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USE OF NUMERICAL MODELS TO FORECAST THE PHENOMENA'S DYNAMICS IN A STORAGE RESERVOIRS

WYKORZYSTANIE MODELI NUMERYCZNYCH DO PROGNOZOWANIA DYNAMIKI ZJAWISK ZACHODZĄCYCH NA ZBIORNIKACH RETENCYJNYCH

Abstract

This article deals with applying numerical finite element models to forecast the phenomena's dynamics in retention reservoirs. Due to the fact of diversity and the dynamism of these phenomena on storage reservoirs, it is important to predict them for a good management of these objects. Historical observations can be a good source of clues for proceeding in a specific case. It is not directly used to all the possibilities. It is important to use a tool, which can estimate probable scenarios. Thanks to the use of simulation programs, it is possible both to predict physical and chemical processes, and to use the solutions to forecasts and research plans.

Keywords: numerical modelling, retention, storage reservoir, hydrodynamic, water quality

Streszczenie

W artykule omówiono zastosowanie modeli numerycznych bazujących na metodzie elementów skończonych do symulacji dynamiki zjawisk na zbiornikach retencyjnych. Ze względu na różnorodność zjawisk, wzajemne powiązania i złożoność dynamiki ich przemian, istotna z punktu widzenia zarządzania akwenem jest ich predykcja. Obserwacje historyczne są niezbędnym źródłem informacji, ale mają ograniczony zakres. Nie można ich wprost przełożyć na całe spektrum zjawisk, ich źródeł i konsekwencji. Ważne jest wykorzystanie właściwych narzędzi, które umożliwiają symulacje dynamiki dla prawdopodobnych scenariuszy. W przypadku akwenów istotna jest ocena zmian fizycznych i jakościowych.

Słowa kluczowe: modelowanie numeryczne, retencja, zbiornik retencyjny, hydrodynamika, jakość wody

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1. Introduction

In Poland, retention reservoirs built on rivers perform several economic functions. Most of these lakes are located in the upper basin of the Vistula and Oder rivers. Some of the reservoirs that are filled with water all the time perform multiple tasks at once like: water supply, flood flow protection and electricity generation. These reservoirs are multifunctional or at least bifunctional. This conjuncture may cause problems because of the first two functions which contradict each other are flood protection and power generation. For the first one, the best solution is maintaining the water surface on the low level, but for the second one conversely – the highest possible level of water is desirable. Most often, the maximal flood reserve isn't maintained because of the profits from the power plant and the water level is lowered only to prepare the reservoir to capture the flood wave. In that kind of situation, the basis is to know how long it takes for emptying or filling in the given values of inflow and outflow. Moreover, the information on when the lake emptying should start and how big outflow should be set is useful.

The second challenge, concerning the reservoir structure and morphology, is deposition of the sediment in the bottom of the lake. On the one hand, the problem can be the reduction of the total water body volume, but on the other, it could have an impact on chemical and biological processes, and ultimately, it may have an effect on the water quality. Therefore, the significant information is: where the particles are deposited; which conditions are appropriate to transport, re-transport and activate them; as well as the information about chemical substances being released. Similar problems could appear when the pollutions are dissolved in water. The propagation of this kind of contamination gets much faster and reaches larger zones. In addition, when the water supply inlet is situated within a lake, the danger is doubled: threat for the environment and for humans. A very important task is the possibility of prediction the way and time of pollution propagation as well as the potential places of accumulation, which can be the source of the eutrophic processes. When the water surface isn't frozen (most time of the year), it is easier to observe the flow dynamics and the transport process. When the lake is ice covered, it is almost impossible. Therefore, all of the mentioned phenomena are very weakly recognised in those conditions.

Correct resolving of the mentioned problems, and providing good ecological potential in a long period of time, requires the support of management of reservoirs using appropriate means to simulate the dynamics of these reservoirs for different scenarios involving: inflow conditions, as well as climate and other changes related to water quality and biology.

2. Background

The proper management of the water storage reservoir requires a broad and detailed knowledge about phenomena diversity and water dynamics. Prediction of the retention reservoir behaviour in a variety of scenarios is a fundamental problem, and also, very difficult one. Modern numerical techniques may be helpful in that matter. Numerical models shall be used for the description of the phenomena dynamics.

This work is based on two methodical and numerical aspects:

1. Approximation of the flow area and its description using the finite element method (FEM). FEM is known and commonly used in many engineering issues related to the simulation of non-stationary processes, such as: tension structures, thermal conductivity or fluids flow. FEM allows for correct mapping of the velocity field of the water surface with variable spatial accuracy dependant on topographic complexity of the water body.
2. Using two-dimensional (planar) equations of unsteady flow. They are approximated to FEM use for describing complex dynamic phenomena like tracing water and sediment transport or other similar fluid dynamics issues. Two-dimensional models – provided right boundary conditions setting, especially at the dam outflow – are appropriate for dynamic simulations of water storage reservoirs. They were successfully used for identifying open and closed dynamic structures within a water body [3].

Numerical models, used in this paper, are included in the SMS package. The Surface-water Modelling Solution (SMS) is a comprehensive simulation environment for hydrodynamic modelling [1]. It allows engineers and scientists to visualise, analyse and understand results of performed simulations easier. The numerical models supported by SMS are able to compute a variety of parameters applicable to surface water modelling. Primary applications of the models include calculation of the water surface elevations and flow velocities for steady-state and dynamic conditions. Other applications include the modelling of: rivers behaviour, contaminate transport, sediment transport, rural & urban flooding, estuarine, coastal circulation, inlet and wave modelling.

A great advantage of SMS is flexible modelling approaches. This simulation environment allows for building conceptual models by using GIS familiar objects like points, arcs and polygons. The conceptual models are high-level and do not rely on meshes and grids. They only need definition of parameters such as bathymetric data, flow rates, boundary conditions or water surface elevations. Moreover, SMS also includes tools that help building computational meshes and grids including their automatic generation and optimization. SMS also provides tools for result analysis and visualisation. This makes results understanding easier and allows involved people to focus on the merits of their problem. Thanks to SMS's support for many computational models, it gives a possibility of cross-comparison of the simulation results obtained with different models [1, 8].

The models used in this paper are AdH [2,15], RMA [4,10] and PTM [6]. All of them belong to the group of two-dimensional depth averaged (also known as “2.5-dimensional”) finite element method modelling tools.

The basic input parameters for this model are:

1. Bed shape (bathymetry).
2. Bed friction parameters (they may be expressed as the roughness coefficients or as the equivalent bed roughness height).
3. Initial water surface level.
4. Water parameters (density, viscosity – the later may be depth dependent).
5. Inflows and outflows (constant, time dependent or surface level dependent).

The models support many other case-specific initial conditions and parameters, for example: wind conditions, atmospheric pressure changes, Coriolis Effect or fixed ceilings (e.g. bridges).

Depending on the performed simulations, appropriate additional parameters must be defined:

- sub-glacial flow – the parameters of the ice cover (its range, density, thickness and roughness) – AdH;
- sediment transport – the parameters of – the particles of the sediment – PTM;
- pollution dissolved in water – the parameters of substances – RMA4.

The main result of the simulation is a two-dimensional planar field of the average horizontal velocity of water. Vertical movement and most of other vertically dependant phenomena are not taken into account [16].

3. Characteristic of the objects of research

The area of research is three retention reservoirs located in southern Poland: Dobczyce, Tresna and Porąbka (Fig. 1). All of these multifunctional lakes have a mountain character and have been made by constructing a dam in a river valley.

The Dobczyce reservoir is situated in Lesser Poland Voivodeship. The lake was created by building a dam on the 60th km of the Raba River, which is one of Vistula river tributaries. The surface of the catchment area is 768 km². The knowledge about pollution propagation in this reservoir is as significant as a function of the Dobczyce Lake itself: the lake is the

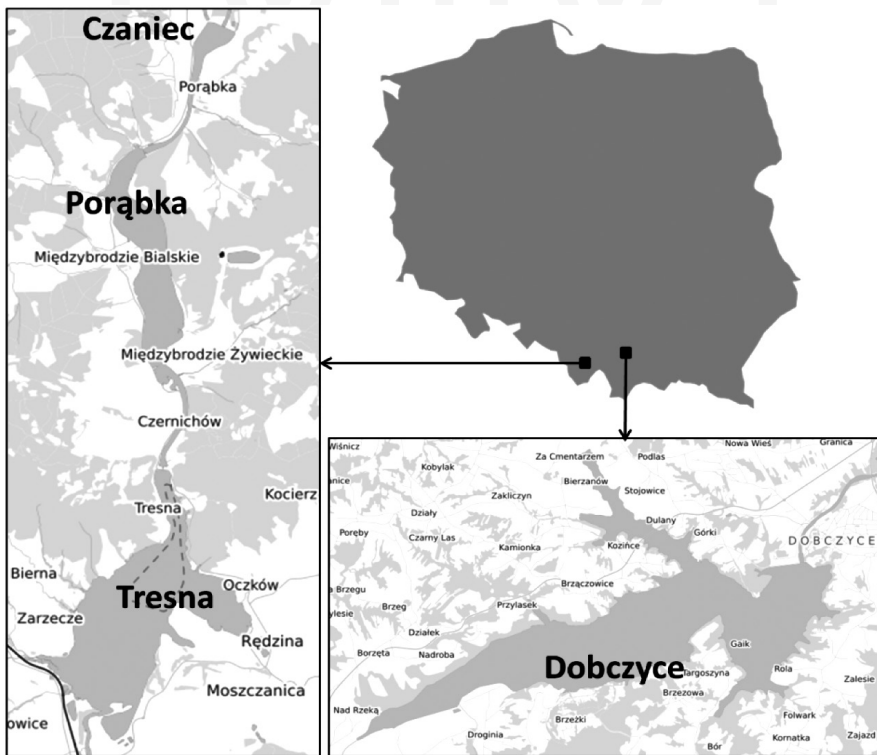


Fig. 1. Geographical location of the reservoirs [1]

primary water source for the nearby city of Cracow. Therefore, the water quality issue is the priority and presently all the lake area is under protection. However, the recreation aspect is discussed now. It is certain that some kind of recreation will be allowed there in the future. Thus, the potential sources of pollution and its contamination propagation should be under control.

The other reservoirs – Tresna and Porąbka are two of three of the Soła Cascade reservoirs located on the Soła River in Silesian Voivodeship. Soła is a right hand tributary of the Vistula River. Soła Cascade is a system of three reservoirs, of which one of the main functions is to provide drinking water for Silesia, Bielsko-Biała, Oświęcim and Kęty, and to provide water to factories and fish farms. The reservoirs are also used to produce hydroelectric power and to mitigate exceptional floods. Reservoirs can reduce the maximum flood flows from 1469 m³/s to the 650 m³/s.

The Tresna reservoir is a mid-size retention reservoir and is the first of three reservoirs forming the Soła Cascade. It is also called the Żywieckie Lake. It has one main inflow – the river – and one outflow located by the dam. The dam is located on the 40th km of the river. Surface of the catchment area is 1 030.0 km².

The Porąbka reservoir, the second one in the system, is also called the Międzybrodzkie Lake. It's the oldest reservoir in the system (built in 1936). The dam is located on the 32.3rd km of the Soła River. Surface of the catchment area is 1 082.0 km². The reservoir is long but not very wide. The Porąbka's reservoir has two hydroelectric power plants. The first one is located on the dam. The second one is located on the eastern shore. This is the second biggest pumped-storage hydroelectricity in Poland.

4. Results

This chapter outlines some representative results of the performed simulations of the phenomena, which take place on the storage lake – Dobczyce. The basis for estimation of all the processes related with the water flow is the two dimensional velocity fields. The map (Fig. 2) shows the direction and magnitude of the velocity vector. The simplest simulation needs information about: the lake bathymetry, inflow discharge, outflow discharge or water surface level and bottom roughness. These parameters can be constant or time varying (excluding the bottom roughness – the parameters describing a material are always set as constant value for the whole simulation). Additional parameters, which can be taken into account, are the factors connected with the environment. The first one is the wind direction and velocity, which in nature is time variable. This effect can have a big impact on the water flow velocity and direction in reservoirs. The simulation result can also include local surface sources of water. By this option, it is possible to take into account local evaporating or rainfall in a given part of the lake. The figure below presents water velocity field for the Dobczyce Lake obtained by AdH simulation model included in SMS package.

The next challenge, especially for the lake with a water supply function, is water quality. Firstly, it can be directly effect of contamination, which enters the water. Secondly, the danger can be caused by substances, which are affected by chemical processes in the appropriate conditions. For example, the eutrophic process is a common phenomenon in small water

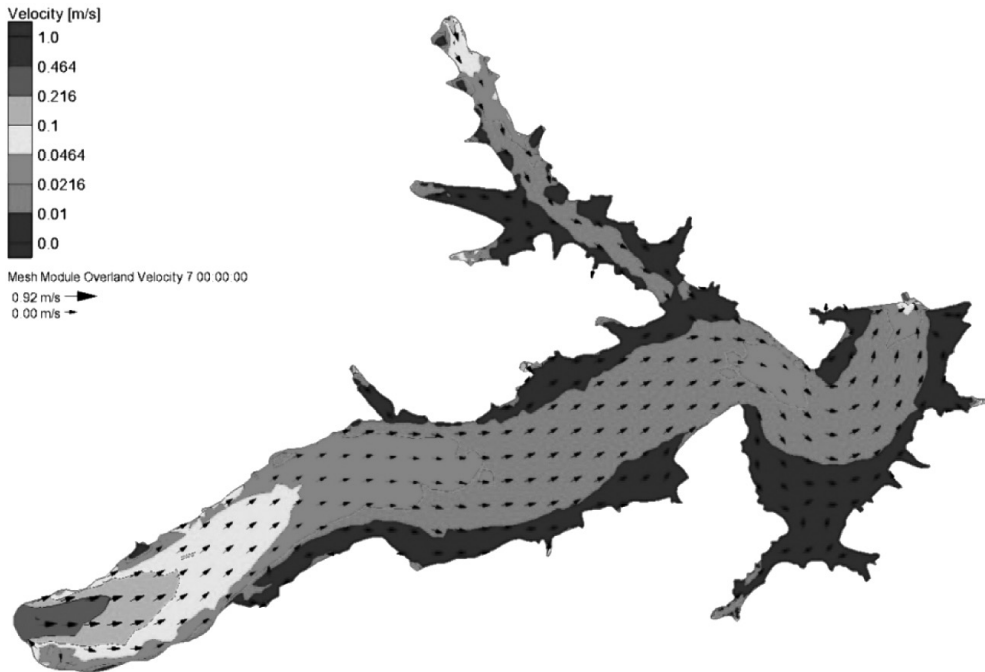


Fig. 2. The planar velocity field for water surface level of 272.6 m a.s.l. and high water flow ($500 \text{ m}^3/\text{s}$) in the Dobczyce Lake

movement places like hollows. Those zones are easy to recognise in situ or can be identify within the solution calculated in AdH, RMA2 [4] or another hydrodynamic model. The situation is more complicated when considering the paths of the dissolved contaminants particles. In this case, there are a lot of variables, which control the processes advection and diffusion in water environment. Beside the parameters describing water dynamics, it is necessary to include data representing the pollution character. The first step is to give some information about the moment when contamination enters and specify the place in reservoir area as well as time duration of potential slick. Next point is to describe the pollutant itself by the following factors: concentration of a given constituent, turbulent mixing coefficient and first order decay coefficient of a given pollutant.

The simulation presented in Fig. 3 was performed under an assumption of rapid leakage of constituents that have the eddy diffusivity parameter ($0.05 \text{ m}^2/\text{s}$) [5, 9, 13] suitable for “industrial” constituents like for example mineral oils. They origin from the zone where industrial-originated contamination may eventually appear: a haven located near the end of the Wolnica bay in the Dobczyce Lake. The presented results contain seven days long simulation and include wind impact taken from real life data of one march week where the most frequent were the southern and western winds.

The total mass of the leaked pollutant is assumed to be ten tons. In this case, it enters the lake during only three-hour event. It results in 926 g/s mass load. The end of the leakage

marks the “zero” moment. Figure 3 presents the propagation of contamination that appeared in place where Wolnica bay connects with the main part of the Dobczyce reservoir. In the beginning, the pollution only disappears in place near place of its sources. Next, after reaching the main basin, the substance unexpectedly is turned back in western part of the lake in direction of Raba inflow. The result, where the polluted substance flows in the direction where the dam outflow is located, would be more expected. However, such a result shows that the wind conditions may be the reason of dynamic situation of flow direction in the water reservoir. During that time, the contamination is dissolved in water. In the part of Fig. 3, which represents the situation after 8 days, it is easy to see that the stain of substance changes direction once again. After 12 days, significantly lower concentration of contamination fill the middle part of the lake and follow east to the dam.

The results of this kind of calculation give information about the potential way of the contamination in the given conditions. It may also show the concentration of pollution in each point as well as present the process of decay of the given substance in time and space. This data may be used in the prediction and analysis of the risk to strategic points on the reservoir, like water supply inlet. That kind of calculation works on two levels. The first one is a hydrodynamic base calculated by RMA2 including important parameters like: inflow, outflow, wind impact etc. The second layer is RMA4 [10] model, which is based on the RMA2 solution. In this step, all data about constituents are being set to characterize potential behaviour of each of the substances in the water environment with specific terms.

The ice cover, appearing on the storage reservoir in the winter, may impact water quality. The simulation results confirm that the ice covering the reservoirs has a significant impact on the water dynamic. Ice cover reduces water velocity, which creates stagnant areas. In addition, the ice cover has a significant impact on reducing water mixing in a storage reservoir [7, 11, 14].

Fig. 4 shows the velocity maps generated by the model for main inflow of $5\text{ m}^3/\text{s}$ and $25\text{ m}^3/\text{s}$ for two scenarios. The map on the left shows the flow velocity field in the reservoir Porąbka, when the water surface is free from ice. Next is a map that presents the scenario when the reservoir is covered with a 0.1 m thick layer of ice. To emphasize the significant impact of the ice cover thickness, the map on the right shows the simulation results for $0,5\text{ m}$ ice cover thickness.

It can be seen that increasing the ice thickness damps the flow velocity. Side currents and whirlpools, that are present for the free surface scenario, tend to disappear when the ice cover is introduced.

A similar situation is presented in Fig. 5. In this case, the flow velocity map was prepared for the Tresna reservoir. As in the previous case, the flow velocity map presents solutions for 0.1 m and 0.5 m values of ice cover thickness. But, in this case, simulations were prepared for $25\text{ m}^3/\text{s}$ and $50\text{ m}^3/\text{s}$ main inflows.

Also, in this example, the velocity field differences are visible. They are smaller in comparison to the Porąbka Lake. The most significant differences in flow velocity fields are visible in the south region of reservoir. It is a place where main inflow flows into the reservoir.

In both cases (Porąbka and Tresna reservoirs) more visible difference concern simulation for lower velocity of the main inflow. In this case, the impact of the roughness coefficient is less significant. In addition, comparing the simulation solutions for the Porąbka and Tresna reservoirs, greater changes are noticeable in the Porąbka reservoir. The Porąbka reservoir

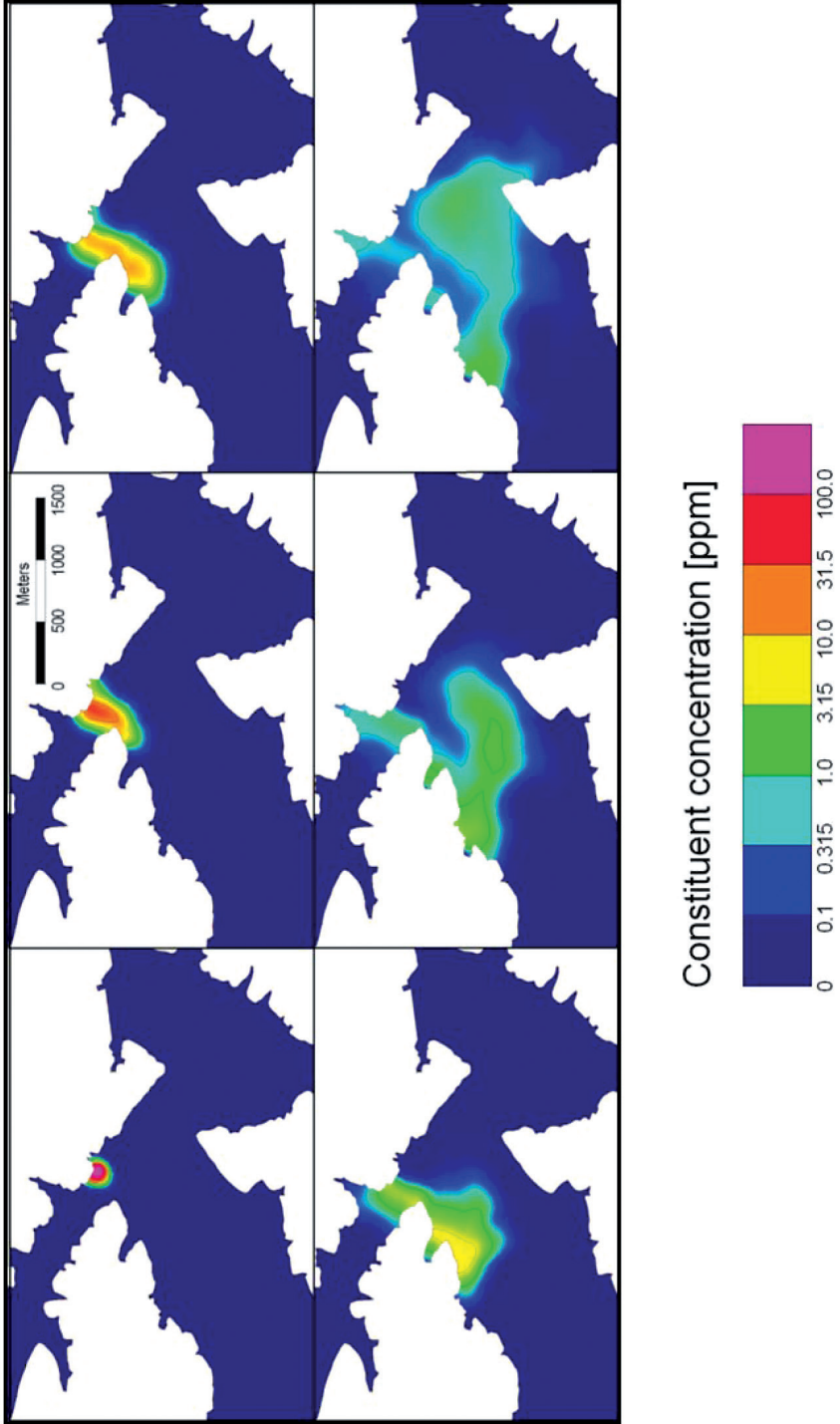


Fig. 3. Process of transporting simulated 3-hour leakage from position the haven after 5h (upper left), 1d (upper middle), 2d (upper right), 4d (lower left), 8d (lower middle) and 12d (lower right) after the “zero” moment

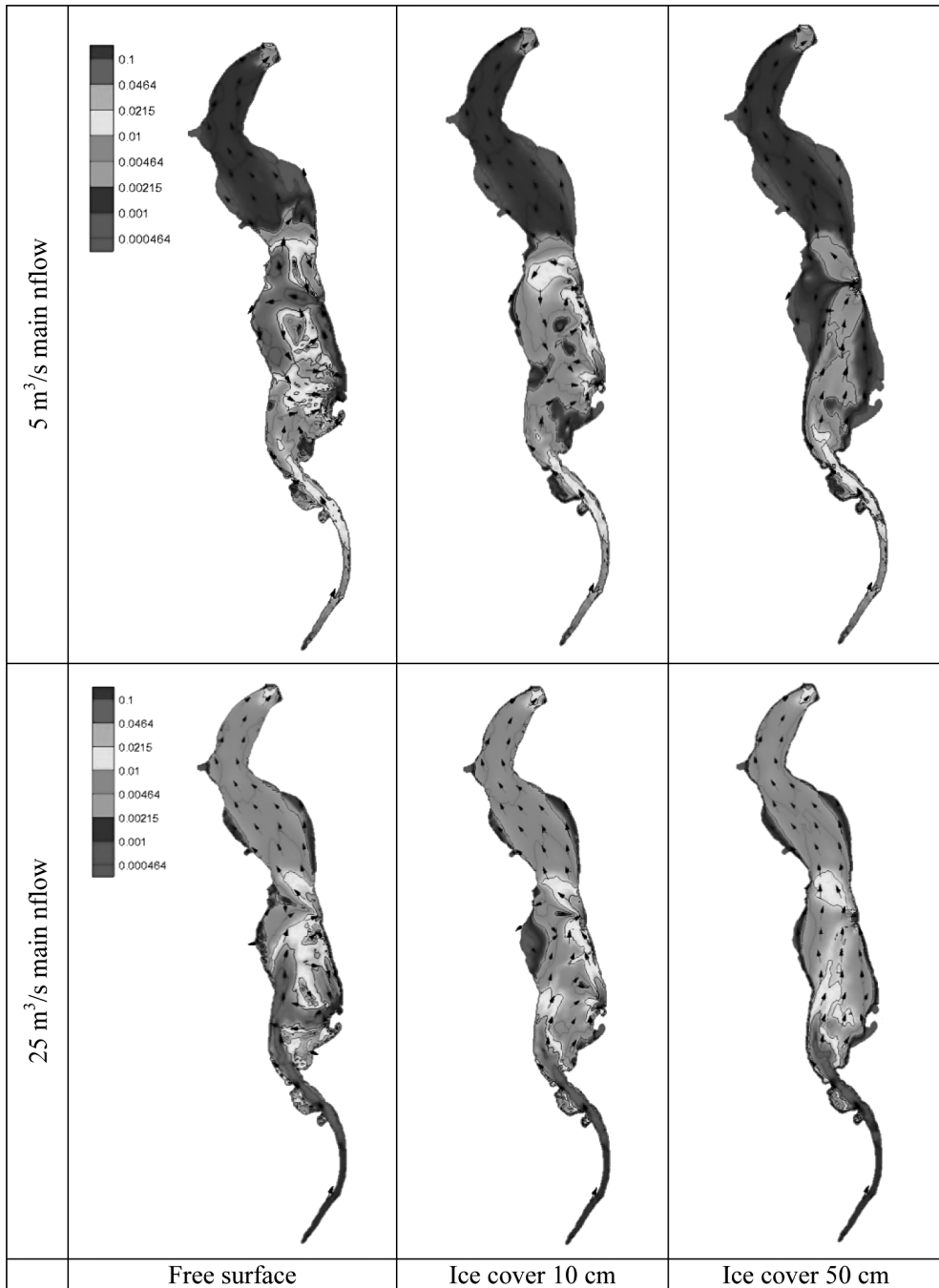


Fig. 4. Flow velocity map comparison for 5 and 25 m³/s main inflow for a different scenarios (with different thicknesses of ice)

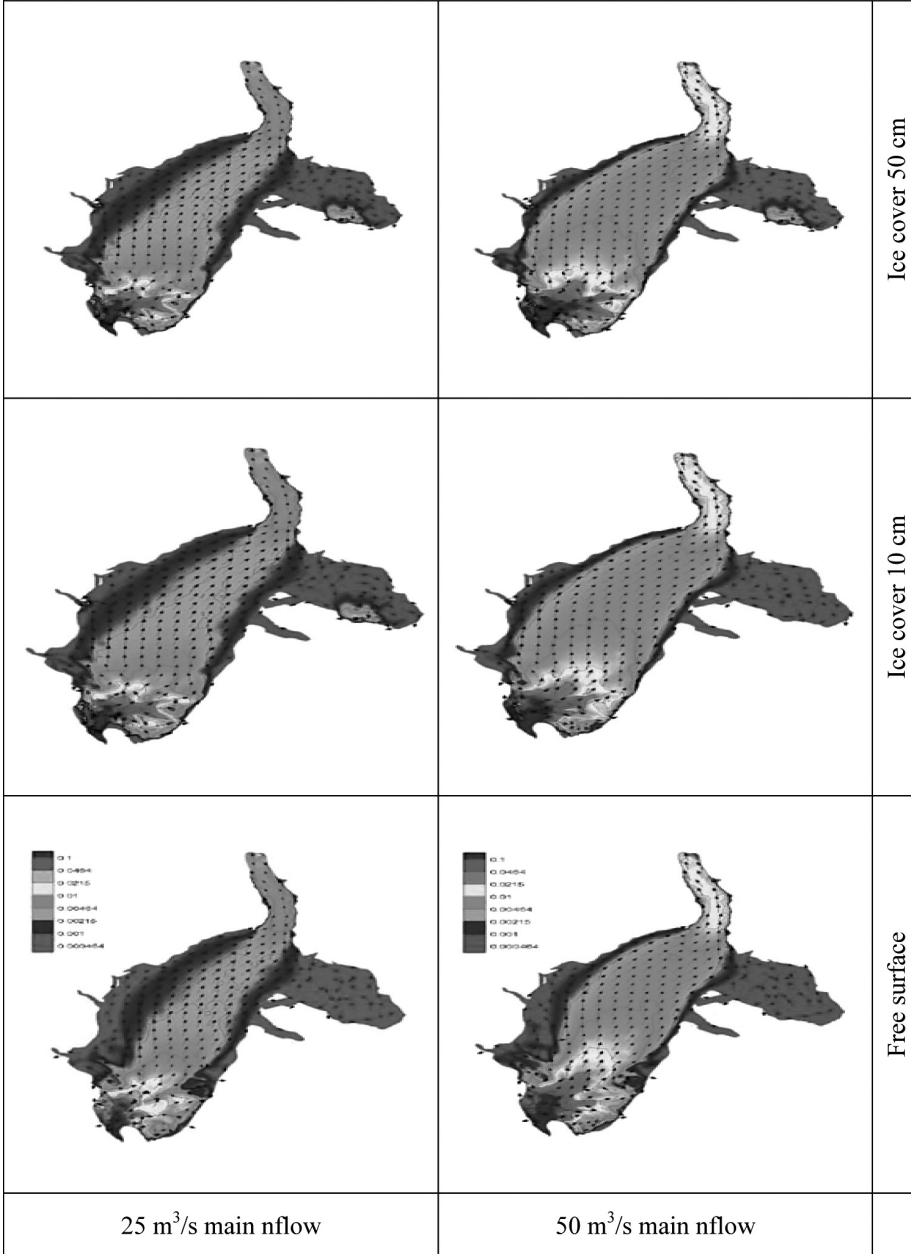


Fig. 5. Flow velocity map comparison for 25 and 50 m³/s main inflow for a different scenarios (with different thicknesses of ice)

depth is much less than in the second reservoir. It is the next factor affecting the dynamics of the flow water under the ice cover.

Summing up the impact of ice cover for hydrodynamics situation in storage reservoir is significant. It changes flow velocity and its direction. The presence of the ice layer causes lake stagnant areas. The currents tend to straighten and whirlpools tend to disappear. Identifying the sub-ice stagnant zones is important because of water quality.

The next challenge, especially for the retention reservoir with flood protection function, is water surface elevation control. The appropriate water surface elevation is determined by the current hydro-meteorological situation. Control of the reservoir operation in flood conditions is a difficult task. It requires keeping a water level that allows to capture a flood wave. This is the reason why it is necessary to know how long it takes for drying and filling the reservoir.

Fig. 6 presents the process of drying and wetting the Tresna reservoir at the rate of $40 \text{ m}^3/\text{s}$ and $200 \text{ m}^3/\text{s}$ for two different water surface levels: 342 and 337.8 m a.s.l. Two figures at the top present the process of drying the storage reservoir for $10 \text{ m}^3/\text{s}$ inflow (left) and $50 \text{ m}^3/\text{s}$ outflow (right). Water level goes down from 342 m a.s.l. to 337.8 m a.s.l. Next water level goes up from 337.8 to 342 m a.s.l. for the same flow rate. This whole process takes almost 600 hours of the model time. The first 24 hours have been added to let the model find a stable operating point, while the remaining time present the all cycle of drying and wetting.

A similar situation was presented on the figures at bottom of the table. The difference is that the inflow value is $10 \text{ m}^3/\text{s}$ and the outflow value is $210 \text{ m}^3/\text{s}$ for drying process. And for the wetting process, inflow is $210 \text{ m}^3/\text{s}$ and outflow is $10 \text{ m}^3/\text{s}$. This time, the whole process takes 200 hours of the model time where the first 24 hours was added to find a stable solution.

The possibility of modelling of storage reservoir wetting and drying process is very helpful. It allows modelling of different scenarios in short time. It gives the knowledge about parts of the lake that was dried and the location of stagnant areas and in next step which area was flooded and how long this process takes.

Fig. 7 shows intermediate steps in the wetting process for:

- water levels from 337.8 to 339 m a.s.l. (first figure);
- water levels from 337.8 to 341 m a.s.l. (second figure).

Inflow value is $50 \text{ m}^3/\text{s}$ outflow value is $10 \text{ m}^3/\text{s}$. Total flow value is small and it is $40 \text{ m}^3/\text{s}$. The wetting process is slow and takes 65 hours so almost 3 days for first figure and 183 hours so more than 7 days in case of the second figure. Water level difference is respectively 1.2 and 3.2 meters.

The similar situation is presented in Fig. 8. It shows wetting of the bed at the same water levels as before (339 and 341 m a.s.l.) at the flow rate of $200 \text{ m}^3/\text{s}$ ($210 \text{ m}^3/\text{s}$ inflow and $10 \text{ m}^3/\text{s}$ outflow). In this case raising the water level about 1.2 meters takes 14 hours and analogously raising about 3.2 meters takes 37 hours. The wetting process is much faster in this case than in the previous case. In addition this situation can predict the results of flood wave entrance into the reservoir.

The process of drying storage reservoir is presented on Fig. 9. It shows fast drying intermediate steps for:

- water level from 342 to 339 m a.s.l. (figure on the left);
- water level from 342 to 341 m a.s.l. (figure on the right).

Flow value difference is significant and it is $200\text{ m}^3/\text{s}$. Inflow value is $10\text{ m}^3/\text{s}$ and outflow value is $210\text{ m}^3/\text{s}$. In the first case, the drying process takes 70 hours and water level difference is 3 meters. In the second case, it lasts 24 hours and the water level difference is 1 meter. In the both cases, the water level drops very fast. This case can reflect the situation when there is a need to quickly empty a reservoir. It happens when a flood wave is coming to the lake. The storage reservoir task is to catch a flood wave, in order to protect a threatened area. The simulation solutions give knowledge about the drying time, which helps elaborate the procedure in similar situation.

The other problem, in case of reservoirs, is the movement of the sediment, which can cause many problems. First aspect is the displacement of the bottom sediment, which is the reason for deepening and shallowing, especially in the places where rivers enter the lake. The water is able to grab the particles of the sediment from the bottom of the lake or river (if the water speed is high enough to grab the particles with given characteristic feature: like size and mass), transport it to another part where floating forces are not so big. The sediment can be deposited, but in other condition, it can be removed once again. It is possible to estimate these

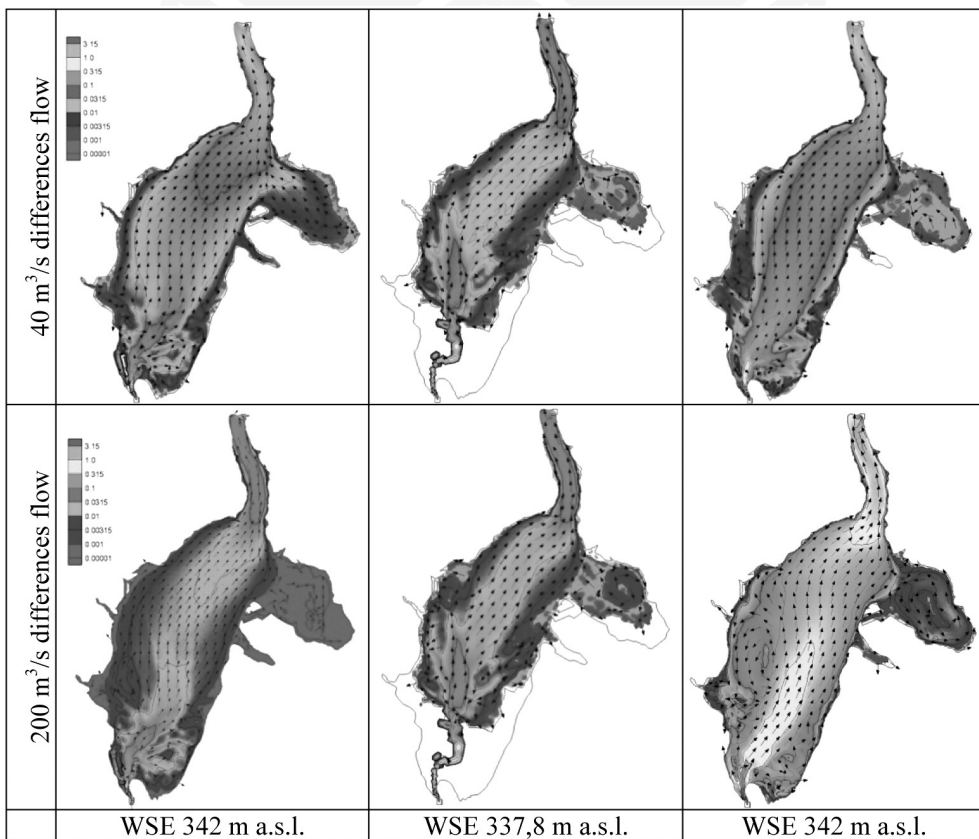


Fig. 6. Unitary flow maps for transient water level from 342 m a.s.l. through 337,8 m a.s.l. and again to 342 m a.s.l. for two different flow

phenomena by the modelling program called PTM [12]. Simulations made in this model give information about the potential places prone to erosion as well as sedimentation. Another solution, available in PTM, is tracking the way of particles introduced to the reservoir. It can be done by choosing the places of sediment source and set basic information about the particles. Fig. 10 presents simple solution of bed displacement for constant inflow of water

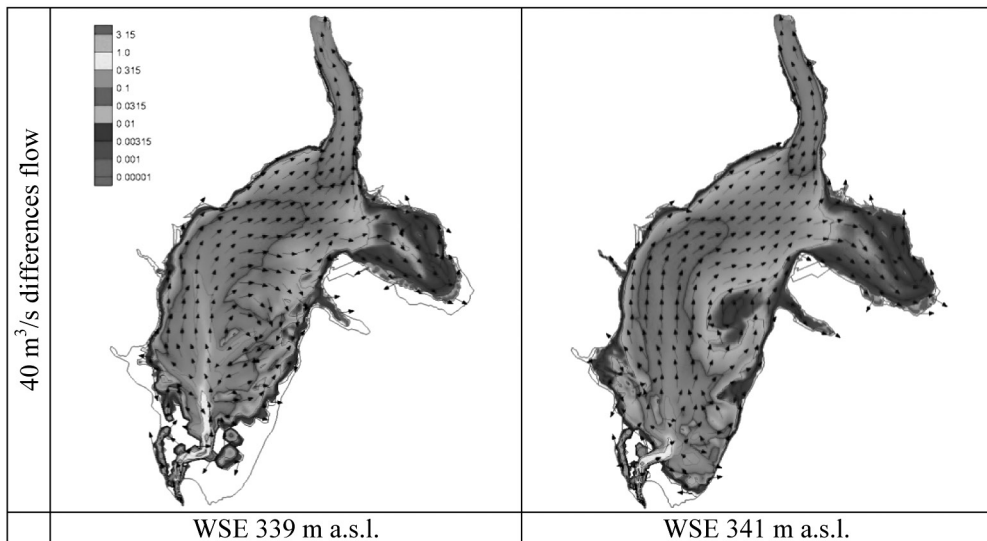


Fig. 7. Unitary flow maps for two different water level for case when the reservoir is slowly wetting

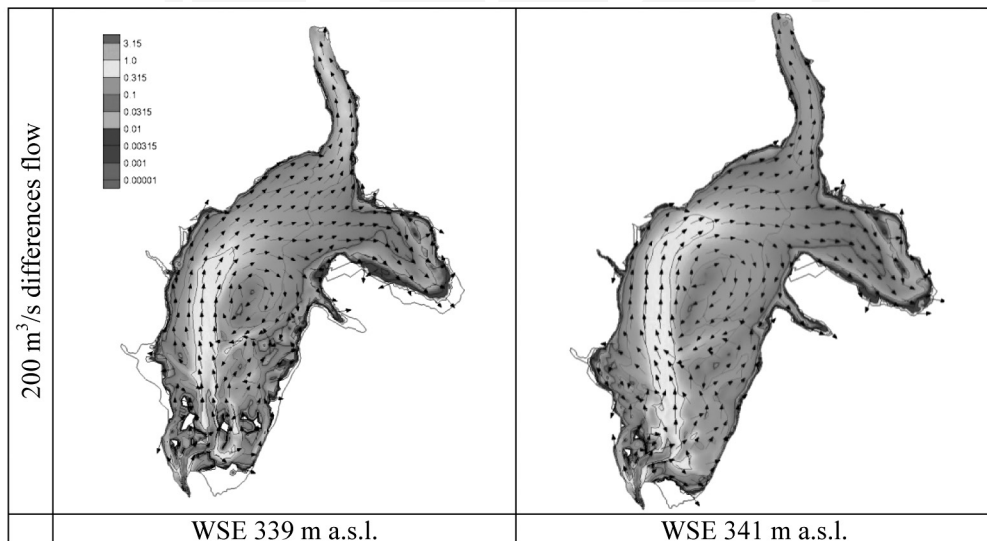


Fig. 8. Unitary flow maps for two different water level for case when the reservoir is fast wetting

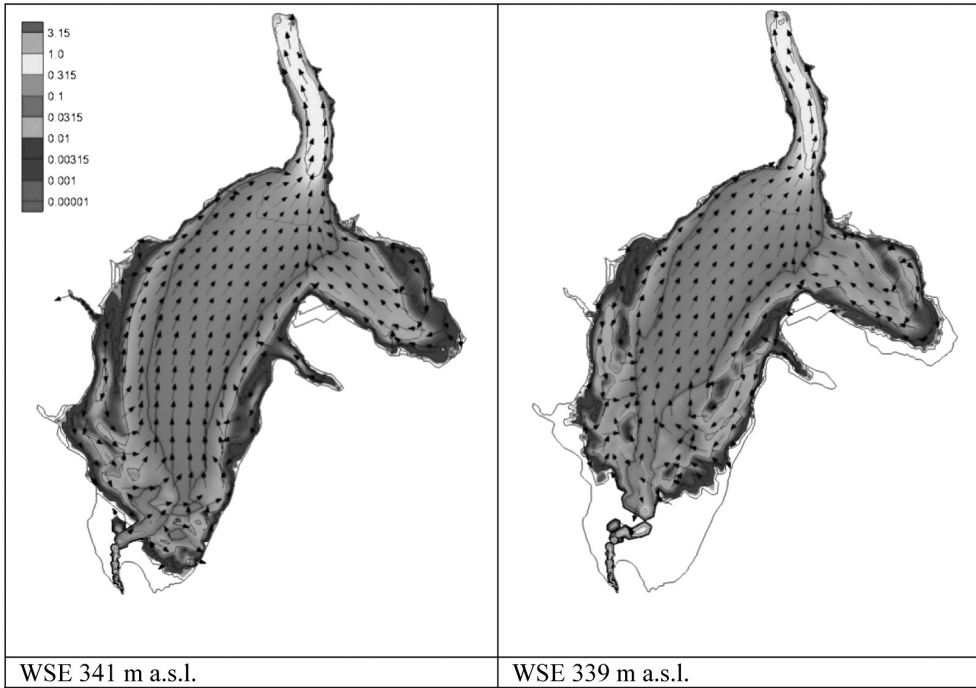


Fig. 9. Unitary flow maps for two different water levels for the case when the reservoir is drying for $200 \text{ m}^3/\text{s}$ differences flow

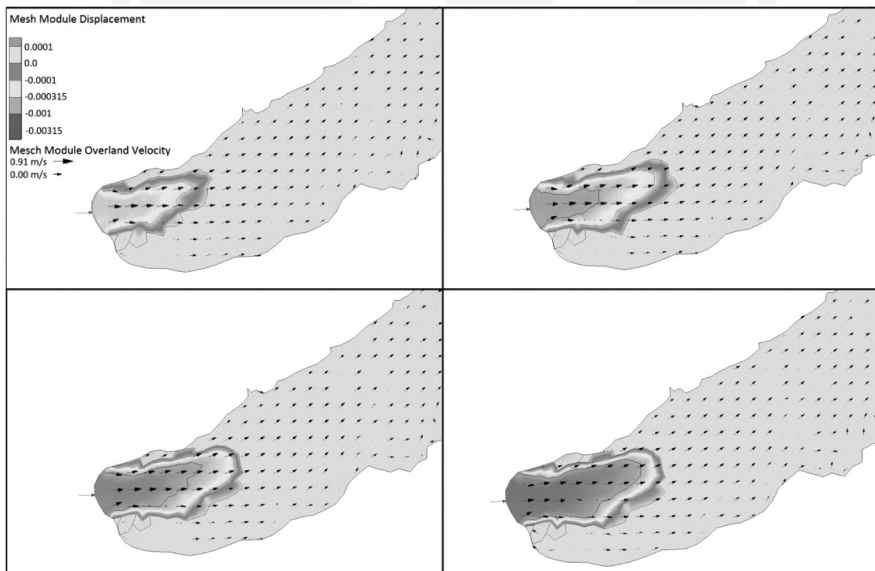


Fig. 10. Process of displacement of the bed in the first part of the Dobczyce Lake after 2d constant inflow (upper left) after 3d (upper right), 4d (lower left), 5d (lower right)

with a discharge of about 500m³/s for five days. It is necessary to mention that this model is in a phase of testing by the authors and the presented result is only an example of its possibilities.

5. Conclusions

The obtained results indicate that, by using the numerical models, it is possible to forecast dynamic phenomena in storage reservoirs and can this be used as a clue to planning in both standard and exceptional operating conditions of the reservoirs. The above-mentioned simulation possibilities can help to choose the appropriate solutions for the physical and quality problems as: pollution propagation, sediment accumulation and transport, flood flow capture preparation, ice cover impact on the water dynamic and water quality.

The numerical solutions are stable and physically justified. The simulation solutions can't be treated like an exact answer for the real problems. It can be only a clue for a good management the reservoir. It is obvious that field research is always recommended and it gives the best view of the situation. However, in large-scale research, as it take place on the water reservoirs, it's often hard, expensive and sometimes even unenforceable. In those cases, only a well-known simulation solution area may outline real the situation.

Nowadays, the aspect of water quality is the priority issue. Because of a large human population, more attention should be put to guarantee basic needs. This problem can't be solved only by using traditional approach. Using the modern simulation techniques is necessary and inevitable. The presented solutions are only a simple example, which focuses on individual problems. It is worth to pay attention to the fact that the simulation environment gives a wide possibility spectrum as the solution of complex problems that contain a group of single phenomena. For example, the observation of the sediment transport, in case of the drying of a reservoir, and finding the dependence between both, it is almost impossible to research in site.

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