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MODELLING OF HEAT CONDUCTION IN THE GROUND

MODELOWANIE PRZEWODZENIA CIEPŁA W GRUNCIE

Abstract

Simple cases of heat conduction in the ground are presented. Systems without interaction with the ground surface are considered. For heating or cooling of the ground by a flat slab, an analytical solution was used. Heat transfer between the ground and a single pipe and several parallel pipes is also considered. In these cases, a numerical solution was used. The analysed problems are of practical importance for modelling and simulation of ground heat exchangers cooperating with heat pumps.

Keywords: process modelling, transient heat conduction, renewable energy sources

Streszczenie

Przedstawiono proste przypadki przewodzenia ciepła w gruncie. Rozważono układy, w których nie występuje oddziaływanie z powierzchnią gruntu. Do ogrzewania lub chłodzenia gruntu przez dużą płaską płytę zastosowano rozwiązanie analityczne. Rozważono również przeniesienie ciepła pomiędzy pojedynczą rurą oraz kilkoma równoległe ułożonymi rurami a gruntem. W tych przypadkach zastosowano rozwiązania numeryczne. Analizowane problemy mają praktyczne znaczenie przy modelowaniu i symulacji gruntowych wymienników ciepła sprzężonych z pompami ciepła.

Słowa kluczowe: modelowanie procesów, niustalone przewodzenie ciepła, odnawialne źródła energii

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1. Introduction

Heat transfer from horizontal pipes located in the ground has been studied for many years because of widespread applications of such systems. They are used in industrial pipelines, district heating, buried power cables, and also in earth tube heat exchangers and ground source heat pumps [1, 2, 3]. Studies on ground thermal behaviour showed that the effect of geothermal heat transferred to the ground from the centre of the Earth on ground thermal behaviour can be ignored, when compared to the solar effect on the ground surface. Therefore, ground-coupled heat pumps use the ground as a solar collector during the heating season and as heat storage during the cooling season.

Heat transfer in the ground, with a heat exchanger installed in it, is a complex process; modelling and designing of the process requires many simplifying assumptions. All models are based on equations of transient heat conduction [4]. The differences lie in the adaptation of a one-, two- or three-dimensional model and the application of different boundary conditions and the initial condition. In some cases, model equations can be solved analytically.

In this work, some simple cases referring to heat conduction in the ground are presented. They are of practical importance for modelling and simulation of ground heat exchangers cooperating with heat pumps. Systems without interaction with the ground are considered. For one-dimensional systems, analytical solutions are known. For a two-dimensional system, a numerical procedure was used.

2. The solution for a semi-infinite body

Heating or cooling of the ground by a slab of large dimensions and a constant temperature is the simplest case. When a slab with a uniform and constant temperature T_0 is located in the ground at a temperature T_i , then the heat transfer occurs perpendicularly to the slab surface. Heat conduction is one-dimensional. In this simple case of transient heat conduction in a semi-infinite body, the temperature of the ground is a function of the distance from the slab surface x and time t . The process is described by the equation:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \quad (1)$$

The initial condition is as follows:

$$t = 0 \quad T = T_i \quad (2)$$

The first boundary condition refers to the surface of the slab:

$$x = 0 \quad T = T_0 \quad (3)$$

The second boundary condition refers to the place suitably distant from the slab surface, which ensures temperature constancy:

$$x \rightarrow \infty \quad T = T_i \quad (4)$$

For initially homogeneous temperature of the ground, the temperature dependence on the position and time is described by the relationship [5–8]:

$$\frac{T - T_0}{T_i - T_0} = \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right) \quad (5)$$

Short process times, i.e. these for which there is no interaction with the ground surface, have been considered. For calculations of ground temperature changes, the data shown in Table 1 were used.

Table 1
Calculations data

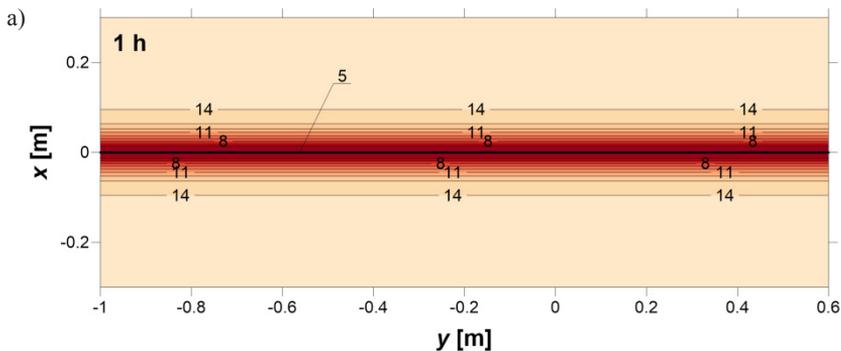
Quantity	Value
Initial temperature of the ground T_i	15 [°C]
Slab temperature T_0	5 [°C]
Ground thermal conductivity k	0.9 [W/(mK)]
Ground density ρ	2600 [kg/m ³]
Ground heat capacity c	900 [J/(kg·K)]

It can be concluded from Table 1 that the slab causes the ground to cool down. The heat transfer occurs symmetrically through the axis of the slab.

The coefficient of thermal diffusivity of the ground is $a = k/(c\rho) = 0.384 \cdot 10^{-6} \text{ m}^2/\text{s}$, which follows from the values presented in Table 1.

The calculation results are shown in Figs. 1a, b and c. Lines of constant temperature (contour lines) for different process times are presented.

The longer the process time, the greater the thermal effect of the slab installed in the ground. For example, after 1 h, an approx. 0.1 m thick ground layer (measured unilaterally) cools down to the temperature of 14°C or less. After 3 h, the thickness of this layer increases to approx. 0.15 m, and after 5 h – to 0.2 m.



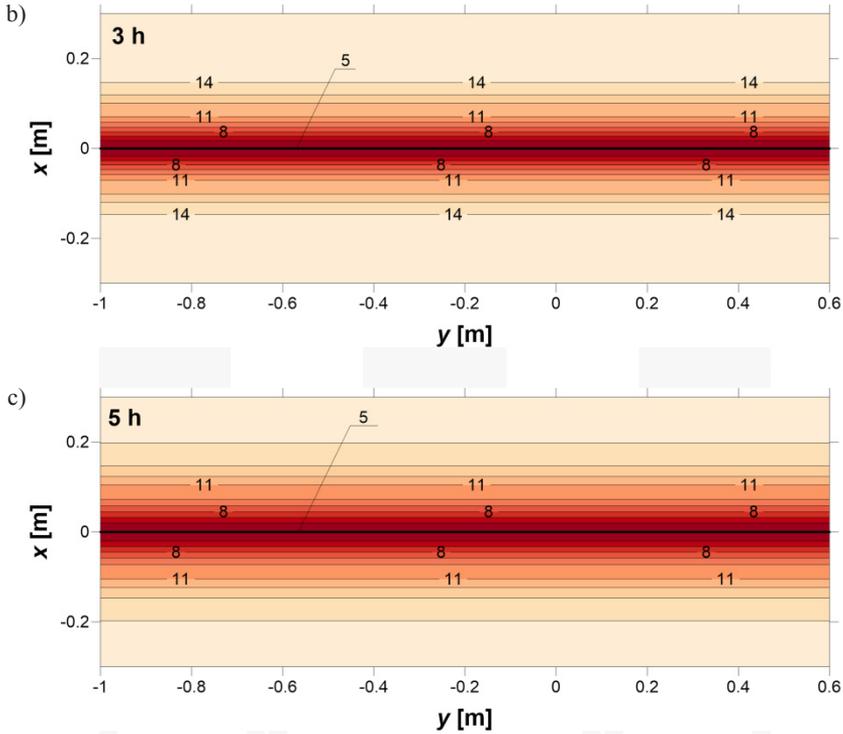


Fig. 1a, b, c. Cooling down of the ground by a slab at a constant temperature

3. The solution for an infinite plate

When the time of heating or cooling of a semi-infinite body (ground) by a flat slab is relatively short, the depth of heat penetration in the ground is limited, and for an analysis of the problem presented in the previous chapter, the ground can be treated as a plate of a finite thickness. The differences between the interpretation of position variables x and z , in both cases, are presented in Fig. 2. For an infinite plate, the following equation is valid:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial z^2} \quad (6)$$

The initial condition (2) and the following boundary conditions were used:

$$z = s \quad T = T_0 \quad (7)$$

$$z = 0 \quad \frac{\partial T}{\partial z} = 0 \quad (8)$$

where s is half of the plate thickness. The solution has the form [5–8]:

$$\frac{T - T_0}{T_i - T_0} = \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{2i-1} \exp \left[-(2i-1)^2 \left(\frac{\pi}{2} \right)^2 \frac{at}{s^2} \right] \cdot \cos \left(\pi \frac{z}{s} \frac{2i-1}{2} \right) \quad (9)$$

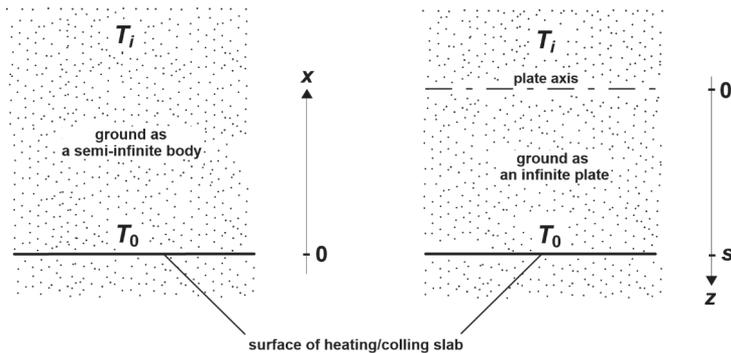


Fig. 2. Symbols for a semi-infinite body and an infinite plate

In Fig. 3, the comparison of temperature profiles obtained from Eqs. (5) and (9) for different process times is presented. In calculations, it was assumed that $s = 0.2$ m. When interpreting the graph, one must take into account that the symbols for the position coordinate are different for a semi-infinite body (x) and an infinite plate (z). For conditions under which the simulation was conducted, after 1 hour, there are no differences in heat transfer in the semi-infinite body and the plate. After 3 hours, some differences in the profiles occur. After 5 hours, these differences are large, especially near the axis of the plate. The differences in temperature profiles for the semi-infinite body and the plate are not important when the temperature front is close to the surface of the plate, i.e. when temperature changes have not reached (yet) the centre of the plate.

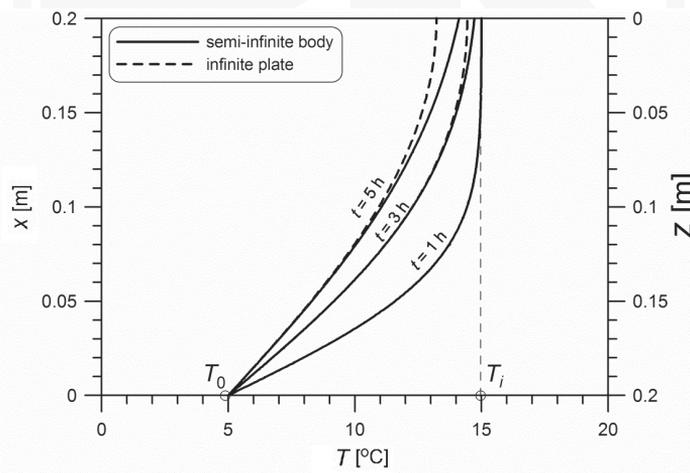


Fig. 3. Comparison of temperature profiles for a semi-infinite body and an infinite plate

4. The solution for an infinite cylinder

A horizontal ground heat exchanger is comprised of a system of pipes with a working liquid flowing inside. When straight pipes are used, they are arranged at some distances from each other. It was assumed that, at the beginning, the ground temperature is uniform. At the beginning of ground cooling (heating), the process can be considered separately for each pipe. The assumption that the temperature of the external surface of a pipe does not change with the time and position was taken. In this case, the heat conduction problem can be treated as one-dimensional. Conduction occurs in a layer of the ground, whose cross-section is shown in Fig. 4.

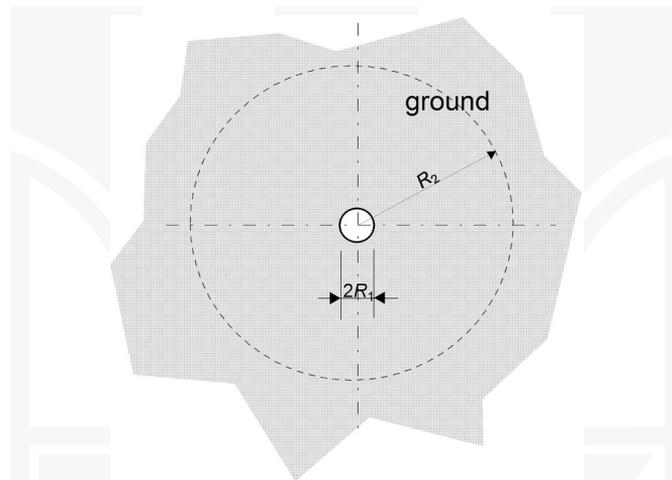


Fig. 4. Heat transfer between the ground and pipe surface

R_1 is the radius of the pipe, which is surrounded by the ground. R_2 is any radius ($R_2 > R_1$) for which it can be assumed that temperature does not change. Therefore, the longer the process time, the larger R_2 must be taken for calculations.

The equation of heat conduction in an infinite cylinder has the form:

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (10)$$

Relationship (2) was taken as the initial condition, while the boundary conditions have the form:

$$r = R_1 \quad T = T_0 \quad (11)$$

$$r = R_2 \quad T = T_i \quad (12)$$

where r is a radial position coordinate ($R_1 < r < R_2$).

In Fig. 5, temperature isolines in the ground around the pipe after 1, 3 and 5 hours of the process are presented. Data from Table 1 – pipe radius $R_1 = 0.019$ m and $R_2 = 0.5$ m – were taken for calculations. The problem was solved numerically with the use of the Crank-Nicolson scheme.

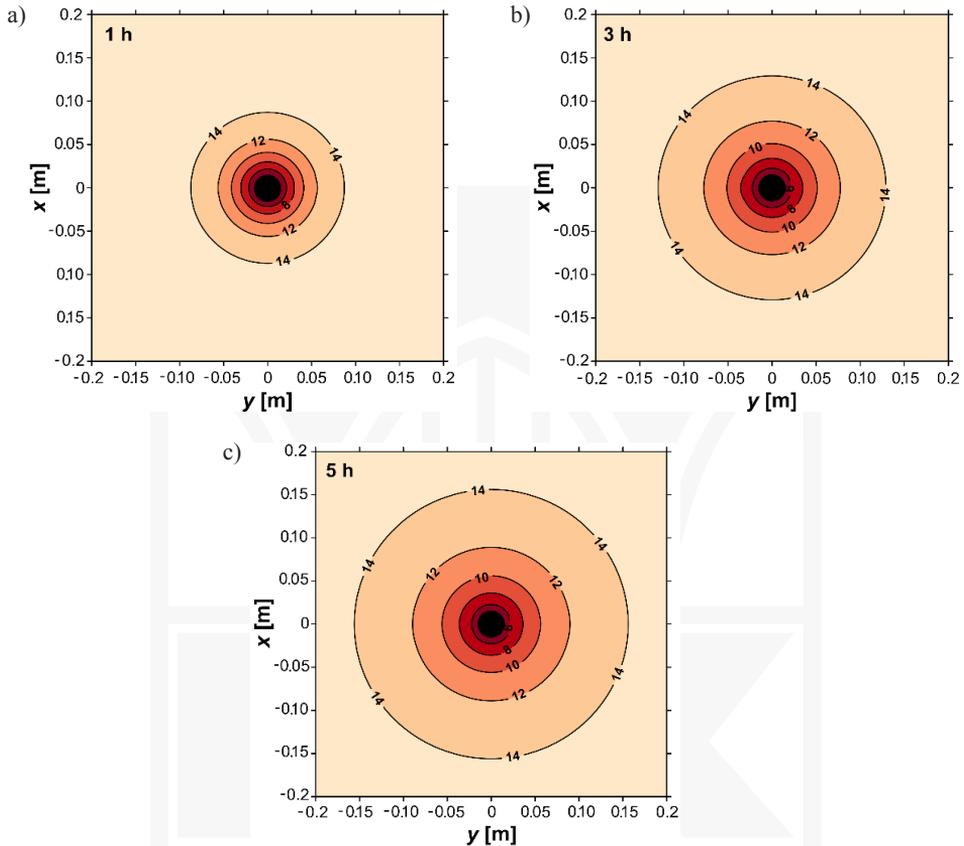


Fig. 5. Maps of temperature isolines in the ground around the pipe with constant surface temperature

As time passes, the ground temperatures decrease because of the relationship $T_0 < T_r$. The lowest temperatures are near the pipe surface (pipe cross-section was marked black). After 5 hours, the temperature front does not reach the radius R_2 , so it was right to choose $R_2 = 0.5$ m for calculations.

5. 2D simulation – application of the Peaceman-Rachford procedure

A system with several parallel pipes of a ground heat exchanger is considered below. Except for the initial period, the description of heat conduction requires considering a two-dimensional system. The equation of heat conduction in a two-dimensional system has the form:

$$\frac{\partial T}{\partial t} = a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (13)$$

In the applied numerical procedure [9], the solution is searched for in a square of a side b (Fig. 6).

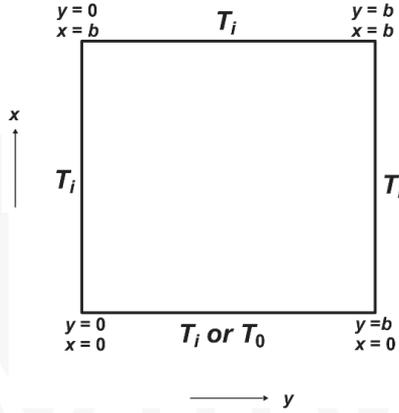


Fig. 6. Interpretation of boundary conditions

The solution has the form of a function of three variables: $T = T(x, y, t)$. Values of the temperature at the sides of the square are determined in the boundary conditions. These conditions are as follows:

$$T(x, 0, t) = T_i \quad (14a)$$

$$T(x, b, t) = T_i \quad (14b)$$

$$T(0, y, t) = f(y) \quad (14c)$$

$$T(b, y, t) = T_i \quad (14d)$$

wherein the following equalities referring to temperatures at vertices of the square:

$$f(0) = T(0, 0, t) \quad (15a)$$

$$f(b) = T(0, b, t) \quad (15b)$$

The aim of the calculations is to determine the temporal and spatial distribution of temperatures of the ground at an initial temperature T_p , which is cooled down by 4 pipes at a 300 mm distance from each other (Fig. 7a). Since the formulation of the boundary conditions for curved surfaces is impossible in the numerical procedure used in this chapter, the real system was substituted by a 'linear' system according to the scheme in Fig. 7b.

The shaded symbols refer to pipe surfaces; in these places, it was assumed, in the boundary conditions, that $T = T_0$, and in the others $T = T_i$. Data for calculations are shown in Table 1.

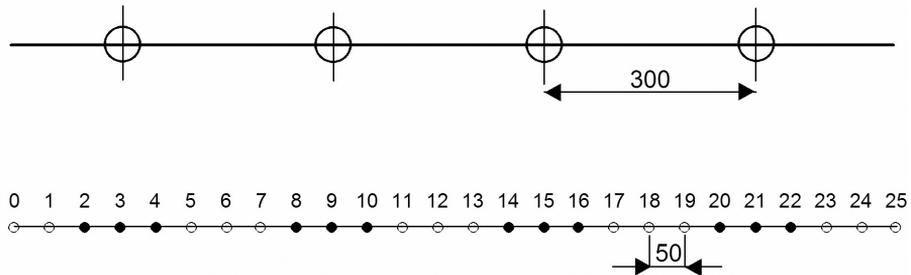
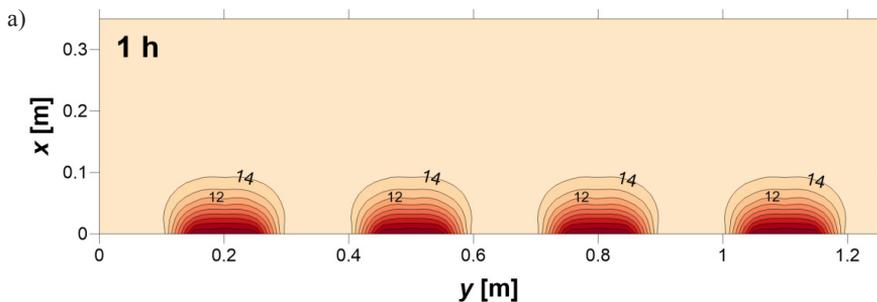


Fig. 7. Explanation of the form of the boundary condition (14c) and function (16)

Function $f(y)$ in the boundary condition (14c) has the form (j – node number):

$$f(y) = \begin{cases} T_i & \text{for } j = 0, 1, 5, 6, 7, 11, 12, 13, 17, 18, 19, 23, 24, 25 \\ T_0 & \text{for } j = 2, 3, 4, 8, 9, 10, 14, 15, 16, 20, 21, 22 \end{cases} \quad (16)$$

In order to solve the problem, the Peaceman-Rachford alternating direction implicit (ADI) method was used. The ADI methods are two step methods. In the first step, a tridiagonal system of equation along the lines parallel to the x axis is solved, while in the second step, an analogical system of equations has to be solved, but along the lines parallel to the y axis. 25×25 nodes were used in numerical calculations. The results of calculations are presented in Figs. 8a, b, c.



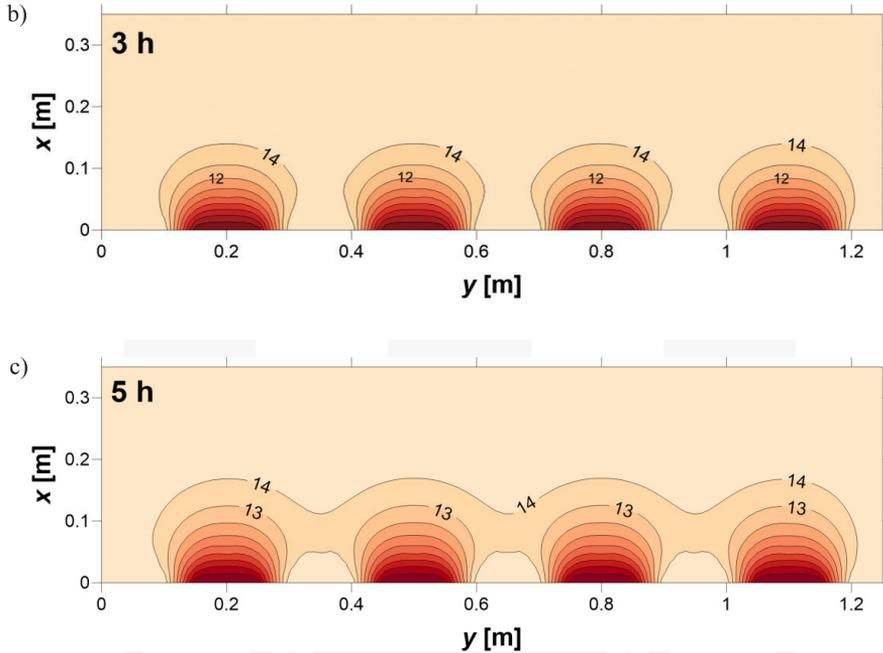


Fig. 8. Results of 2D simulation with the use of the Peaceman-Rachford procedure

The figures refer to the cross-section of the system of pipes. Contour lines for the ground are presented; temperatures are expressed by numbers (in °C). Position coordinates in the ground are on the axes. The pipes are far enough from the ground surface to ensure that there is no interaction with the surroundings. As can be seen, at the beginning, each pipe cools the ground individually, independently of the other pipes. However, after some time, temperature fronts connect with each other and an increasing part of the constant temperature line is more and more rectilinear and parallel to the y axis. Hence, the system behaves similarly as during cooling of the ground by an infinite plate. This similarity becomes greater, the greater the number of pipes and the denser they are arranged.

6. Conclusions

- The described models represent simple systems for heating and cooling of the ground. Under real conditions, there are systems that differ in, for example:
 - a) Interactions with the ground surface.
 - b) Periodic work of the heat exchanger.
- For short times of heating/cooling, the ground can be treated as a finite thickness plate (instead of a semi-infinite body). However, for heating/cooling of the ground by a system of parallel pipes, solutions for a single pipe can be used.
- It was noticed for simulation of heat conduction for a system of parallel pipes of a ground heat exchanger that, after some time, the temperature isolines are similar to straight lines.

This gives a basis for using one-dimensional equations of heat conduction for the modeling of horizontal ground heat exchangers.

- The presented calculations can be used as a special case of heat transfer in the ground, which is useful for testing complex models that refer to transient heat conduction in ground heat exchangers.

References

- [1] Al-Khoury R., *Computational Modeling of Shallow Geothermal Systems*, CRC Press, 2012.
- [2] Rubik M., *Pompy ciepła w systemach geotermii niskotemperaturowej*, Multico Oficyna Wydawnicza, Warszawa 2011.
- [3] Sedova M., Adamovsky R., Neuberger P., *Analysis of ground massif temperatures with horizontal heat exchanger*, Research in Agricultural Engineering, Vol. 59(3), 2013, 91–97.
- [4] Kupiec K., Larwa B., Gwadera M., *Heat transfer in horizontal ground heat exchangers*, Applied Thermal Engineering, Vol. 75, 2015, 270–276.
- [5] Carslaw H.S., Jaeger J.C., *Conduction of heat in solids*, second ed., Clarendon Press, Oxford 1959.
- [6] Crank J., *The Mathematics of diffusion*, Oxford University Press, 1956.
- [7] Cengel Y., Ghajar A., *Heat and mass transfer: fundamentals and applications*, McGraw-Hill, 2010.
- [8] Staniszewski B., *Wymiana ciepła – podstawy teoretyczne*, WNT, Warszawa 1979.
- [9] Marciniak A., Gregulec D., Kaczmarek J., *Basic numerical procedures in Turbo Pascal for your PC*, Nakom, Poznań 1992.