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SHAPING CURVED STEEL ROD STRUCTURES

KSZTAŁTOWANIE KRZYWOLINIOWYCH, STALOWYCH KONSTRUKCJI PRĘTOWYCH

Abstract

The paper addresses the problem of the efficient shaping of curved steel rod structures conducted based on the Enneper surface. The proposed parametric design process consists in linking the geometric shaping of grid models with their structural analysis and optimization, which is realized through the application of design tools working in the Rhinoceros 3D software. The mechanical performance of lattices covered with glass panels is evaluated and the structures are pre-dimensioned. The presented aesthetic grid shells with good structural characteristics may constitute original coverings. However, the analysis, which targets an early stage of the design, aims at providing design guidelines to facilitate communication between architects and civil engineers.

Keywords: parametric modeling, structural analysis, rod structures, Rhinoceros 3D, Grasshopper, Karamba 3D

Streszczenie

W artykule podejmuje się problem efektywnego kształtowania krzywoliniowych, stalowych konstrukcji prętowych utworzonych na bazie powierzchni Ennepera. Zaproponowany parametryczny proces kształtowania polega na powiązaniu geometrycznego kształtowania siatkowych modeli z ich analizą strukturalną i optymalizacją, realizowanymi za pomocą narzędzi projektowych pracujących w środowisku Rhinoceros 3D. Oceniono mechaniczne właściwości kratownic pokrytych taflami szklanymi, a struktury wstępnie zwymiarowano. Zaprezentowane powłoki kratowe o dobrych właściwościach strukturalnych mogą stanowić oryginalne przekrycia, natomiast analiza strukturalna ukierunkowana na wczesny etap projektowania ma na celu dostarczenie wytycznych projektowych ułatwiających komunikację między architektami i inżynierami.

Słowa kluczowe: parametryczne mdelowanie, analiza strukturalna, struktury prętowe, Rhinoceros 3D, Grasshopper, Karamba 3D

1. Introduction

Architectural and civil engineering design during the last decade was inspired not only by various possibilities offered by digital technology, but also by other disciplines, such as mathematics and physics. The so-called "parametric design thinking" has been introduced, which stands for the ability to understand and construct complex and parametrized operations, which make a designed form respond and evolve [1–4].

Along this line of thought, the paper discusses a novel parametric approach to the conceptual design of steel grid structures. Grid shells are usually long span structures composed of a lattice of a single layer or multilayer members, which form a curved surface. The application of grid shell structures increased in the past decade, as they are an effective means of covering space. Grid structures can be used in various vertical and overhead applications, as well as in order to form complete building enclosures. They have a great potential to be applied both in readapting existing spaces and creating new aesthetically interesting and pleasing structures. Grid shell structures can be made of a wide range of materials, from wood to steel. The most popular are layered geodesic domes, which are shaped using the procedures of sphere division into triangles elaborated by Fuller [5]. However, the parametric study in this paper is conducted on the example of spatial lattice created basing on an Enneper surface. Recently, optimization in the parametric environment has been a subject of interest to many researchers [6,7]. Most contributions in this field are concerned with the optimization of geometric properties of meshes approximating free-form structures. Many works address the problem of the planarity of grid faces. However, we have not found research works considering the Enneper surface as a base for grid structure modeling. Our approach to grid shell design is to link geometric shaping of grid spatial forms with their structural analysis and optimization. A similar approach was applied in the case of designing roof shells composed of repetitive concrete modules of Catalan surfaces presented in [8]. The goal of our research is to elaborate universal scripts in order to create digital grid structure models of various shapes and various grid patterns, as well as to develop a methodology to calculate the change of the load-bearing capacity of grid structures depending on their curvature, the number of supports and topology. The geometric shaping of grid shells embraces the following phrases: parametric description of the models, assigning values to parameters in accordance with given requirements and constraints, generating alternatives as well as evaluating them.

2. Materials and methods

2.1. Characteristics of the applied software

Digital architecture has profoundly changed the processes of design and construction. Thanks to the application of new digital design tools, new forms can be examined much more comprehensively. The design approach to parametric modeling of grid structures proposed in this research is realized by the application of versatile tools: Grasshopper, and Karamba

3D, working in the same modeling software- Rhinoceros 3D. Grasshopper is generally used to algorithmically generate the desired framework and form the model, modify it, visualize and finally analyze it. However, the structural analysis is carried out using Karamba 3D. This plugin makes it extremely simple to combine parameterized complex geometric forms, load calculations and finite element analysis. Rhino/Grasshopper gives plenty of flexibility for design exploration. Due to this fact, it is very convenient during the initial design process. Grasshopper is one of the most commonly used generative design editors. It allows designers to build both simple and complicated complex forms. Moreover, during the generation of models, their geometric shapes are presented, whereas the mathematical description is hidden. Parametric design refers to the use of parameters, which are various constraints and dependencies. However, they are substantially responsible for determining the relationship between design intentions and design possibilities.

2.2. Geometric properties of the Enneper surface

Analytical definition of the geometric characteristics of the designed forms is becoming of more and more increasing interest and importance today. It concerns both the visual representation focused on the aesthetic appearance of structures and the analytical representation relating to the analysis of their geometric properties. The Enneper surface is a minimal and self-intersecting surface [9, 10]. However, it can take different shapes depending on the choice of parameters that define it. It also displays intrinsic curvature, so it should be resistant to certain deformations. Due to this fact, it is worth considering as a base surface for grid structures.

Enneper surfaces as three-dimensional objects in three-dimensional space can be described mathematically by simple equations with three space variables (x, y, z). However, for the need of the creation of Grasshopper's algorithms, the surfaces should be described by two parameters (u, v), which determine their shapes. The Grasshopper's toolbox provides components, which allowed us to analyze and evaluate various Ennaper surface shapes. As the base surface for the generation of spatial lattices, we have chosen a regular, non-intersecting shape of the Enneper's surface, of which the horizontal projection is a circle (Fig. 1).

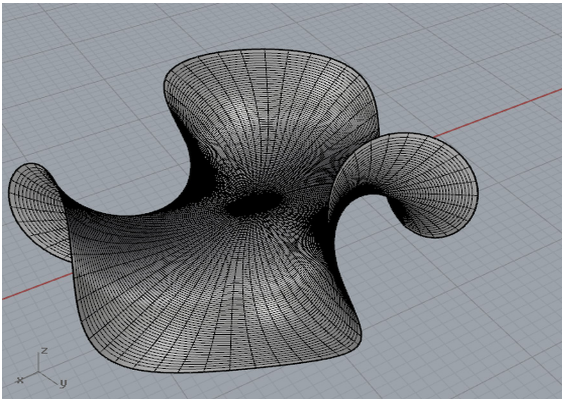


Fig. 1. The Enneper surface's shape used for formation of lattices (by J. Dźwierzyńska)

A lot of works analyze the geometric properties of Enneper surfaces, but little is known about the application of these surfaces in the construction industry. However, tensioned fabric structures in the form of an Enneper surface are studied in [11].

2.3. Grid shell structures

The term "grid shell" is used in the paper alternatively with the term "lattice shell". It is defined as a structure composed of a network of straight members, creating a single layer, which forms a shell surface. It is a type of structural discrete system in which the axes of bars form a specific spatial geometric grid [12]. Covering panels, if applied, act only as a load. There are both aesthetical and structural benefits of the application of grid shells. They not only create beautiful spaces by segmenting the existing space by their discretized topology, but also exhibit additional stiffness due to their curvature. Moreover, the grid structural system constitutes a means to minimize the visual mass of the structure. The load bearing capacity of the grid shell is directly related to its corresponding mesh topology. In general, triangular meshes are more rigid and stronger than the meshes of other types. However, polygonal meshes also have some structural advantages, as they are torsion-free structures [13]. Due to the fact that triangulated grid structures can easily approximate free-form shapes, we have chosen this kind of structures for further considerations.

3. Results

3.1. Assumption of the grid topology

In general, the robustness of the grid shell structure is strictly related to the distribution of load along its beams. The more uniform the distribution, the stronger the structure. Due to this fact, the grid pattern should affect the load-bearing capacity of the structure. In our research, several representative grid shells, formed on the base of the same Enneper surface, have been subjected to structural analysis. In order to conduct further calculations, we assumed that each

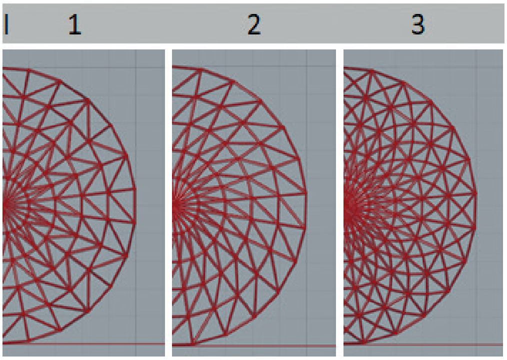


Fig. 2. Considered types of grid patterns (by J. Dźwierzyńska)

grid shell covered a round place whose radius was equal to 5.0 m. Moreover, each grid shell was of the same height of 4.5 m or 2.0 m. The considered grid shells differed due to various grid patterns applied. The rectangular projection of half of all the grid coverings is shown in Fig. 2.

Each of the patterns constitutes a triangular mesh with the same division into u=5 segments along the radius and a division into v=25 segments along the boundary circle. Due to this fact, each grid shell is composed of 126 nodes included in the Enneper surface and 350 faces being triangles.

3.2. Analysis of the static behavior of the grid structures

3.2.1. *Characteristics of the structural models*

In order to perform the structural analysis by means of Karamba 3D, first, we created parametric geometric models of each grid structure by means of Grasshopper's form generative algorithmic modeling tools. Next, these geometric models were converted into structural models.

Due to the fact that the FEM simulation represents physical objects as a collection of discrete components or elements, the geometry of grid shells was presented by meshes. In order to study the structural behavior, the following characteristics have been considered:

- the way of supporting the structure,
- material – steel resistant against stress and external actions,
- cross sections,
- assembly,
- load properties [14].

Depending on the value of the parameter applied during the generation of the Enneper surface, we could achieve various shapes of this surface. However, we have analyzed only those surfaces, which could constitute coverings of a round place. For each of the shapes, we applied various ways of supporting (three-, four- or five-point supports located in the grids' boundary nodes) Fig. 3–5.

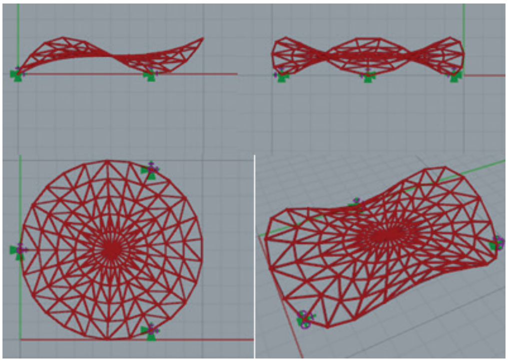


Fig. 3. Representation of a grid shell with three supports (by J. Dźwierzyńska)

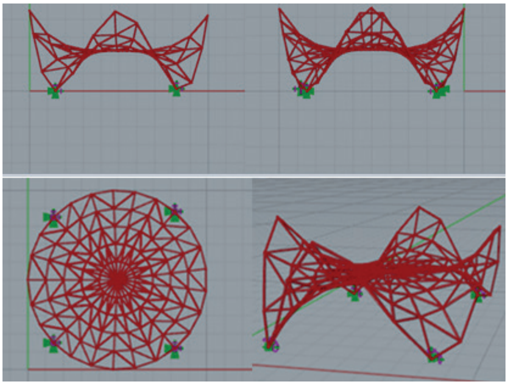


Fig. 4. Representation of a grid shell with four supports (by J. Dźwierzyńska)

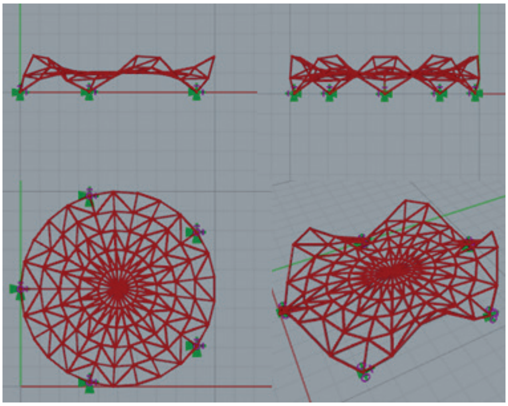


Fig. 5. Representation of a grid shell with five supports (by J. Dźwierzyńska)

Due to this fact, for further consideration, we have taken three different grid shapes. However, each of these shapes could be characterized by three various topologies: 1, 2, 3 (Fig. 2). Additionally, in order to perform further calculations, we have assumed two possible heights of the structures: 2.0 m and 4.5 m.

Preliminary, the cross sections of all pipes were assumed to be 10.16 cm, whereas the thickness – 0.36 cm. All pipes were rigidly joined by welded joints. The structure was subjected to a combination of dead loads and live loads.

3.2.2. *Performance of the first simulation*

The first simulation by means of Karamba 3D was performed for all kinds of steel grids (of various patterns and various supporting systems). The grids were subjected to a dead load, which was a self-weight of the spatial steel grid structure. The dead load acted on the structure in the negative *z* direction (Fig. 6).

The structural behavior was analyzed both for the steel grid of 2.0 m in height and of 4.5 m in height. We have taken into account the minimal mass of the structure and the maximum

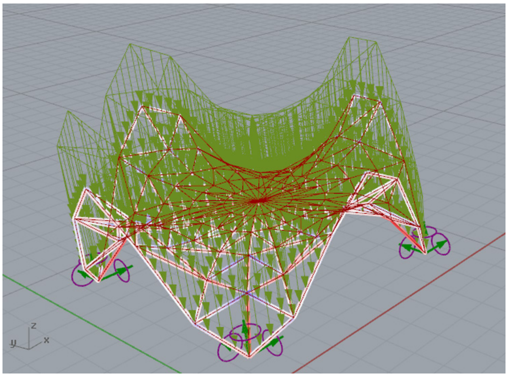


Fig. 6. Dead load acting on the structure (by J. Dźwierzyńska)

displacement, both at end-points and mind-points of the mesh elements. The achieved results of the performed analysis are presented in Table1 and Table 2.

Table 1. Results of the structural analyses for the spatial grid of pattern 1 and of a height equal to 2.0 m

Number of supports	Radius/ Thickness [mm]	Mass [kg]	Maximum displacement [mm]
3	101.6/3.6	3061.20	23
4	101.6/3.6	3081.91	27
5	101.6/3.6	3102.01	31

Table 2. Results of the structural analyses for the spatial grid of pattern 1 and of a height equal to 4.5 m

Number of supports	Radius/ Thickness [mm]	Mass [kg]	Maximum displacement [mm]
3	101.6/3.6	3269.32	16
4	101.6/3.6	3345.67	17
5	101.6/3.6	3412.84	21

Based on the above results, the structure of topology 1 has been chosen as the structure with the smallest node deformations and the smallest mass.

In the case of grid structures of topology 2 and 3, the assumed radii of 10.16 cm for the beams' circular cross sections were not sufficient to carry their own weights. In order to increase the load-bearing capacity of these structures, it was necessary to compact the meshes or increase the cross-sections of beams. However, both actions significantly increased the mass of the structures. Due to this fact, the structure with the grid pattern of type 1 has been chosen as the most effective.

3.2.3. *Performance of the second simulation*

Next, the first order static analysis was performed for the chosen structure taking into account permanent and variable loads. Permanent loads constituted an own load, which was the sum of the weight of the lattice structure and the weight of the covering glass panels. However, the variable loads were loads from snow and wind. Both snow and wind loads can have a considerable effect on grid shell structures. These loads are calculated in the form of pressure coefficients acting over the surface of the shell. We assumed a snow pressure of 1,3 KN/m^2 and a wind pressure of 1 KN/m2, and applied different possible load combinations. The most unfavorable combination of loads turned out to be a combination taking into account permanent loads and variable loads of snow and wind. It was possible to predict the behavior of the shell grid structures under loads and calculate deflections. This was thanks to the application of Karamba 3D components, which enabled to create static models based on parametric geometric models of grid structures prepared before. In order to specify various types and different load combinations for the models, we have oriented self-loads and snow loads globally to the system of axes x, y, z, whereas wind loads locally to the mesh. The procedure for calculating nodal forces from surface loads in Karamba 3D consisted of a calculation of the resultant load on each face of the given mesh and the distribution of loads among nodes. The achieved results of the performed analysis are presented in Table3 and Table 4.

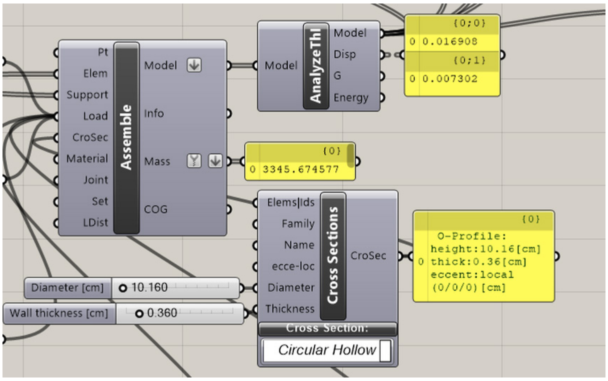
Table 3. Results of the structural analyses for the grid shell of pattern 1 and of a height equal to 2.0 m

Number of supports	Radius/ Thickness [mm]	Mass [kg]	Maximum displacement [mm]
3	101.6/3.6	3061.20	33
4	101.6/3.6	3081.91	38
5	101.6/3.6	3102.01	44

Table 4. Results of the structural analyses for the grid shell of pattern 1 and of a height equal to 4.5 m

Number of supports	Radius/ Thickness [mm]	Mass [kg]	Maximum displacement [mm]
3	101.6/3.6	3269.32	27
4	101.6/3.6	3345.67	24
5	101.6/3.6	341.841	29

Analyzing the obtained results, we can state that, in the case of grid shells of a height equal to 2.0 m, the increase in mass causes a slight increase in deformation. There is no such regularity for structures of a height equal to 4.5 m. In this case, an increase in the mass of the structure with four supports does not cause an increase in deformation. The results of Karamba's analysis for this case are presented in Fig. 7.



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{0;0}
0 0.016908
{0;1}
0 0.007302
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{0}
0 3345.674577
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{0}
  O-Profile:
  height:10.16[cm]
0 thick:0.36[cm]
  eccent:local
  (0/0/0)[cm]
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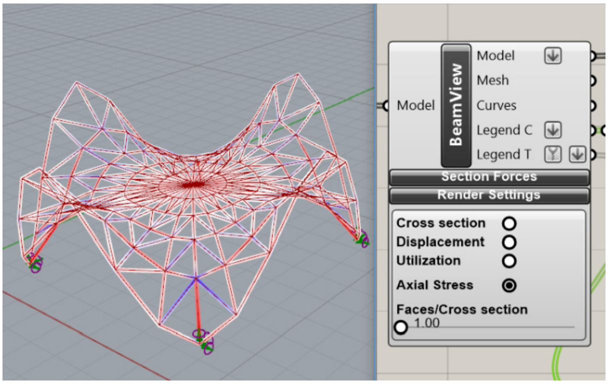
Diameter [cm] 10.160

Wall thickness [cm] 0.360

Cross Section: Circular Hollow

Fig. 7. Calculations of the mass and maximum displacement for the grid shell of 4.5 m in height and with four supports (by J. Dźwierzyńska)

Maximum utilization of beams was achieved in the case of the structure with four supports and it was equal to 93%. However, the minimum utilization for the same structure was equal to 12 %. Due to the big difference between these values, it was recommended to increase the beams' cross-sections at supports as well as slightly reduce the cross sections of other beams, followed by recalculating the new model. Another solution to cause the loads to be distributed more evenly throughout the structure was increasing the density of the grid pattern at supports or increasing the number of supports. That is due to the fact that the greatest axial stress for each structure occurred in the beams located at supports, which is presented in Fig. 8.



Cross section ○
Displacement ○
Utilization ○
Axial Stress ●
Faces/Cross section 1.00

Fig. 8. The distribution of axial stresses in the grid shell with four supports (by J. Dźwierzyńska)

3.2.4. *Static behavior of spherical cap grid shells*

In order to evaluate the performance of grid structures based on the Enneper surface in comparison to the performance of other structures, we have created Grasshopper models of spherical cap grid shells. We have applied the same grid pattern in the models as the pattern 1 of the Enneper grid structures, as well as the same cross-sections (Fig. 9). Next, the model was converted into a structural model and a structural analysis assuming the most unfavorable combination of loads was performed.

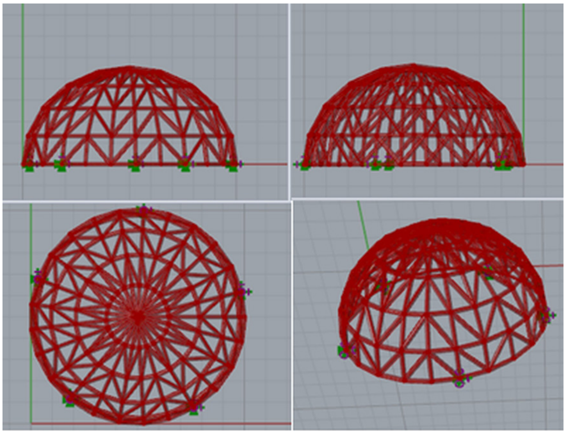


Fig. 9. Representation of a spherical cap grid shell with five supports (by J. Dźwierzyńska)

In fact, the shape of the above grid structure was not spherical, but ellipsoidal, owing to the height of the structure. The grid's height was equal to 2.0 m or 4.5 m, whereas the radius of the covered round place was equal to 5.0 m. We have calculated both the mass and the maximum deflection for the structure being supported by five supports. The results are presented in Table 5.

Table 5. Results of the structural analyses for the grid structure based on a spherical shape with pattern 1 applied

Number of supports	Height of the structure [m]	Mass [kg]	Maximum displacement [mm]
5	2.0 m	3519.7	5
5	4.5 m	4155.46	2

Comparing the obtained results for the grid structure created based on a spherical surface with the results achieved for the grid structure created based on the Enneper surface, we can state that both structures are stable with the assumed load; however, the latter structure is lighter.

4. Conclusions

The elaborated scripts for the parametric shaping of grid shells created based on the Enneper surface work well. They seem to be especially useful as they allowed us to create various geometric models of grids and analyze their structural performance using various boundary conditions. An implementation of structural analysis at the early stage of design is very beneficial. The conducted analysis showed that the load-bearing capacity of the grid shell is directly related to its corresponding mesh topology and enables one to choose the grid pattern with the best structural characteristics. In all cases of the analyzed forms, decreasing the span-to-height ratio (becoming steeper) improves the load-bearing capacity of the grid structure.

However, our intention was not to design a specific structure, but to show a practical framework of a parametric design approach, which consists in testing the mechanical performance of the designed forms at the initial stage of design. Our method is fairly general, and it can be applied for generating various grid structures. However, the results of the analysis concerning grid shells created based on the Enneper surface showed that they are structures with good statics properties and may be an interesting proposal when it comes to covering large areas.

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