoked with the same probability.

Once the groups are established, the participants are awarded for their participation. For each agent, the total income is represented as $W_A + W_C$, where W_A is the share from a group the agent has established, and W_C is the sum of the shares from other groups where the agent was included. With the trait p, the agent has an influence over the part W_A but has no control over W_C as this depends on other agents and their p. The cooperation emerging from the game proposed in this paper regards this exact interdependence of the agents. The next section will discuss in detail the problem of fair or unfair cooperation.

The last step in an agent's lifetime is reproduction. Each agent presents its genetic trait and income. The genetic trait $p = \frac{k}{N}$ is different from the one that the agent receives from its predecessor due to a stochastic process and an interference from fluctuation q (1). A new

generation of *M* agents is created. For each new agent, its trait *p* is taken from an agent of the departing generation, chosen by the means of geometric probability where the income is the weight.

The games of dilemma are typically characterised by four parameters: income as a traitor *T*, income in fair cooperation R, income in no cooperation P and income when being cheated *S*. A simplified example of only two players A and B will be helpful: let the income function per player be $\frac{\sqrt{n}}{n}$, where *n* is the number of participants in the given group. If both A and B work alone, then each earns *P* = 1. If *A* makes *B* a participant, then both of them earn $\frac{\sqrt{2}}{2} < 1$, but *B* has the option to reciprocate. If so, then both will end the round with $R = \sqrt{2} = 1.41$. Otherwise, *B* cheats *A* by not returning the favour, then *B* earns $T = 1 + \frac{\sqrt{2}}{2} = 2.41$ and *A* earns only $S = \frac{\sqrt{2}}{2} = 0.71$. As shown, this example has the configuration of $T > R > P > S$, exactly like the prisoner's dilemma. It means that this problem is a social dilemma: by going into cooperation, A may improve its income at the risk of losing considerably if *B* cheats.

In the actual system, we generalise profit function as n^{α} for $0 < \alpha < 1$. In this interval of α, the property of social dilemma is retained. An agent X assuming a group of size *k* will receive its share:

$$W_A = \frac{k^{\alpha}}{k} \tag{2}$$

In order to receive fair return, the agent X must be included in other groups set by agents who have X in their neighbourhood, such that the sum of shares W_C would be equal to:

$$W_R = k^{\alpha} - \frac{k^{\alpha}}{k} \tag{3}$$

If $W_C < W_R$, then we can consider the agent X to be cheated, otherwise, X is a cheat. However, even if $W_C < W_R$, it can be such that $W_A + W_C > 1$; therefore, it is not guaranteed that always $P > S$. Certainly, $T > R > P$ for $k > 1$. It is important that in such a system, it is no longer an absolute statement if the cooperation is fair or not. It is relative to the wager, established by the agent setting a group of size *k*. This shows how different this system is compared to known games of dilemma. The iterated prisoner's dilemma investigates cooperation among *n* players, but participation is always in pairs. Stag-hunt and snowdrift are games that allow *n*-player participation, but the number is arbitrary. In the system presented herein, players independently choose the size of the participation pool.

The above considerations are correct for $0 < \alpha < 1$. In case of $\alpha \geq 1$, the participation is disproportionally profitable (the larger the group, the better the income); therefore, agents

trying to gain advantage by cheating are facing a different dilemma to the risk described above. For this reason, the problem in $\alpha \geq 1$ is not in the scope of this paper.

As shown, the outcome of the system is dependent on the values of p which are being carried over the generations. An agent with a lower p relative to the other agents should accumulate greater income than the others. This will result in greater offspring count bearing that low value of p, but cheating in the next generation will be tougher due to a lower average value of p. We expect this dependency to provide a point of equilibrium between cheaters and altruists.

5. Experiment

As already stated, the process of creating a group of participants out of the neighbourhood is stochastic and is governed by two variables p and q, where p is the agent's property inherited from the direct predecessor, and q is a population-wide variable. The result of this process (1) is a set of participants, whose count k will determine the agent's direct share W_A (2), the fair sum of indirect shares W_R (3) and ultimately, the new value of p dubbed $p' = \dfrac{k}{N}$ that will be passed to the next generation provided that the agent will be successful in reproduction. Because the value of p is key in the behaviour of this system, we will now try to estimate its value. First, we will calculate the expected value of formula (1), which we shall call a vertex:

$$vertex(p,q) = B(p)xorB(q) = p \cdot (1-q) + (1-p) \cdot q = p + q - 2pq \qquad (4)$$

The value of this function has two functions. Firstly, it is the expected value of probability that one particular agent will be added to the group of a given agent A. Secondly, as the formula (1) is executed N times by the agent A in a stochastically independent way, it determines p'. This is the case because the expected value of $k = vertex(p,q) \cdot N$ and $p' = \dfrac{k}{N}$; therefore, $p' = vertex(p,q)$.

However, the vertex function covers only part of the lifetime of agent. As mentioned before, the evolutionary process shapes the population of agents in a manner according to their income. This process we will call genetix: $(p_1 \dots p_M) = genetix(p_1 \dots p_M; \alpha)$ which brings us to the full model of the process:

$$(p_1 \dots p_M) = vertex(genetix(p_1 \dots p_M; \alpha), q) \qquad (5)$$

The above equation treats agents independently – this is only important if their $p - s$ differ due to the stochastic process. In the analytical approach, we will focus on the average value of p that characterises the entire population simultaneously. For the average value, we will reduce (5) to the scalar equation:

$$p = vertex(genetix(p, \alpha), q) \qquad (6)$$

We will start with an analysis of the vertex function:

$$p = vertex(p, q) = p + q - 2pq \tag{7}$$

This equation has a fixed point of $p = 0.05$. For $0 < q < 0.5$, the equation has monotonic convergence towards the fixed point. For $0.5 < q < 1$, the equation has oscillating convergence towards 0.5 which introduces a certain level of instability in the process (5). For this reason, this paper will not regard $q > 0.5$ and as q represents mutation, it is unlikely that any real-world evolutionary process would rely on such a high level of mutation.

In order to analyse how the genetix function affects the value of p with the influence of α, we must perform the following steps:
1. Produce the Poisson distribution of agents with their values of p.
2. Assign the expected value of income for each class of distribution.
3. Transform the distribution in a way that an evolutionary process would do.
4. Recalculate the new value of average p out of the transformed distribution.

The first step is achieved as following $P_{0...N}$ is a vector in which the k-th coordinate represents the probability that an agent has k cooperators.

$$P_k = \binom{N}{k} \cdot p^k \cdot (1-p)^{N-k} \tag{8}$$

In the second step, we will calculate the expected value of income weighted by frequency P_k. In $R_{0...N}$, the k-th coordinate represents the fraction of the whole income pool that is in possession of the k class. It is important to note that out of k, we are only able to calculate the agent's income part W_A. In this manner, it is impossible to calculate the W_C part, as it requires information not available in such plain partitioning. If the population is well-mixed, then all agents should receive a similar amount of W_C. In such cases, this analysis is correct. In the numerical data, it is shown that this accuracy depends on certain circumstances, giving us the ability to determine whether or not the population is well-mixed at a given time.

$$R_k = P_k \cdot \frac{(k+1)^\alpha}{k+1} \tag{9}$$

From the third step, we will receive the distribution as it should be reshaped by the evolutionary process. The R_k are frequencies weighted with respective incomes. $S_{0...N}$ normalises those values to 1, which means that the k-th component bears the probability that a new agent will receive the genetic trait $p = \dfrac{k}{N}$.

$$S_k = \frac{R_k}{\sum_{i=0}^{N} R_j} \tag{10}$$

In the last step, we will recalculate the new average p by weighting $\dfrac{k}{N}$ components of S.

$$p' = \sum_{k=0}^{N} \frac{S_k \cdot k}{N} \tag{11}$$

p' is value of the scalar version of the genetix for arguments of p and α. Fig. 1 presents Poisson distributions regarding the genetix function with $N = 10$. The initial plot, labelled $p = 0.5$, is derived from equation (8). Further plots, labelled with different values of α are derived from (10) after transformations with respective values of α. As it should be expected, the transformation causes the plots to lean left from the original plot, representing a tendency of going towards $k = 0$. For $0 < \alpha < 1$, the equation $p = genetix(p, \alpha)$ tends monotonically towards $p = 0.0$. The tendency becomes stronger with a lowering value of α – this means that the genetic selection is a process that disrupts cooperation.
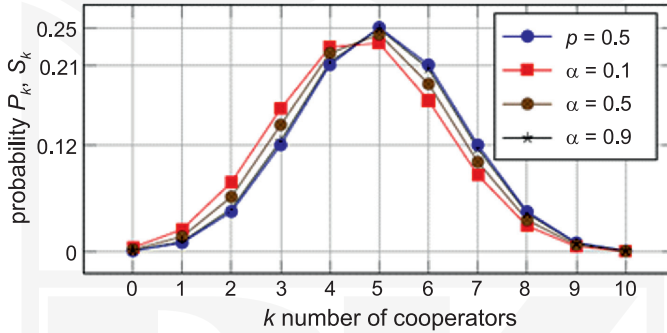
Fig. 1. Different values of α deform the initial distribution of $p = 0.5$

Keeping in mind equation (6) as the simplest model of our system, we learn that its behaviour is an equilibrium of two opposite factors: $vertex(p, q)$ tends to bring cooperation to the level of $p = 0.5$, while $genetix(p, \alpha)$ tends to disrupt cooperation down to $p = 0.0$. For a given q and α, Fig. 2 presents the average value of p, calculated from 200 iterations
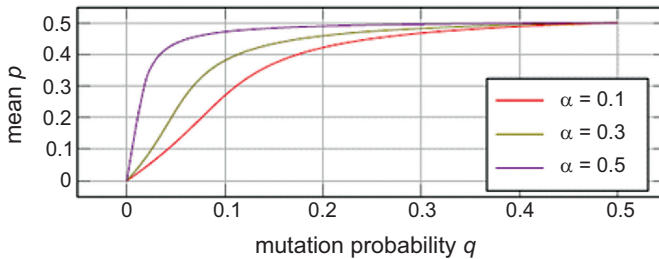

Fig. 2. Analytically calculated mean p for different values of q and α

of (6). Each iteration represents one generation in the system. The plots validate the previous analysis: the higher the q, the stronger p tends to 0.5, and the lower the α, the stronger p tends to 0.0.

We will now investigate the role of mutation in sustaining the cooperation. For fixed values of q and α, we run 200 iterations of (6) where in the first 100 iterations, q is set at a certain level and then changed to zero – this effectively reduces formula (1) to only $B(p)$. With $q = 0.0$, vertex function (7) becomes an identity function. By doing so, we aim to discover if the trait p will sustain the cooperation on its own. As should be expected from the behaviour of the genetix function, the answer is negative. Without fluctuation, there is no mutation and the willingness to cooperate disappears. Fig. 3 and Fig. 4 present the course of this analytical experiment for initial values $q = 0.25$ and $q = 0.05$ respectively. As for the dynamical properties, the higher the rate of decline, the lower is the value of α– this ensures that it is the property of genetic selection that disrupts the cooperation.
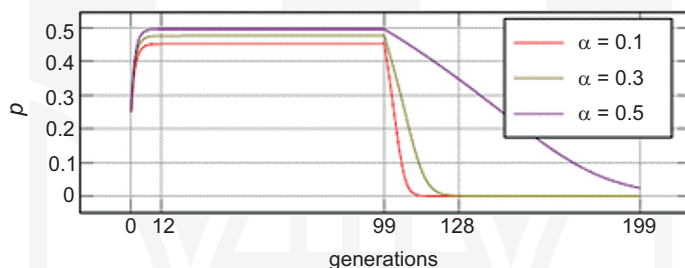

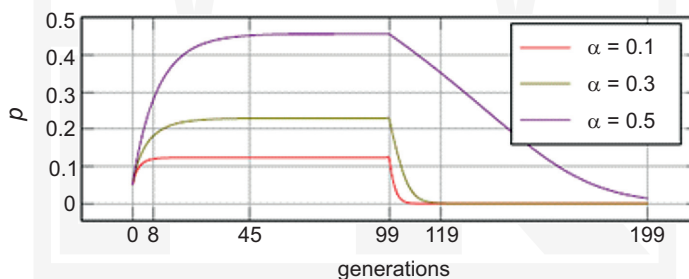Fig. 3. Analytical solution for $q = 0.25$ in 100 generations and $q = 0.0$

Fig. 4. Analytical solution for $q = 0.05$ in 100 generations and $q = 0.0$

6. Results

In this section, we are presenting the results from running the actual system, as described in section 3. It appears that in a steady state, the system behaves quite like is expected from theoretical considerations with vertex and genetix functions. However, transient states require further attention.

Fig. 5 presents an average value of p from a numerical experiment of 200 generations of $M = 1000$ agents each. The experiment was performed independently for each value of q

(step 0:01) and α (as in the legend), it is the numerical counterpart of theoretical plots from Fig. 2. The plots are nearly identical – this shows that the analytic approach is correct at this scope. The mathematical difference is that plots in Fig. 2 represent an average of 200 scalar values of p out of (6) while plots in Fig. 5 represent an average of a population of 1000 per generation then averaged for 200 generations.
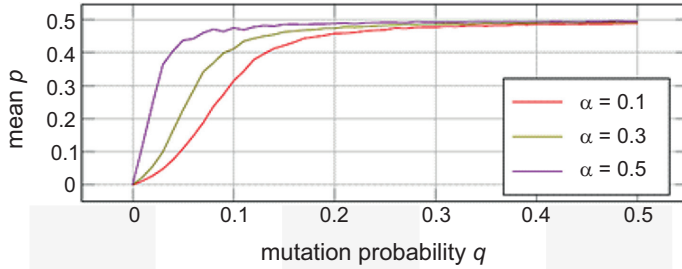

Fig. 5. Mean p as a function of q and α

With closer observation, Fig. 6 and Fig. 7 provide the average values of p for a specific q. The value of p presented in those figures is an average of population of $M = 1000$ in the given generation. These plots are numerical counterparts of plots from Fig. 3 and Fig. 4 respectively. It appears as though plots analytically follow the obtained curves very closely.
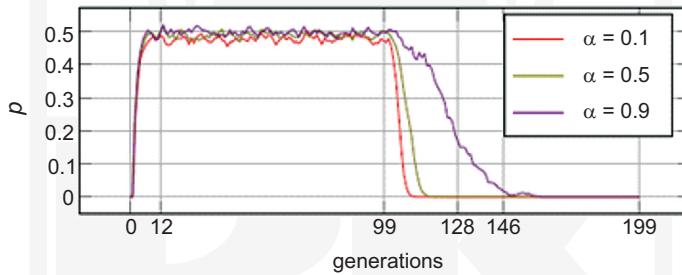

Fig. 6. The results of running the simulation with $q = 0.25$ for generations 0–99 and subsequently, $q = 0.00$
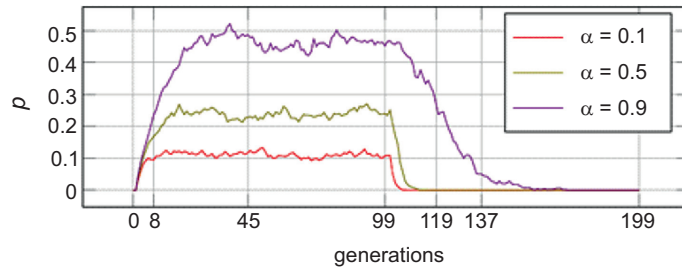

Fig. 7. The results of running the simulation with $q = 0.05$ for generations 0–99 and subsequently, $q = 0.00$

The correlation coefficient between them varies from 93.23% to 99.2% with a median of 98.47%.

6.1. Transient states

In steady states, the numerical results follow the theoretical plots very closely. Additionally, the tendency of change in transient states is correctly predicted, but the exact dynamics of change escapes the theoretical approach from equation (6). Fig. 8 juxtaposes selected corresponding plots from Fig. 3 and Fig. 6 in order to highlight the differences. The interesting part is to the right of generation no. 99, where q was set to 0.0. The plot for $\alpha = 0.1$ shows that for a few iterations, the system behaves as if nothing happened, but the cooperation then falls down so abruptly that it even overtakes theoretical prediction. For $\alpha = 0.9$, the changes also initially ignored but then, p starts to follow the theoretical prediction and falls down very quickly. That last part resembles the theoretical curve for $\alpha = 0.7$, what suggests that the effective α was lower than the nominal α.
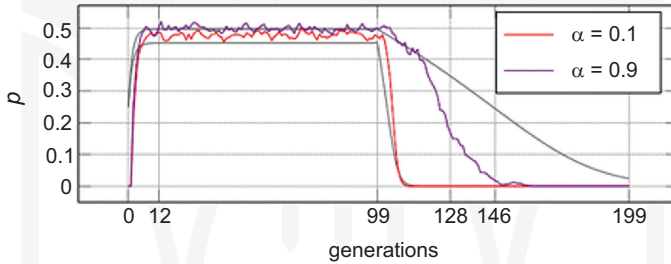

Fig. 8. Comparison of analytical and numerical solutions for $q = 0.25$ in 100 generations and subsequently, $q = 0.0$

We attribute this discrepancy to the fact that the true system simulated here has $M = 1000$ independent agents, each with its own property p, while the theoretical approach is based on a single value of p which mimics what we believe is an average value. The distribution of agents with different values of p is well-balanced, as assumed by theoretical analysis. However, in transient states, such a balance is apparently disturbed. Closer observation is appropriate – firstly, we will divide the population into four classes depending on their
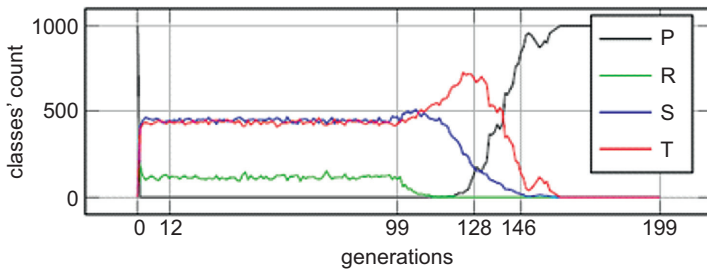

Fig. 9. Classes' count for $q = 0.25$ in 100 generations and subsequently, $q = 0.0$

placement in the process of cooperation. The classes are established in accordance with the nomenclature used in section 3 and named respectively. 'P' is the class of agents which do not establish cooperation nor are they included in other agents' groups; 'R' is the class of agents which receive fair returns to their wager; 'S' is the class of agents which receive less than fair returns, i.e. they the losers; 'T' is the class of agents which receive more than a fair return, i.e. they are the cheats.

Fig. 9 presents the count of classes for $M = 1000$ and initial $q = 0.25$ until the 100th generation. Before discussing the transient part, let us focus on the steady part. As seen, the fraction of fairly treated agents (R) is only about 15% of the total. The remainder is divided roughly in half between those which exploit and those which are exploited. This equilibrium complies with the assumption of a well-mixed population, what proves that the vertex-genetix analysis is correct is this state of the system.
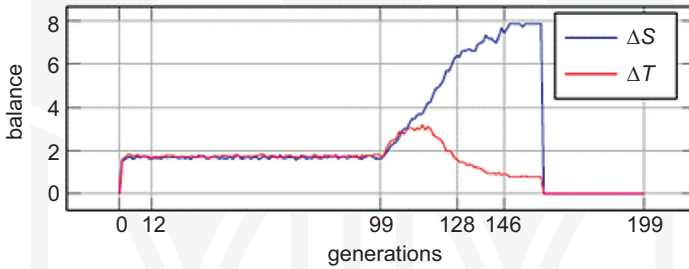

Fig. 10. Balance for $q = 0.25$ in 100 generations and subsequently, $q = 0.0$

In the transient state, where the cooperation is going to disappear, this equilibrium is destroyed, hence the population is no longer well mixed. The number of exploited agents lowers successively, while the number of cheaters grows until it reaches certain point. The intrinsic question is where the extra profit is coming from if the number of givers is diminishing. The answer is provided by Fig. 10. We introduce a variable balance: $\Delta = |W_C - W_R|$. For cheats, this means how much they gain over and above the fair value; for those who are exploited, this means how much they lose from a fair value. As seen in the figure, the balance for the S class grows up to extreme value where $W_C = 0.0$ – this means that each member of the S class cannot count on any reciprocation. The amount of profit generated by the S class is fully absorbed by the T class. Without any reciprocation, the cooperating agents are dying off. In other words, cooperation becomes a deadly genetic disease.

7. Conclusions

The system presented herein is a new evolutionary game that includes a social dilemma. It emphasises basic evolutionary phenomena: mutation and inheritance, over the strategies exercised in the prisoner's dilemma and other such games. It also provides another degree of freedom – players are able to establish their own number of participants. The question of fair and unfair cooperations has an answer relative to each player's wager, that is set as a number of its participants.

Statistical fluctuation introduces mutation into the inheritance. With the initial population of non-cooperating agents, mutation is able to introduce cooperation. As has been shown, mutation is important in keeping the established cooperation, being a counterforce to selection, that prefers cheaters, thereby is a disruptive force for cooperation.

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