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TYPOLOGY OF FRACTIONAL NUMERALS IN TURKIC LANGUAGES¹

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Abstract

This paper analyzes fractional numerals in Turkic languages and classifies them into seven types based on morphological criteria. These types are then divided into three paradigms, the Paradigm of Origin (PO), the Paradigm of being Inside (PI) and the Paradigm of Belonging (PB), according to the underlying logic of the constructions. The emergence of each paradigm is also discussed, the conclusion being that they are of different origin.

1. Introduction

1.1. State of the art

Fractional numerals seem to be the least researched type of numerals in the Turkic languages. There is a lack of publications focusing solely on fractions, while the general literature on numerals rarely includes them. Examples of studies on numerals which omit fractions are Clauson (1959) and Stachowski (1997). Koşaner (2016) writes about fractional numerals but limits himself to Turkish, and analyzes a completely different aspect than that examined here. Bachtijarova (2015) and Sultanbaeva (2016) enumerate fractions in Bashkir, but fail to provide any additional remarks. The only comparative study of Turkic fractions of which I am aware is SIGTJa (198–200), but unfortunately the data is limited, and the analysis required greater depth.

¹ I owe my most sincere thanks to prof. Marek Stachowski (Jagiellonian University) for his immense help with the preparation of this paper.

My main sources were the published grammars of individual languages. However, regrettably, the material in most grammars is scarce. Additionally various grammars of the same language may give entirely different information. Whereas it is possible that this might mean that some of the data is simply incorrect, this is currently impossible to determine, and so data from all the sources consulted will be included in the article. This data is bolstered by arithmetic textbooks,² whenever it was possible to gain access to suitable works. Regrettably, not every Turkic language in Russia is used within the educational system, and despite a thorough search through governmental and educational sites relating to the languages used,³ it was extremely difficult to obtain a satisfactory number of textbooks. Russian-Turkic dictionaries of mathematical terms failed to provide further information, as they include general terms, such as "fraction", "numerator", "denominator", etc., but no examples of fractional constructions.⁴ An interesting work, which might have pertinent insights on the topic is Salčak (1973), but it was impossible to gain access to the full text. Wikipedia was consulted under the entry "fractions" in the available Turkic languages, but no additional data was found, as all the examples were constructed using digits rather than words. The incomplete and contradictory nature of the data in the grammars under discussion suggests that the original texts would have been a better source, but such a study is well beyond the scope of this paper. The lack of a particular construction in a single manual does not mean this construction is not in use. Also original texts could only act as primary sources if an exhaustive survey of the mathematical materials as well as extensive field work was carried out, which is currently not possible, especially if it was to include every language in the paper. Thus, the textbooks will be considered secondary sources, acting as confirmation of certain constructions that are used in school education. It is also hoped the present study will motivate a greater interest in fractional numerals among grammarians, so that future reference works may address this area of grammar with more caution.

1.2. Types of fractional constructions

A fractional construction is composed of a numerator (N) and a denominator (D). These are present in all types of constructions; in some constructions N is seemingly omitted, but in these cases it is by default equal to one. Most constructions build D and N upon cardinal numerals, to which additional suffixes and words are added. This will be considered as standard, and thus it will not be mentioned each and every time. The only exception are the ordinal constructions [1.2.6]. The constructions will be presented in an abstracted form, but suffixes of case, number, etc. can be added. When inflecting a fractional numeral, suffixes are added to N, while

² I would like to thank the anonymous reviewer for this valuable suggestion, as well as all the other suggestions and comments.

³ For the full list see TJa.

⁴ For example see Salpagarov (2006).

D remains unchanged. The default syntax of Turkic fractions is that D precedes N. Certain constructions contradict this principle, and these will be highlighted. Occasionally an auxiliary word is used in addition to the numerals. Although the lexemes used differ between languages, the meaning is always the same: 'part'. Depending on the language, it can be added to either D or N, or both. Should the segment to which it is attached take on a case suffix, the suffix is instead adjoined to the auxiliary word.

Below is a survey of all constructions present in the collected data. Only their basic structure will be presented in this instance, while further analysis can be found in section 3. Examples adduced here are taken from the latter part of the paper. Citations can be found in appropriate sections below. The constructions are divided into seven types, each comprising a number of subtypes:⁵

1.2.1. Locative constructions (L)

- a) *Pure locative* D is in the locative, N in the nominative. Tur. *ücte bir* '1/3'.
- b) Locative + a possessive suffix
 A third person possessive suffix is added to N.
 Nog. beste üşi '3/s'.
- c) Locative + an auxiliary word Krym. üç payda bir pay '1/3'.
- d) Locative in a postposition
 A postposition išinda (= Tur. içinde) is added to D. N remains in the nominative.
 An auxiliary word is also added to both D and N.
 Sal. uš t'iŋ išinda iški t'iŋ '²/3', lit. 'inside of 3 parts 2 parts'.

1.2.2. Post-locative constructions (PL)⁶

D is in a case which formally is not the Common Turkic locative, but which acquired the meaning of a locative case. N is in the nominative.

a) Dative Yak. altağa biir '¼'.

⁵ There is one more construction, present in Salar, but it is ambiguous and so is not included in the classification.

⁶ There might be a concern that if the only subtype classified within this type is the dative, the whole type should be instead called dative. I do not agree, as the distinctive feature of this construction is its locative meaning in a genetically non-locative case. Which particular case is used is secondary. If in a certain language the genitive case acquires a locative meaning and is used in fractions, it too would be classified in PL. Likewise, a dative construction could exist in a genuine dative meaning, but this would not be classified here.

- 1.2.3. Ablative constructions (A)
- a) Pure ablative
 D is in the ablative, N in the nominative.
 Trkm. *üčden iki* ⁵²/₃².
- b) Ablative + a possessive suffix
 A third person possessive suffix is added to N.
 Kum. ekiden biri ¹/₂².
- c) Ablative + an auxiliary word Chuv. pilěk pajran pěr paj ¹/5².
- 1.2.4. Genitive constructions (G)
- a) Pure genitive
 D is in the genitive, while N is in the nominative and gains a third person possessive suffix.
 Sr. onnuŋ piri ⁽¹/10²).
- b) Genitive + an auxiliary word Sr. on pölüktiŋ pir pölügi '1/10'.
- c) Genitive without the possessive suffix A possessive suffix in N is omitted. Suyg. törtüŋ vyš '¾'.
- d) Genitive without the possessive suffix, with reversed syntax A possessive suffix in N is omitted and N precedes D. Alt. beš jetinin ⁶⁵/7².
- 1.2.5. Constructions with derivational suffixes (S)
- a) -*čuk*

The diminutive suffix -*čuk* is added to D. N is placed before D, unless it is equal to one, in which case it is omitted. This construction is only used with decimal fractions.

Tuv. beš ončuk '5/10'.

b) -lig

An adjectival suffix of quality *-lig* is added to D. N precedes D. Khak. *pir törttig* ¹/₄².

c) -lik

A suffix of abstracta *-lik* is added to D. N precedes D. Karbal. *eki onluq* ²/10².

- 1.2.6. Ordinal constructions (O)
- a) Ordinal numeral + cardinal numeral
 D is an ordinal numeral, N is a cardinal numeral. N precedes D.
 Gag. bir üčünžü ¹/₃.
- b) Ordinal numeral + an auxiliary word D is composed of an ordinal numeral and an auxiliary word. The word takes on a possessive suffix. If N is equal to 1, it is omitted. If it is greater than 1, it can be marked by placing a cardinal numeral with the word kere 'x times' before D.⁷ Kar. dört kere altynžy pajy '4', lit. 'four times (his) sixth part'.
- 1.2.7. Deverbal constructions (DV)
- a) Gerundial constructions

It can be argued that gerundial constructions are not fractional numerals, but rather a description of a mathematical operation. The grammars consulted for this article deem them numerals, and so they will be treated as such in the paper, but the possibility of a different interpretation has to be taken into consideration. These constructions are created with a prior gerund, or a gerund of means. D has the form of "dividing x by y". N is the result of that operation undertaken z times. There are two types of gerundial constructions, differing in the way N is formed.

1a) Double gerund

N is a cardinal numeral in the accusative, with a gerund of the verb "to take". Bash. *nämäne utyðya bülep unan* <u>egerme berðe alyp</u> ^(21/30), lit. '**dividing (it)** D <u>N</u> by 30, <u>take 21</u>'.</sup>

- 2a) Gerund + an auxiliary word
 N is a cardinal numeral with a word "part".
 Karbal. bešni eki etip bir ülüšü '2.5', lit. '5 divided by 2, one part'.
- b) Participial construction

D is a cardinal numeral with a participle "made". Yak. *bies gymmyt biir* = '¼', lit. '5 made, 1', i.e.: 'a whole divided into 5 parts, one part taken'.

1.3. The issue of locative/ablative

The contemporary locative has the meaning of 'being inside of', and synchronically the L is understood in this manner as well. But in Old Turkic the locative case covered both this meaning and the meaning of the ablative, i.e. 'moving out of' (Räsänen

 $^{^{7}~}$ At least in one of the two languages using this construction. The second, however, is uncertain (Shor).

1957: 61; Erdal 2004: 173–174). Taking into account the semantics and the widespread use of the A today, the likely original meaning of the L was also 'moving out of'. Other researchers share this view (Dimitrev 1948: 93; Räsänen 1957: 90–91).

1.4. Objective of the work

The current work aims to gather a wide corpus of Turkic fractional numerals, analyze each fractional construction, establish a typology, and propose a hypothesis on the origin of the constructions. The authors of SIGTJa derive the L, A, and G from ProtoTurkic, while other constructions are thought to be later innovations. The L is supposed to be the oldest, while the A and G were formed in "a later stage of the protolanguage" (SIGTJa 199). This view is difficult to support, as the ablative case was not present in ProtoTurkic. It formed only in the Old Uyghur period (Erdal 2004: 173–174). The genitive case was present in ProtoTurkic, but convincing arguments, which will be discussed below, mean a different explanation of the origin of the G should be proposed.

2. Overview of languages

In this section the constructions found in individual languages will be examined. Only modern tongues are included. Unfortunately, I was not able to gather data on all the Turkic languages, hence some are not included. For the purpose of the presentation the languages were divided into subgroups.⁸ The order is as follows: Oghuz, Kipchak, Karakhanide, South-Siberian, Yakut and Chuvash, with the languages within subgroups introduced alphabetically. The constructions will also be presented in a set order: L, A, G, then the remaining constructions.

Oghuz

Azeri

- L altıda dörd '4/c' (Sevortjan 1971: 75).
- 2. A (Nakhichevan dialect): *ikidän bir* ¹/₂' (SIGTJa II 198).⁹

⁸ I acknowledge the controversies concerning the classification of the Turkic languages, however they are largely irrelevant to the issue discussed in the paper, hence I decided to settle on the most basic classification.

⁹ The authors cite here "НГДГАЯ: 106", but this is missing from the bibliography of their work.

Gagauz

- 1. L *beštä iki '₂*/₅' (Pokrovskaja 1964: 124).
- 2. O

bir üčünžü '¼' (Arabadži 1959: 146).

Salar

There are three constructions in Salar; L, A, and one unclassified (Tenišev 1976: 123–124).

1. L (in a postposition)

uš t'iŋ išinda iški t'iŋ '2/3', lit. 'inside of 3 parts 2 parts'.

The origin and initial meaning of this construction are unclear. It has been stated that the L originally carried the meaning of the ablative and is to be interpreted as such in the majority of cases. In Salar the situation is somewhat different. No other language forms fractions with a postposition, which suggests a Salar innovation. It is unknown whether this arose before or after the split between the locative and ablative cases. A post-split formation could imply an instance of the L with an original locative meaning, which is an interesting possibility.

- 2. A (in two variants)
 - a) pure *jiʒiten alʒ'i '%*/.
 - b) with the auxiliary word *t'iŋ* added to D, or both D and N. *jiʒi t'iŋten alʒ'i '%*'.

uš t'iŋten iški t'iŋ '²/₃'.

The use of an auxiliary word, putting aside the etymology of this particular lexeme (\leftarrow Chinese), can be either of native or foreign origin. On the one hand, auxiliary words are used in several Turkic languages, on the other, Chinese renders fractional numerals with the construction *fen zhī* 'his part' (Rimmington 2006: 18; CGW), so the Salar auxiliary word could also originate from its influence. The former seems more likely, as the Salar construction is absent of possessivity, which is present in Chinese in the form of *zhī* (Yabla).

3. Other

iški pir vaχyr ¹/₂['], lit. 'two one looks(?)'.¹⁰

Tenišev offers the following meaning (described as "lit."): 'one subordinate to two' (Rus. *единица подчинена двум*). Unfortunately as I am unsure of the meaning and structure of this construction, it is difficult to discuss it further.

¹⁰ *Vaχyr* = Tur. *baktr*? This interpretation was suggested by prof. Marek Stachowski (Jagiellonian University). However, the construction remains unclear.

Turkish

 L üçte bir '1/3' (Stachowski 2007: 157).

Turkmen

1. A

üčden iki ²/₃ (Clark 1998: 163–164; Bajdžanov 2008: 81–82).

Kipchak

Baraba-Tatar

1. A

pästän üč ³/₅ (SIGTJa II 198; Dmitrieva 1966: 162).

Bashkir

Bashkir uses two constructions (Dmitrev 1948: 93-94):

- 1. A bištän ös '³⁄₅'.
- 2. DV (gerund)

nämäne utyðya bülep unan egerme berðe alyp ^{'21}/30', lit. 'dividing (it) by 30, take 21'.¹¹ In Bashkir the *-ep* gerund carries the meaning of an action directly prior to the main action. It is an adverbial of means, not time, and corresponds to the Tur. *-arak*, not *-ip*.

Crimean-Tatar

Crimean-Tatar consists of four dialects, of which the southern is classified as Oghuz, with the middle, the Dobruja, and the northern as Kipchak (Jankowski 2010: 37–47).

Three fractional constructions are used (ibid..: 248).

- In literary language (based on the middle dialect, but with a strong Turkish influence) the L is used: *ücte bir* ¹/₃.
- The northern and Dobruja dialects (which were heavily influenced by Nogay) use the A: ücten bir ~ üsten bir '1/3'.
- Jankowski (2010), citing (Mahmut 1975: 147) also mentions the G, but fails to specify the dialect in which it is present: úşníň ekísí ⁵²/s².

¹¹ I owe the transcription, translation, and grammar notes to dr Barbara Podolak (Jagiellonian University).

Karachay-Balkar

1. A

ondan bir '1/10' (Filonenko 1940: 51).

Some sources provide examples with a double possessive suffix:

törtten üčüsü '¾' (SIGTJa gives *üčüšū*, but this seems to be in error – Baskakov gives the form with <s>, as does SIGTJa in the following example); *žetiden bešisi '5*⁄⁄' (SIGTJa II 198; Baskakov 1976: 167).

Baskakov (ibid.) also adduces whole sentences, e.g.:

segizden üčüsünden ese, altydan bešisi köpdü '5% is more than 3%'.

Forms with a double possessive suffix only include two numbers: *beš* '5' and *üč* '3'. These examples are brought up in the sources adduced above, but the issue of the possessive suffix is not discussed. Filonenko (1940: 51) omits any mention of a possessive suffix, whereas, Chabičev (1966: 221) suggests that a possessive suffix is optional without specifying whether it is geminated.

ekiden bir ~ ekiden biri '½' (Chabičev 1966: 221).

Perhaps the origin of a possessive suffix in a fractional numeral is the junction of a fraction and a noun, e.g.:

zamanny (sic!) ekiden biri 'half the time', lit. '1/2 of the time'.

It is important to underline the lack of a repeated possessive suffix in the example. Originating in this construction, the possessive suffix could have subsequently been re-analyzed as belonging to the numeral itself and now is used even without an adjacent noun in the genitive,¹² e.g.:

bir saunu ekiden birin qoratsang ekiden biri qalady 'if you subtract ½ from a whole, ½ remains' (Chabičev 1966: 169).

2. G

törtnü ekisi `2/4' (Chabičev 1966: 221).

 S (-lik) (Baskakov 1976: 167) eki onluq ²/10². According to Baskakov, it is mainly used in literary language.

4. DV (gerund)

bešni eki etip bir ülüşü '2.5', lit. '5 divided by 2, one part'.

Karaim

Neither (SIGTJa) nor (Németh 2011) provide any information on fractions in Western Karaim, with the former stating that the language does not include fractional numerals. In Crimean Karaim the O is used (with an auxiliary word). If N is greater than 1, it is marked by a cardinal numeral + *kere* 'times'.

¹² This explanation could possibly also apply to other instances of the L and A with a possessive suffix, but such a discussion is beyond the scope of this study.

üčünžü pajy ¹/₃', lit. '(his) third part'. *dört kere altynžy pajy* ⁴/₆', lit. 'four times (his) sixth part' (Prik 1976: 87).

Musaev (1977: 40) also claims fractional numerals are absent in Western Karaim, but at the same time adduces a construction similar to that above, apparently not viewing it an actual fractional construction. It differs in the lexeme used, with two lexemes possible, *želek* and *ül'us*'. The possibility of magnifying N is not mentioned.

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üč'uńč'u želegi '¼3'.
ekińči ül'us' '¼3'.
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Karakalpak

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    A
altydan tört '4/6' (Baskakov 1952: 229).
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Kazakh

- A (Archangelskij 1927: 24; Muhamedova 2016:241). üşten bir ¹/₃' The A is the construction found in arithmetic textbooks (A'bilqasymova 2017: 78).
- G (Archangelskij 1927: 24; KAZ) *üštiŋ biri* ¹/₃².
- 3. S (-*lik*) (SIGTJa II 199) *bir üštik* ^{'1}/₃'.

Krymchak

 L (with the auxiliary word *pay* added to both N and D) üç payda bir pay '1/3' (Güllüdağ 2005: 229).

Kumyk

Four constructions are present in Kumyk (Dmitrev 1940: 74-75).

- L. The author cites a grammar by Batyrmurzaev, but unfortunately fails to provide appropriate references. onda eki ⁶²/10².
- 2. A (with a possessive suffix) *ekiden biri* '1/2'.
- G üčnü(sic!) ekisi ²/₃².
- 4. DV (gerund)

*ettini eki bölün bir paj '7*2' lit. 'dividing 7 by 2, one part', cf.: *ettini üč bölün eki paj aldym* '14'3' lit. 'dividing 7 by 3, I took 2 parts'. Note that *bölün* 'dividing; having divided' is a gerund, not a participle in Kumyk. It can mark both prior and simultaneous action (Dmitrev 1940: 139).

Kyrgyz

- A törtön uč ⁴³/₄ (GKJKS 18; Abduldaev 1987: 181). This construction is used in textbooks (Bekboev 2015: 110).
- 2. G

üčtön(sic!) *ekisi* ²/₃ (Abduldaev 1987: 181).

Nogay

- L (with a possessive suffix) beste üşi '3/5' (Akbaba 2007: 637).
- 2. A *altydan bir '1*⁄c' (Baskakov 1940: 79).
- 3. G *dörttiñ biri ¹/*4² (Atay 1998: 125).
- S (-lik) (Atay 1998: 125) bir onluk '1/10'. yırma bes yüzlik '25/100'.

Siberian-Tatar

1. G

ikeneŋ pere ¹/₂² (Sagidullin 2014: 36).

The author notes that N is in the genitive and D takes the possessive suffix, but this would appear to be a mistake. In fact, the opposite is true, as seen in the example.

Tatar

 A (Burbiel 2018: 134) altydan biš '5%'. The textbooks confirm this construction (Vilenkin 1992: 165).

Karakhanide

Saryg-Uyghur

 G (without a possessive suffix) törtüŋ vyš '¾' (Tenišev 1966: 22).

Uyghur

- A tötten üč ^{'3}/₄' (Tömür 2003: 136).
- 2. G *ikkiniŋ biri ʿ1*/2' (Kajdarov 1966: 13).

Uzbek

 A (Kononov 1948: 121) *učdan ikki ²/*₃². The textbooks confirm this construction (Haydarov 2015: 119).

South-Siberian

Altay

Altay uses four constructions, three of which are mentioned in Nevskaja (2017: 169–171).

- G (with the auxiliary word *ülü*, added to both D and N, or only to N) tört ülünin eki ülüzi ⁵/₄. jetinin beš ülüzi ⁵/₇.
- G (without a possessive suffix, and with reversed syntax) beš jetinin ⁶√⁷.
- 3. O (in two variants)
 - a) with a cardinal numeral *bir ekinci* ¹/₂ (Dilek 2007: 1040).
 - b) with the auxiliary word ülü ekinci ülüzi '1/2'. Nevskaja fails to specify if N greater than 1 can also be expressed using this construction.
- 4. S (-*lik*) (Dilek 2007: 1040; Rassadin 1978: 129) *beš altylyk* ⁵%.

Khakas

Khakas uses two constructions (Baskakov 1975: 413-414).

- 1. G altynyŋ pizi '5⁄ć'.
- S (-lig) pir törttig '¼'.

Shor

- 1. G (in two variants) (Dyrenkova 1941: 108)
 - a) pure onnuŋ piri '¼10'.
 - b) with the auxiliary word pölük added to either N or D, or both N and D. onnun pir pölügi '1/10'. on pölüktin piri id. on pölüktin pir pölügi id.

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Tofalar

1. O (Rassadin 1978: 129) *üš törtüškü '3*⁄4'.

Tuvan

Tuvan uses two constructions (Ischakov 1961: 208–209).

- 1. G (in two variants)
 - a) pure *üštüŋ ijizi* ^{°2}/3'.
 - b) with the auxiliary word chū beštiŋ iji chūzu '²/s'. This variant is only used in colloquial speech.
- 2. S (-*čuk*)

beš ončuk '5⁄10'.

This is only used in decimal fractions and is said to be a recent innovation.

Yakut

Yakut uses two constructions:

- PL (Kirişçioğlu 2007: 1262) altağa biir '1⁄c'.
 -ğa is formally a dative suffix, which with time acquired the function of the locative (ibid.: 1259).
- DV (participle) (Landmann 2016: 42–43) gymmyt (< gyn- 'to make' (= Tur. kıl- id.) + participial suffix -byt (= Tur. -mış id.). bies gymmyt biir = '1/5', lit. '5 made, 1', i.e.: 'a whole divided into five parts, one part taken'

This construction is used in education (JVU). Kirişçioğlu also cites DV, although in his example N is in the dative-locative case:

alta gımmıt biire '¼'.

Contrary to the previous construction, the dative meaning seems more appropriate. This could have an implication on the exact semantic interpretation of the construction, and change the literal translation into something similar to 'a whole divided into five parts, what remains of which adds up to one'. This, of course, could also be an over-interpretation. The general logic of the construction remains the same, so perhaps this issue is not of great significance.

Chuvash

1. O

There is a difference of opinion as to whether N is in the short or the long form of a cardinal numeral,¹³ with the latter premise supported by Krueger (1961: 176), Andreev (1966: 51), Degtjarev (1991: 25).

pĕrre tăvattăměš(ĕ) '1/4' (Krueger 1961: 176).

In contrast Matveev (1919: 51–52) and Rezjukov (1959: 116) favour the short form. *pěr pillěkměš*(\check{e}) ^{'1/5'} (Matveev 1919: 51).

- 2. A (in two variants)
 - a) with the auxiliary word *paj* (Matveev 1919: 51) *pilěk pajran pěr paj* ¹/₅².
 - b) with a possessive suffix (Ersoy 2007: 1311): ikkëren përi '1/2'.

3. Typology

3.1. Paradigm types

Fractional constructions are formed according to a certain logic when imagining fractions, with a unique type of this logic named a paradigm. The moment of formation is decisive. A construction can later be re-analyzed and change the way it is understood by the speakers, but this classification will only take into account the original meaning, which permits an understanding as to why the construction has taken the shape it currently exhibits. One paradigm can encompass many constructions, varying in their origin and morphological or syntactic makeup. The connecting factor within a paradigm is the aforementioned logic. One language can utilize constructions from different paradigms, as is the case in many Turkic languages, which is the result of language development and borrowings.

The examined material allows three distinct paradigms to be proposed: the Paradigm of Origin (PO), the Paradigm of being Inside (PI) and the Paradigm of Belonging (PB). In the PO N **comes from**, **is taken from** D, e.g.: "from 3 I have taken 1" '1/3'. The 1 was formerly a part of 3, but it was taken from it. In the PI N **is inside of** D, e.g.: "inside of 3 there is 1" id. In the PB N **belongs to** D, e.g.: "one third", "one of three" id.

In the following discussion each paradigm is analyzed, and additionally an attempt is made at sketching the areal groupings and proposing the possible origins of the paradigms and singular constructions.

¹³ The short form is used for the attributive, while the long form for abstract meaning. Numerals in the long form can be a predicate, a subject, or an object (Krueger 1961: 174).

3.2. Paradigm of Origin

3.2.1. Locative constructions

The L can be classified in this paradigm due to its original ablative meaning (see 1.3). Almost all the instances of the L lie within the sphere of the former Ottoman influence, with Salar being the only outlier. It seems probable that the L was spread among these languages together with the Ottoman mathematical works. To prove this, appropriate historical data would need to be examined, but it can be used as a working hypothesis. As Salar is also an Oghuz language, the following model can be proposed: the L is an Oghuz archaism, which was later spread by Ottoman-Turkish to the neighbouring Kipchak languages. It was also preserved in distant Salar, although it developed somewhat differently, resulting in a construction with the locative in postposition.¹⁴ It was lost in Turkmen due to its relative isolation from other Oghuz languages and the influence of the Kipchak and Karakhanide languages. The L was also initially preserved in Yakut, where it later changed into PL.

3.2.2. Ablative constructions

The A is the most widespread of all the construction types. It is found mainly in Kipchak, but also in Karakhanide and Turkmen. In light of what has been said regarding the locative – ablative connection, it is possible to suggest that the A developed from an earlier L, replacing it in Kipchak and Karakhanide. The speakers preserved the meaning of the original construction, while adjusting its grammatical form. The A is also used in Chuvash, where it can either be native, resulting from a development parallel to, but independent of Common Turkic, or a borrowing from the neighbouring Kipchak languages. The Chuvash construction differs in the employment of an auxiliary word, but this addition does not appear to be a major change. At the same time the lexeme used, *paj*, is also employed as an auxiliary word in the neighbouring Kipchak languages, albeit in different constructions. Thus, a Kipchak influence seems very probable. It might also be useful to study the local Uralic languages to examine whether they could have had an influence upon Chuvash.

3.2.3. Deverbal constructions

Deverbal constructions are rare and at the same time varied. Except for Karachay-Balkar and Kumyk, which, due to their high degree of similarity and geographical proximity, can be linked to each other, the other constructions seem to be entirely independent developments.

On morphological or genetic grounds, DV is not connected to the L or A, but its semantics allow us to classify it within the PO. Gerundial constructions are a very descriptive way of expressing fractions, which raises questions about the degree of their grammaticalization, as they could simply be a description of a mathematical

¹⁴ But compare Section 3.3 below.

operation. At the same time, the participial construction in Yakut seems to be fully grammaticalized, as it is used in education as the main fractional construction. The question as to whether there was any external stimulus which triggered the rise of these constructions has yet to be answered.

3.3. Paradigm of being Inside

The only construction described in the current paper which can be classified in the PI is PL. As has been noted above, in classifying the constructions into paradigms, the moment of formation is decisive. When the L formed, it was understood as an ablative construction. Later, after the split between the ablative and locative cases, some languages re-invented their fractional construction with the original logic in mind, thus forming the A, while others kept the archaic locative construction. With time the speakers probably forgot its initial meaning, and it is possible that today the L is understood by its users in a strictly locative sense. As the form of the construction did not change, it is still classified within the PO, according to the original logic. But, if speakers were now to re-invent their fractions according to their current understanding, a subsequent construction would be an example of the PI. This is exactly what happened with PL. However, currently only Yakut seems to have taken this next step in the evolution of the L, hence its construction is the only one classified here, but it is entirely conceivable that in the future other languages utilizing the L will follow suit. The Salar locative in a postposition construction could also possibly be classified as the PI, but until the time of the constructions emergence can be determined, it cannot be decisively said if its original meaning was locative or ablative.

3.4. Paradigm of Belonging

3.4.1. Genitive constructions

The semantics of the genitive correspond to the concept of "belonging", and thus it is only natural to classify the G within the PB. The construction is basically a definite compound, which is, as such, a native element. The use of a definite compound in a para-fractional context is also typical, e.g. the Tur. *gazetecilerin ikisi* 'two of the journalists' (Stachowski 2007: 95). Nevertheless, a para-fractional meaning is not yet a fractional numeral, just as the Pol. *dwóch z trzech dziennikarzy* 'two of the three journalists', lit. 'two from the three journalists' is not a fractional numeral. Despite having this construction, Polish has never developed a fractional numeral of the PO type. The G is very widespread among the Turkic languages, but the majority of idioms using the G are found in areas under the influence of Russian; that is in present-day Russia itself and Central Asia, with the two exceptions being Uyghur and Saryg-Uyghur. This means that only the Turkic languages which had contact with Russian, with two exceptions, grammaticalized the definite compound as a fractional numeral. The Russian influence seems very likely, as the G clearly resembles the Russian fractional construction. Rus. две третих '2⁄3' (Bogdanov 2009: 317). Tuv. üštüŋ ijizi '2⁄3'.

The constructions differ in syntax, the presence of a possessive suffix and the absence of the plural in Turkic, but these differences are to be expected due to the general rules of Turkic grammar. A definite compound requires a possessive suffix. If plurality is signified by numerals, the noun stands in the singular case. D is expected to precede N. Additionally, Russian uses ordinal numerals, while Turkic languages use cardinal numerals. The definite compound in Turkic is formed with nouns, which might be the reason for using a cardinal (a substantive) over an ordinal (an adjective) numeral. Applying these rules to the adaptation of the Russian construction results in the G. However, certain subtypes of the G do not follow these rules, namely those subtypes without the possessive suffix and with reversed syntax. These bear an ever closer resemblance to the Russian construction. While a standard G appears to be an attempt at adapting the Russian construction using native tools, these subtypes might be considered as direct calques from Russian, especially the Altay construction. Interestingly enough, this construction is also found in Saryg-Uyghur, which is less likely to be susceptible to Russian influence due to its lack of proximity. The question of how the G arrived into Uyghur and Saryg-Uyghur remains to be answered and is a concern regarding the aforementioned interpretation. With further historical data it may be found that the G is native after all, or at the very least that there are two possible sources of this construction. This possibility cannot be ruled out, but in my opinion the hypothesis provided above is for the moment the more plausible.

3.4.2. Constructions with derivational suffixes

The semantics of the -*čuk* and -*lik* suffixes are somewhat problematic, as neither is explicitly connected to fractions. Having said that, both suffixes are used to derive nouns, and so can be understood as a 1/x fraction, i.e. *onluk* '1/10', lit. 'a tenth part'. N in these constructions is formed by multiplying that fraction; *beš onluk* '5/10', lit. 'five tenth parts'. Thus, even though at first glance it seems that N precedes D, contrary to the Turkic rule, in reality, the Turkic syntax is maintained. The *-lig* suffix is different, because it derives adjectives, not nouns. If understood in this way, *beš onlug* '5/10' would have to be translated as 'five tenth; five of ten', not 'five tenth parts'. If this is the case, then the syntax, despite appearing similar, is in fact to the reverse of the *-čuk* and *-lik* constructions, and in opposition to the Turkic rule. Nevertheless, it is possible to interpret it similarly to other S constructions, as Baskakov and Inkižekova-Grekul (1975: 407) state that adjectives with *-lig* are not easily distinguished from nouns (ibid.: 407).¹⁵

¹⁵ This information is presented in the paragraph on the suffix *-lig*. It is not mentioned in the discussion of the fractional construction with *-lig*, which is treated solely as an adjective of quality (Baskakov and Inkižekova-Grekul 1975: 413–414).

The S construction also appear to be a Russian borrowing, for reasons similar to the G. It is only used in Russia and Central Asia, and the constructions resemble the Russian construction even more so than the G, as the Rus. $\partial Be \ mpemux$ '2/3' can be understood as 'two of the third parts', which corresponds to -*čuk* and -*lik*, while -*lig* resembles it in that D is an adjective.

3.4.3. Ordinal constructions

The O likewise seems to be a Russian borrowing. The O with a cardinal numeral is a word for word (aside from the lack of a genitive) translation of the Russian construction. The O with an auxiliary word is slightly different, but as has been stated above, this does not seem to be a significant difference. Outside Russia the O is used in Moldova and by the Karaims. The O in Moldova is most easily explained by the Russian influence, as the country was very greatly influenced by the Russians in the Soviet era. The O in Karaim could. of course. also be a result of contact with Russian, but a Polish influence seems at least equally probable, as the Karaims occupy a terrain which for most of their early inhabitation belonged to Poland. The Polish fractional construction is almost identical to the Russian, so linguistic criteria alone cannot be used to resolve this issue.

4. Summary

Based on the geographical distribution and inner shape of the examined fractional constructions, the following interpretation can be put forward. The Paradigm of Origin seems to be the native Turkic paradigm. The most archaic construction appears to be the locative construction, which has been preserved in Oghuz, while in Kipchak and certain other languages it evolved into the ablative construction. Deverbal constructions are a later innovation in individual tongues, but this too operates according to the Paradigm of Origin logic. The Paradigm of being Inside is the next step in the evolution of the locative constructions. Currently there is only one distinct example of this paradigm known, with a further example yet to be proved definitively. The Paradigm of Belonging most probably emerged later due to the influence of Russian, or more broadly a Slavic influence. Genitive constructions are three different ways of adapting the Russian (Slavic) construction, with some of the subtypes being direct calques, while others are more sophisticated adaptations, utilizing native tools.

It is possible to suggest three linguistic areas based on the analyzed material:

• *Ottoman* – an area comprising languages influenced in the past by Ottoman-Turkish, in which the L is one of, or indeed the only, construction used. It encompasses areas around the Black Sea, that is from Moldova and Crimea in the west, to the Caucasus in the East, as well as Azerbaijan and Turkey in the south.

- *South-Siberian* in the Sayan-Altay region only the PB is used, completely replacing the native PO, which might stem from Russian having been the predominant language of education and science in the region. The native mathematical terminology of the languages involved was not very developed and had to be translated from Russian (Tovuu 2020).
- *Kumyk-Karachay* these languages share rare constructions generally not seen elsewhere, i.e. a possessive suffix in the A and the same gerundial construction. As in this instance only two languages have been influencing each other, this is not so much a linguistic area, but rather a situation of a two-way contact. None-theless, it seems useful to draw attention to these languages, as future studies could determine the direction of influence and the exact nature of this relation.

The current work does not aspire to be a definite answer to the problem at hand, but rather a first step in the direction of understanding Turkic fractions. I tried to gather as much data as possible, so that even if conclusions drawn in the article can be questioned, it will still be of value for future scholars. I would be delighted for other researchers to build upon this work, as well as correct any mistakes and inaccuracies. This paper deals exclusively with synchronic data, but it will be important in the future to examine diachronic data as well, so that historical conclusions can be re-evaluated. A comparison of Turkic data with data from other language families might also prove fruitful and yield general typological observations.

Abbreviations

Languages

Alt. = Altay	Pol. = Polish
Bash. = Bashkir	Rus. = Russian
Chuv. = Chuvash	Sal. = Salar
Gag. = Gagauz	Sr. = Shor
Kar. = Karaim Karbal. = Karachay-Balkar Khak. = Khakas Krym. = Krymchak Kum. = Kumyk Nog. = Nogay	Suyg. = Saryg-Uyghur Trkm. = Turkmen Tur. = Turkish Tuv. = Tuvan Yak = Yakut
140g. – 140gay	lak – lakut

Terms

A = ablative construction	O = ordinal construction
D = denominator	PB = Paradigm of Belonging
DV = deverbal construction	PI = Paradigm of Being Inside
G = genitive construction	PL = post-locative construction
L = locative construction	PO = Paradigm of Origin
N = numerator	S = construction with a derivational suffix

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