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## THE EFFECT OF CHANGING LOADS ON THE DEFLECTION OF RC BEAMS REINFORCED WITH RIBBED BARS

### WPLYW OBCIĄŻEŃ ZMIENNYCH NA UGIĘCIA BELEK ŻELBETOWYCH ZBROJONYCH PRĘTAMI ŻEBROWANYMI

#### Abstract

The paper presents a way of taking account of the effect of changing loads on the deflection of RC beams reinforced with ribbed bars. The proposed solutions were verified experimentally, based on the results of tests performed by the author. The beams were subjected to multiple repeatable loads. The deflection increment dependent on the value of the basic parameters characterising the changing loads (cycle maximum load  $M_{max}$ , stress ratio  $R$ , number of load cycles  $N_i$ ). The increment of the deflection of RC beams reinforced with ribbed bars subjected to changing loads increases with increases in the number of load cycles  $N_i$  ( $\log N_i$ , to be more accurate), and is greater for higher values of ratio  $\kappa$  and lower values of ratio  $R$  ( $\kappa = M_{max}/M_n$ ;  $R = M_{min}/M_{max}$ ;  $M_{min}$  – minimal moment of cycle;  $M_{max}$  – maximal moment of cycle;  $M_n$  – failure moment under short-term loads). The relations for the number of load cycles  $N_i$  are similar.

**Keywords:** changing loads, deflection of beams, ribbed bars, RC beams

#### Streszczenie

W pracy podano sposób uwzględniania wpływu obciążeń zmiennych na przyrost ugięć belek żelbetowych zbrojonych prętami żebrowanymi. Podane rozwiązanie poddano weryfikacji doświadczalnej. Wykorzystano przy tym wyniki własnych badań doświadczalnych. Badane belki były poddane obciążeniami wielokrotnie powtarzalnym. Przyrost ugięć uzależniono od wartości podstawowych parametrów charakteryzujących obciążenia zmienne (wartość obciążenia maksymalnego cyklu  $M_{max}$ , współczynnik asymetrii cyklu  $R$ , liczba cykli obciążenia  $N_i$ ). Przyrost ugięć belek żelbetowych zbrojonych prętami żebrowanymi poddanych obciążeniom zmiennym wzrasta wraz ze wzrostem liczby cykli obciążenia  $N_i$  (a ściślej  $\log N_i$ ) i jest większy dla większych wartości współczynnika  $\kappa$  oraz dla mniejszych wartości współczynnika  $R$  ( $\kappa = M_{max}/M_n$ ,  $R = M_{min}/M_{max}$ ;  $M_{min}$  – moment minimalny cyklu,  $M_{max}$  – moment maksymalny cyklu,  $M_n$  – moment niszczący przy obciążeniach doraźnych). Podobne są zależności dla danej liczby obciążeń  $N_i$ .

**Słowa kluczowe:** obciążenia zmienne, ugięcia belek, pręty żebrowane, belki żelbetowe

## 1. Introduction

Reinforced beams have been investigated by researchers of reinforced concrete ever since this type of structure came into use. Both standard recommendations and the literature on the subject, however, have dealt primarily with short-term loads (immediate) or – much less frequently – long-term loads (now called quasi-long-term) [6, 7, 9–12]. The research works that cover the effect of changing loads on reinforced beam deflections are exceptionally rare. This results not only from both the economic and time-related costs of tests, but mainly from the fact that the number of structures subjected to repeated changing loads which may affect the deflections of the elements in question is rather limited.

Changes to the deflection of reinforced concrete beams under changing loads are caused by several factors including:

- ▶ the formation and propagation of microcracks in the zone under tension between the cracks,
- ▶ the widening of cracks due to larger sections of loss of concrete-rebar bond (particularly in the zone of predominant action of bending moment, and – to a much lower extent – to shearing forces) [1, 2, 4, 5],
- ▶ a lower value of concrete modulus of elasticity under the loads in question (3). The extent of the second of the aforementioned factors depends on the type of reinforcing bars (plain, ribbed).

This paper presents a method of taking into account the effect of changing loads on the deflection of RC beams reinforced with ribbed bars. The proposed solutions were verified experimentally, based on the results of tests performed by the author. The beams were subjected to multiple repeatable loads. The deflection increment was dependent on the value of the basic parameters characterising changing loads (cycle maximum load, stress ratio, the number of load cycles). Beam deflections were not the primary purpose of the experiments described in [3]. Nevertheless, on their basis, certain conclusions can be drawn and the formula covering the effect of parameters and the number  $N$  ( $\log N$ ) i.e. the number of load cycles on RC beams deflection can be verified.

## 2. Experimental section

### 2.1. Test specimens

The tests were performed on seventeen reinforced concrete beams of dimensions and reinforcement as shown in Fig. 1. Each beam was 15 cm wide. In one group of beams (Fig. 1a) in the zone of action of bending moment of a constant value from the programme load the omission of stirrups is intentional to avoid introducing inclusions that would change the beam cracking image [3]. In the other groups of beams, stirrups and top longitudinal bars were applied along the entire length of the beams (Figs. 1b & 1c).

In the article, the results of tests performed on beams reinforced with ribbed bars made from 18G2 steel are presented (the tests were done when the given denotation of steel was valid). All the beams were reinforced with bars with a diameter of 16 mm ( $\mu = 1.48\%$ ). In each series, the beams were cured in natural conditions (the production plant and the laboratory: temperature  $t = 18 \pm 3^\circ\text{C}$ ; air relative humidity  $\text{RH} = 60 \pm 10\%$ ).

The beams were made from concrete with a mean compressive strength ' $f_{\text{cm}}$ ' and tensile strength (splitting tests) ' $f_{\text{ctm}}$ ' of:  $f_{\text{cm}} = 27.96 \text{ MPa}$ ,  $f_{\text{ctm}} = 2.78 \text{ Mpa}$ . The standard deviation values were  $s_{\text{cm}} = 1.09 \text{ MPa}$ ,  $s_{\text{ctm}} = 0.12$ . Young's modulus of elasticity of the concrete was on average 28.09 GPa. The mean tensile strength of the steel was 529.23 Mpa, the yield point was 390.43 MPa.

The test period for each beam was up to four days and this is why concrete shrinkage and creep were not included. With such a short test time, the effect of these rheological factors is negligibly small and was disregarded in the analysis.

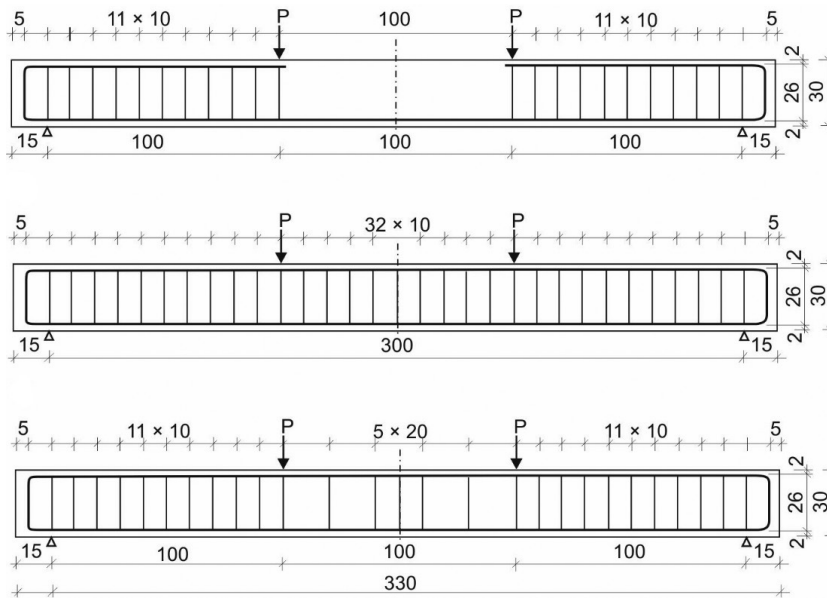


Fig. 1. Dimensions [cm] and beam reinforcement method: a – with no stirrups in the middle zone; b – with stirrups at larger spacing in the middle zone; c – with stirrups at constant spacing along the entire length of beam

## 2.2. Program and test description

The beams were subjected to multiple repeatable loads: two concentrated loads applied on one third of the beam span. The ZD-40 PU testing machine was used. The loading variation frequency was constant at 6.67 Hz.

The loading cycle parameters adopted were:

- ▶ ratio  $\kappa$  of maximum bending moment in cycle ( $M_{\text{max}}$ ) to failure moment ( $M_n$ );  

$$\kappa = M_{\text{max}} / M_n$$

- ▶ stress ratio  $R$ , equal to minimum moment ( $M_{\min}$ ) to maximum moment ( $M_{\max}$ ) ratio;  
 $R = M_{\min} / M_{\max}$
- ▶ The values of bending moments  $M_{\min}$ ,  $M_{\max}$ ,  $M_n$  correspond to loads: minimum  $P_{\min}$ , maximum  $P_{\max}$  and failure load  $P_n$ .

The values of ratio  $\kappa = M_{\max} / M_n$  were adopted as 0.45 and 0.60. The stress ratio  $R = M_{\min} / M_{\max}$  was: 0.15; 0.30; 0.60. The values of failure moments  $M_n$  for particular types of beams were determined experimentally by static tests performed on identical specimens (identical reinforcement, identical concrete mix).

The deflections were measured at the midpoint of the beam spans (and in support sections for verification) with dial test indicators of 0,01 mm scale. The measurements were taken after a certain number of load cycles  $N_i$  for the minimum load  $M_{\min}$  ( $P_{\min}$ ) and maximum load  $M_{\max}$  ( $P_{\max}$ ) of the cycle. In the analysis, deflections measured for the cycle maximum load  $M_{\max}$  ( $P_{\max}$ ) were taken into account.

### 3. Analytical solution of the task

To begin the solution of the task, the recommendations of Eurocodes [9, 10] were adopted. In general, beam deflection  $a$  can be calculated from formula [9]:

$$a = \alpha_k \frac{M_{Sd,lt} l_{eff}^2}{B_{\infty}} \quad (1)$$

where:

- $\alpha_k$  – coefficient dependent on load configuration and support conditions,
- $l_{eff}$  – effective span,
- $M_{Sd,lt}$  – value of bending moment; for the in calculation of real deflections adopted as characteristic value;

$$B_{\infty} = \frac{E_{c,eff} I_{II}}{1 - \beta \left( \frac{M_{cr}}{M_{Sd,lt}} \right)^2 \left( 1 - \frac{I_{II}}{I_I} \right)} = B \quad (2)$$

- $\beta$  – coefficient (for ribbed bars  $\beta = 1.00$ ),
- $M_{cr}$  – cracking moment,
- $M_{Sd,lt}$  – moment of inertia of reduced cross section (uncracked cross section – phase I),
- $I_{II}$  – moment of inertia of cracked section (phase II),
- $E_{c,eff}$  – effective modulus of elasticity of concrete equal;

$$E_{c,eff} = \frac{E_{cm}}{1 + \phi(\infty, t_o)} \quad (3)$$

- $E_{cm}$  – secant value of concrete mean modulus of elasticity,
- $\phi(\infty, t_o)$  – creep coefficient (included in the case of long-time loads).

In this context, it should be emphasised that adopting the coefficient  $\beta = 0.5$  in formula (2) does not generate deflection calculation results compatible with those of experimental tests.

The effect of changing loads on the modulus of elasticity can be described by formula [3]:

$$E_{cmN_i} = E_{cm} \left( 1 - 0,30 \frac{N_i}{N} \right) \quad (4)$$

where:

$N_i$  – number of load cycles for which the value of modulus of elasticity is being determined,

$N$  – limit number of load cycles corresponding to concrete fatigue compressive strength.

In the case of the beams analysed, applying the given load parameters and the values of stress in the zone of concrete under compression, resulting from these parameters, this effect can be disregarded since  $N_i \ll N$ .

A detailed analysis of the effect of changing loads on RC beam deflection was preceded by a comparison of the values of deflections calculated from formulae (1) and (2) with the values of coefficients  $\beta = 1.0$  and  $\beta = 0.5$ . The corresponding bending moments were calculated for the geometric and load parameters characteristic of the tested beams. The ratio of the calculated deflections  $a(\beta = 0.5)/a(\beta = 1.0)$  did not exceed 5%, which was several times less than indicated by experimental tests. Consequently, it can be stated that the effect of changing loads on RC beam deflection cannot actually be taken into account by changing the value of coefficient  $\beta$  from 1.0 to  $\beta = 0.5$ .

Keeping valid the general formulation of the equation provided in norms, it would be easiest to include the effect of load variation on RC beam deflection by introducing an additional coefficient  $\gamma_f$  in the second term of the denominator in formula (2). The value of this coefficient should depend on the parameters and the number of load cycles.

A general formula for the calculation of  $\gamma_f$  was adopted as:

$$\gamma_f = 1 + b(1 - cR)^d \log N_i \quad (5)$$

where:

$R$  – stress ratio,

$N_i$  – number of load cycles,

$b, c, d$  – coefficients determined on the basis of experiment results.

The values of parameters  $b, c$  and  $d$  which would yield a good compatibility between calculated beam deflections and those obtained from tests could not be determined by way of an analysis of the tests performed by the author. Therefore, a formula describing the complete increment of deflections is proposed below. This formula takes the form:

$$\gamma = 1 + b(1 - R)^c (0,5 + \kappa)^d \log N_i \quad (6)$$

where (as in the previous formula):

$R$  – stress ratio,

$\kappa$  – maximum cycle load to failure load ratio,

$N_i$  – number of load cycles,  
 $b, c, d$  – coefficients whose values can be adopted from test results.

To determine the values of the coefficients in formula (5) the results of tests on group of beams whose load parameters were:  $\kappa = 0.25, 0.45, 0.60, 0.75$  and  $R = 0.15, 0.30, 0.50, 0.60$ . The results of tests on the other beams were used for the verification of the correctness of the final formula.

Using the test results, the values of parameters for formula (5) were determined as:  $b = 0.036, c = 1.18, d = 1.30$ . The final formulation was adopted:

$$\gamma = 1 + 0,036(1 - R)^{1,18} (0,5 + \kappa)^{1,30} \log N_i \quad (7)$$

Formula (7) must have a practical restriction, since at  $N_i \rightarrow \infty$  the value of the coefficient also increases to infinity. The number of cycles  $N_i = 10^7$  can be adopted as such a restriction. In the conditions of effective loads, this number is practically impossible to reach.

#### 4. Analysis of solution and its experimental verification

The results of the analysis of formula (7) are shown in Figs. 2–6.

Figure 2 illustrates the experimental verification of formula (7). ‘The results of tests on the beams used for verification (those that were not employed for the determination of parameters for formula (6)) are indicated on the graphs plotted on the basis of this formula.

Figures 3–6 refer to the effect of particular parameters of load cycles and their number on coefficient  $\gamma$ . The graphs in Fig. 3 indicate a deflection increment with increases in the number of load cycles  $N_i$  at higher values of ratio  $\kappa$  (for a given value of  $R$ ). The effect of stress ratio  $R$  is inverse; the lower its value, the greater the effect of changing loads on RC beam deflection, which is proved by the plots in Fig. 4.

On the basis of the analysis results presented graphically in Fig. 5, it can be stated that the deflection increment after a specified number of load cycles  $N_i$  (in Fig. 5,  $N_i = 10^6$  was adopted) increases with increases in the value of ratio  $\kappa$ . On the basis of the plots in Fig. 9, in turn, it can be concluded that with the increase of  $R$ , the increment of RC beam deflections under changing loads decreases.

#### 5. Remarks and final conclusions

The aim of this paper was to propose a formula for including the effect of changing loads on the deflection of RC beams reinforced with ribbed bars. The proposed formula was verified in experimental tests performed by the author (see Fig. 2). The test scheme was also employed for the specification of the values of coefficients for equation (6). It should be emphasised that for the calibration of the coefficients, the results for different beams were used than for the assessment of the correctness of formula (7). However, the correctness of formula (7) can be verified by the results of independent investigation.

Regardless of the degree of formula (7) accuracy with regard to quantity, it can be used as a basis for the formulation of unequivocal remarks and conclusion in terms of quality. The analysis of graphs and results of tests shown in Figs. 2-6 proves that the increment of the deflection of RC beams reinforced with ribbed bars under changing loads is affected by parameters ( $\kappa = M_{\max}/M_n$ ,  $R = M_{\min}/M_{\max}$ ;  $M_{\min}$  – minimal moment of cycle,  $M_{\max}$  – maximal

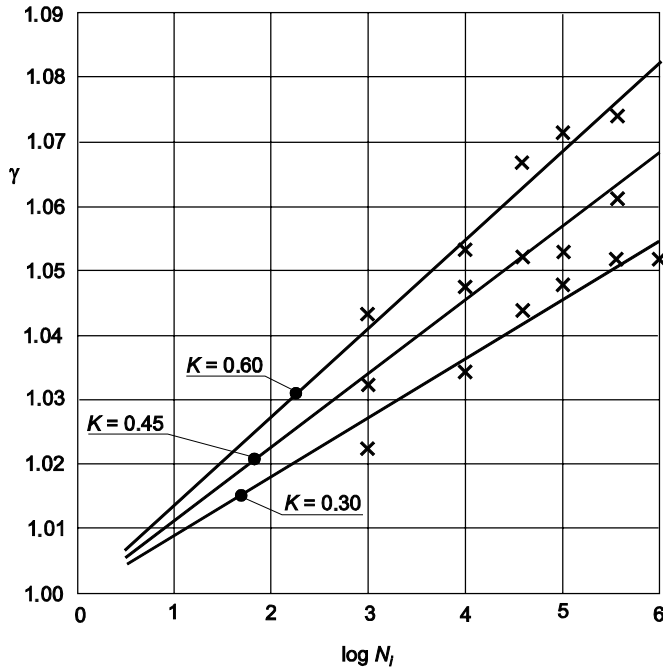


Fig. 2. Experimental verification of formula (7)

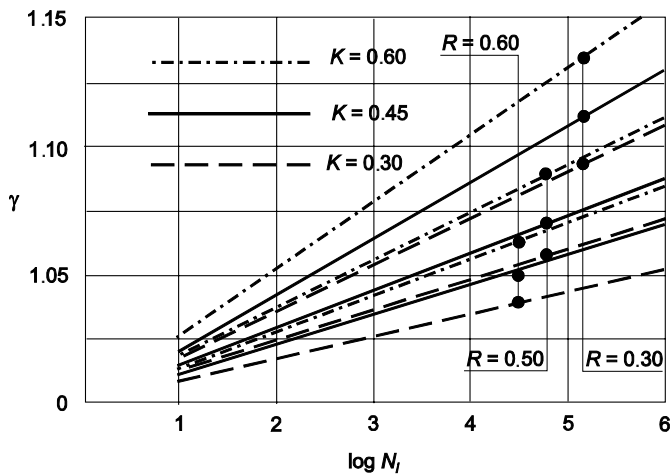


Fig. 3. Effect of the number of load cycles  $N_i$  and ratio  $\kappa$  on deflection increment

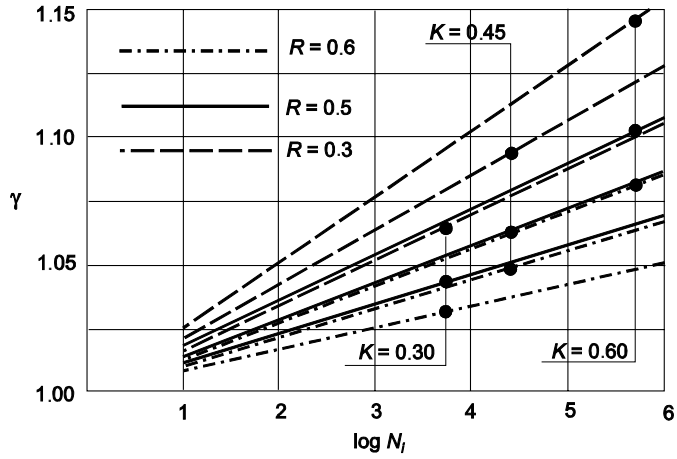


Fig. 4. Effect of the number of load cycles  $N_i$  and ratio  $R$  on deflection increment

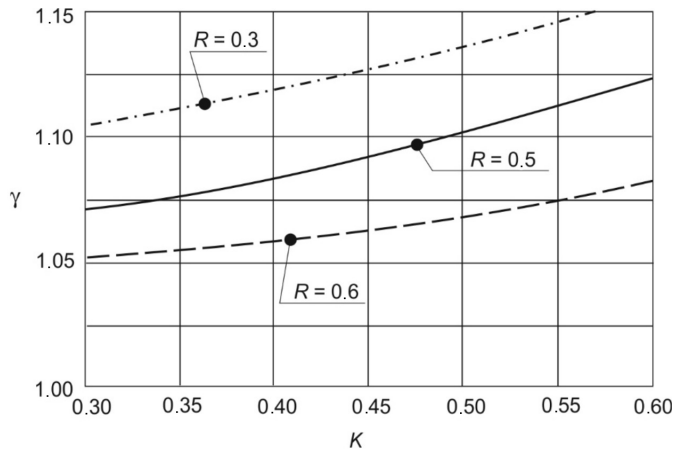


Fig. 5. Effect of ratio  $\kappa$  on deflection increment for some values of ratio  $R$  for  $N_i = 10^6$

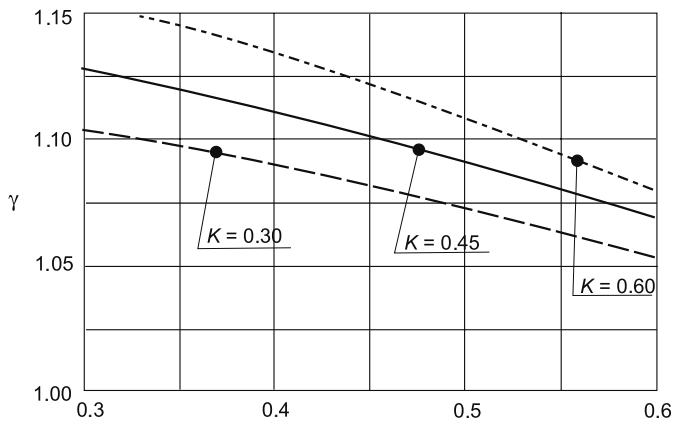


Fig. 6. Effect of ratio  $R$  on deflection increment for some values of ratio  $\kappa$  for  $N_i = 10^6$



moment of cycle,  $M_n$  – failure moment under short-term loads) and number of load cycles  $N_i$  (to be more precise,  $\log N_i$ ).

The increment of deflection of RC beams reinforced with plain bars subjected to changing loads increases with increases in the number of load cycles  $N_i$  ( $\log N_i$ , to be more accurate), and is greater for higher values of ratio  $\kappa$  (see Fig. 3) and lower values of ratio  $R$  (see Fig. 4). The relationships for the number of load cycles  $N_i$  are similar (see Figs. 5 & 6).

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