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MATHEMATICAL MODEL OF THE SIX ELECTRODE PULSE CURRENT ELECTRIC ARC FURNACES

MODEL MATEMATYCZNY SZEŚCIOELEKTRODOWEGO PIECA ŁUKOWEGO PRĄDU IMPULSOWEGO

A b s t r a c t

In the article based on the fundamental laws of physics and mathematics has been formulated mathematical model to analyze the electromagnetic waveforms in electrical power systems with non-linear loads, such as arc furnaces.

Keywords: six-electrode electric arc furnace

S t r e s z c z e n i e

W artykule na podstawie fundamentalnych praw fizyki i matematyki został sformułowany model matematyczny do analizy przebiegów elektromagnetycznych w układach zasilania energią elektryczną odbiorów nieliniowych, np. pieców łukowych.

Słowa kluczowe: sześcioelektrodowy elektryczny piec łukowy

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1. Introduction

Modern electrical networks containing power lines with different voltages form rigid, hierarchical structures that make experimental research on their electrical states difficult and extremely dangerous. The same is true for power supply systems for large loads that contain magnetic core machines, machines in which commutation occurs, electrical power converters, and electromagnetic state controllers for converters. The complexity of electromagnetic connections between the branches of the circuits of such systems makes it obvious that writing a system of equations of the electromagnetic state is very difficult in terms of assessing electromagnetic waveforms using mathematical modeling [2, 3]. Usually, analysis of steady states in electrical systems, in which electromagnetic relationships are not taken into account, the system of equations is written using the node-voltage method. This helps to greatly reduce the system of equations compared to the traditional methods or even, frequently, the mesh current method [1, 2]. However, analysis of transient electromagnetic processes in the power supply systems for various loads, in which the electromagnetic relationships between the separate branches or parts of the circuit are very important, requires the system of equations to be written in contour coordinates (contour currents and contour magnetic fluxes) [3, 4]. Due to the limited capability of analytical methods for solving large systems of equations and the availability of modern computers, effective software, and numerical methods, the easiest way to solve systems of differential equations is through numerical methods [4–6].

Another important property of the electromagnetic circuits in modern power supply systems is the significant non-linearity of the characteristics of their individual components. This non-linearity leads to errors and instabilities in calculations that concern systems of differential equations. In such a case, care should be taken to select an appropriate numerical method for solving a rigid system of equations [4–6].

Mathematical experiments have to be repeated multiple times during research in order to obtain the results required for a given task, which is why it is especially important to use automated software that is based on adequate mathematical models of the objects in question. If such software is unavailable, it needs to be developed and adapted to modern software and technical means.

Impact assessments of electric arc furnaces (EAFs) on power supply systems indicate that within the last few decades, DC EAFs have begun to compete with three-phase AC EAFs. Compared to three-phase EAFs, power supply systems are much more complicated due to the use of rectifiers, i.e., semiconductive parts with non-linear current–voltage characteristics that lead to current deformations in the power supply system.

Despite the prevalence of DC EAFs over AC EAFs due to the negative effect on power supply systems and the environment, the application of the former is limited by certain factors. The most significant of these are multiple overvoltages that occur as a result of the technical gap related to the high current of the arc. Short technical breaks in operation require DC EAFs to be shut down directly under load using switches, which are installed on the side of the primary coil of the furnace transformer. This leads to overvoltages between the terminals of the switches and parts of the EAF power supply system. The proposed power supply system for an impulse current EAF can terminate the current of the arc during a technical break by raising the electrodes, after which the furnace shuts down in a no-load state. This helps to practically eliminate overvoltages occurring due to the termination of the no-load current, which in turn helps to considerably extend commutation breaks of the switches, thus improving the operation conditions for the insulation of the primary component of an EAF. However, the working conditions for a given subsystem require extensive research, primarily on quasi-steady states. For a given stage, this research can be done – as an exception – through mathematical modeling. Consequently, an important technological task arises to create an adequate mathematical model of a power supply system for a DC EAF that would enable research on the electromagnetic processes and show differences in them in quasi-steady and commutation states [8–10].

2. Mathematical model of six electrode EAF

Figure 1 shows a diagram of the proposed power supply system for a six-electrode pulse current EAF, in which the arc currents can be terminated by raising the electrodes [10]. The three-phase electric system, by means of an overhead power line, a mains transformer, and a cable line, powers the furnace transformer, in which the secondary coil is connected through a short-distance network and a power module for one-phase AC-DC rectifiers to a group of anode and cathode electrodes (cathode electrodes 1’, 2’, 3’ and anode electrodes 1”, 2”, 3”), located over the feed surface in the furnace chamber.

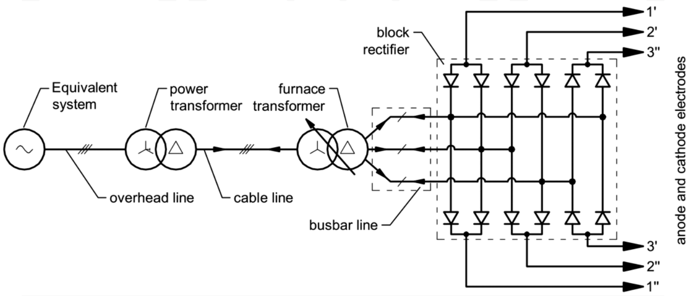

Fig. 1. Schematic of the power supply pulse current EAF

To allow the system of equations of the electromagnetic state to be written, Fig. 2 shows an equivalent circuit, in which the voltage source has a limited power, and the overhead and cable lines are substituted for quadripoles that contain components with linear characteristics. Due to the limited length of the cable lines, the partial capacity and partial conductivity between the cables and between the cables and the frame (screen) are not taken into account. The models of the mains transformer and the furnace transformer divide the magnetic fluxes into the main flux and the leakage flux. The characteristics of non-linear elements i.e., transformers, diodes and electric arc are given in [4, 6, 9, 11].

In the equivalent circuit, the branches with the non-linear components $X_{\mu 1}$ and $X_{\mu 2}$ are the branches of the main magnetic flux routes of the mains transformer and the furnace transformer, whereas the linear components X_{01} and X_{02} are the branches of the magnetic

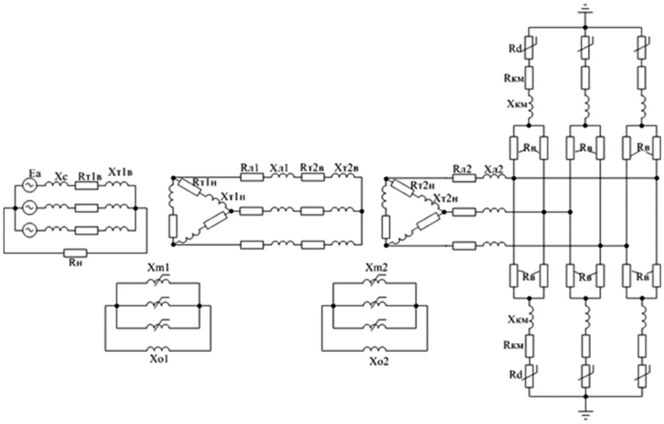

Fig. 2. Equivalent circuit of the power supply pulse current EAF

flux routes outside the magnetic circuit of the transformers. Note that the model must describe the processes in both the magnetic and the electric contours at the same time, as a single electromagnetic process in the power supply system. Due to the complexity and multidimensionality of the equivalent circuit, the system of equations of the electromagnetic state should be created automatically. This can be easily achieved based on matrix and vector mathematical operations. To this end, a graph of the electromagnetic circuit needs to be drawn. Afterwards, the previously designed algorithms should be used to draw the mesh incidence matrix, which in turn will allow for an automatic creation of the system of equations. Fig. 3 shows the graph of the electromagnetic circuit for the equivalent circuit in question (Fig. 2). The dashed lines indicate the chords of the graph (the number of chords corresponds to the number of independent contours). The solid lines indicate the branches of the graph tree.

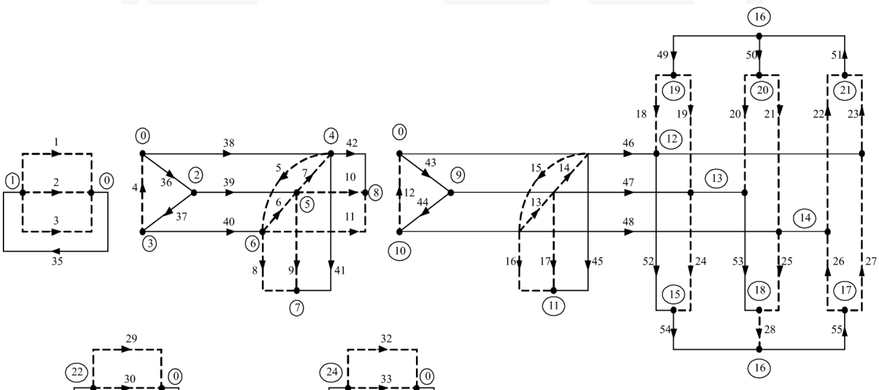

Fig. 3. Graph for electromagnetic circuit shown in Fig. 2

Based on the connection table of chords and branches for electric and magnetic circuits using a special algorithm is formulated lower part of incidence matrix [4], which for consider scheme is shown in Fig. 4. Entries in this matrix being zero was marked by dots, and by the units was marked branches of the tree graph by witch is closed that or other edge, where +1 represents branch for the current direction is the same as the direction of contour current, and –1 when the direction of the current in this branch is in opposite to the direction of the contour current. The upper part of the incidence matrix is a diagonal and the number of diagonal entries corresponds to the number of independent edges.

$$.\mathbf{\Gamma} = \begin{Vmatrix}
1&1&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&. \\
.&.&.&1&-1&.&1&1&1&1&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&. \\
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.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&1&1
\end{Vmatrix}$$

Fig. 4. Lower part of incidence matrix

The number of rows lower part of incidence matrix is equal to the number of independent nodes equivalent circuit, and the number of columns corresponds to the number of independent loops of electromagnetic circuit.

In the general case for shown electromagnetic circuit based on Kirchhoff's circuit laws [1–3], write the following mathematical model:

$$\mathbf{\Pi} \cdot \begin{pmatrix} \vec{i} \\ \vec{\psi} \end{pmatrix} = 0;$$

$$\mathbf{\Gamma} \cdot \begin{Vmatrix} \mathbf{R} & \\ & \mathbf{0} \end{Vmatrix} \times \begin{pmatrix} \vec{i} \\ \vec{\psi} \end{pmatrix} + \mathbf{\Gamma} \cdot \begin{Vmatrix} M & K_{12} \\ K_{21} & -R_{\mu} \end{Vmatrix} \times \begin{pmatrix} \dfrac{d\vec{i}}{dt} \\ \dfrac{d\vec{\psi}}{dt} \end{pmatrix} + \mathbf{\Gamma} \times \begin{pmatrix} \vec{u}_{ne} + \vec{u}_C - \vec{e} \\ 0 \end{pmatrix} = 0. \tag{1}$$

The voltage and current of the capacitors are connected to each other by the equation:

$$\mathbf{C} \cdot \frac{d\vec{u}_C}{dt} = \vec{i}. \tag{2}$$

where:

$\mathbf{\Pi}, \mathbf{\Gamma}$ – topological matrixes electromagnetic circuit graphs of power system,

$\mathbf{R}$ – diagonal matrix resistance circuits,

M – static leakage inductance matrix of transformer, inductors and the others branches of equivalent circuit,

$K_{12} = \|0 \quad W_1^{-1}W_2\|$ – winding ratio matrix of delivery and furnace transformers,

W_1, W_2 – matrix of the number of turns primary and secondary windings of transformer,

R_μ – diagonal identity matrix of the dynamic reluctance (referred to the square of the number of turns primary windings of transformers),

$\vec{i}, \vec{\psi}$ – vectors-columns of currents and mutual flux of phases primary windings of transformers,

$\frac{d\vec{i}}{dt}, \frac{d\vec{\psi}}{dt}$ – vectors derivatives by time with current of electrical circuits and mutual flux of magnetic circuits,

$\vec{u}_{ne}$ – vector of voltage on the nonlinear resistive elements,

$\vec{u}_C$ – vector of voltage on the capacitors,

$\vec{e}$ – vector of phase electromotive force,

$\mathbf{C}$ – capacity matrix of electrical circuits.

The final mathematical model of power network of pulse current EAF writing with taking into account backward differentiation formula, are given [3, 4]:

$$(d\vec{y}/dt)_{k+1} = a_0 h^{-1}\vec{y}_{k+1} + h^{-1}\sum_{s=1}^{p} a_s \vec{y}_{k+1-s} \qquad (3)$$

where:

$\vec{y}$ – integrating function of vector,

$a_0, ..., a_s$ – factors of approximating series, which define a matrix given in [3–6],

p – order of method,

h – integration step.

Taking into account formula (3) mathematical model of the power supply (given by equations 1 and 2) will be writing in the differential form [4, 6].

$$\begin{aligned}
&\mathbf{\Pi} \cdot \begin{pmatrix} \vec{i}_{k+1} \\ \vec{\psi}_{k+1} \end{pmatrix} = 0; \\
&\mathbf{\Gamma} \cdot \begin{Vmatrix} \mathbf{R} + a_0 h^{-1}\mathbf{M}_{k+1} & a_0 h^{-1}\mathbf{K}_{12} \\ a_0 h^{-1}\mathbf{K}_{21} & -a_0 h^{-1}\mathbf{R}_{\mu 12} \end{Vmatrix} \times \begin{pmatrix} \vec{i}_{k+1} \\ \vec{\psi}_{k+1} \end{pmatrix} + \\
&+\mathbf{\Gamma}\left(h^{-1} \cdot \begin{Vmatrix} \mathbf{M}_{k+1} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & -\mathbf{R}_{\mu k+1} \end{Vmatrix} \times \sum_{s=1}^{p} a_s \begin{pmatrix} \vec{i}_{k+1-s} \\ \vec{\psi}_{k+1-s} \end{pmatrix} + \begin{pmatrix} \vec{u}_{ne\,k+1} + \vec{u}_{C\,k+1} - \vec{e}_{k+1} \\ 0 \end{pmatrix} \right) = 0; \\
&a_0 h^{-1}\mathbf{C}_{k+1}\vec{u}_{C\,k+1} + h^{-1}\mathbf{C}_{k+1}\sum_{s=1}^{p} a_s \vec{u}_{C\,k+1-s} = \vec{i}_{k+1}.
\end{aligned} \qquad (4)$$

Voltage of capacitor in $(k+1)$ step is defined as:

$$\vec{u}_{C\,k+1} = a_0^{-1} h \mathbf{C}_{k+1}^{-1} \vec{i}_{k+1} - a_0^{-1} \sum_{s=1}^{p} a_s \vec{u}_{C\,k+1-s} \tag{5}$$

which substitute to the second equations of equations (4) and get:

$$\begin{aligned}
&\mathbf{\Pi} \cdot \begin{pmatrix} \vec{i}_{k+1} \\ \vec{\psi}_{k+1} \end{pmatrix} = 0; \\
&\mathbf{\Gamma} \cdot \begin{Vmatrix} \mathbf{R} + a_0 h^{-1} \mathbf{M}_{k+1} + a_0^{-1} h \mathbf{C}_{k+1}^{-1} & a_0 h^{-1} \mathbf{K}_{12} \\ a_0 h^{-1} \mathbf{K}_{21} & -a_0 h^{-1} \mathbf{R}_{\mu 12} \end{Vmatrix} \times \begin{pmatrix} \vec{i}_{k+1} \\ \vec{\psi}_{k+1} \end{pmatrix} + \\
&+\mathbf{\Gamma} \left(h^{-1} \cdot \begin{Vmatrix} \mathbf{M}_{k+1} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & -\mathbf{R}_{\mu\,k+1} \end{Vmatrix} \cdot \sum_{s=1}^{p} a_s \begin{pmatrix} \vec{i}_{k+1-s} \\ \vec{\psi}_{k+1-s} \end{pmatrix} + \begin{pmatrix} \vec{u}_{ne\,k+1} + a_0^{-1} \sum_{s=1}^{p} a_s \vec{u}_{C\,k+1-s} - \vec{e}_{k+1} \\ 0 \end{pmatrix} \right) = 0;
\end{aligned} \tag{6}$$

After spreading system of nonlinear equations (5) in a limited Taylor series [4], we get a mathematical model described by the currents and mutual magnetic fluxes of primary windings of transformers.

$$\begin{aligned}
&\mathbf{\Pi} \cdot \begin{pmatrix} \Delta\vec{i}_{k+1}^{(l)} \\ \Delta\vec{\psi}_{k+1}^{(l)} \end{pmatrix} = \mathbf{\Pi} \cdot \begin{pmatrix} \vec{i}_{k+1}^{(l)} \\ \vec{\psi}_{k+1}^{(l)} \end{pmatrix}; \\
&\mathbf{\Gamma} \cdot \begin{Vmatrix} \mathbf{R}_{k+1}^{(l)} + a_0 h^{-1} \mathbf{M}_{k+1}^{(l)} + a_0^{-1} h \mathbf{C}_{k+1}^{-1} & a_0 h^{-1} \mathbf{K}_{12} \\ a_0 h^{-1} \mathbf{K}_{21} & -a_0 h^{-1} \mathbf{R}_{\mu\,k+1}^{(l)} \end{Vmatrix} \times \begin{pmatrix} \Delta\vec{i}_{k+1}^{(l)} \\ \Delta\vec{\psi}_{k+1}^{(l)} \end{pmatrix} = \\
&\mathbf{\Gamma} \cdot \begin{Vmatrix} \mathbf{R}_{k+1}^{(l)} + a_0 h^{-1} \mathbf{M}_{k+1}^{(l)} + a_0^{-1} h \mathbf{C}_{k+1}^{-1} & a_0 h^{-1} \mathbf{K}_{12} \\ a_0 h^{-1} \mathbf{K}_{21} & -a_0 h^{-1} \mathbf{R}_{\mu\,k+1}^{(l)} \end{Vmatrix} \times \begin{pmatrix} \vec{i}_{k+1}^{(l)} \\ \vec{\psi}_{k+1}^{(l)} \end{pmatrix} + \\
&+\mathbf{\Gamma} \left(h^{-1} \cdot \begin{Vmatrix} \mathbf{M}_{k+1}^{(l)} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & -\mathbf{R}_{\mu\,k+1}^{(l)} \end{Vmatrix} \cdot \sum_{s=1}^{p} a_s \begin{pmatrix} \vec{i}_{k+1-s}^{(l)} \\ \vec{\psi}_{k+1-s}^{(l)} \end{pmatrix} + \begin{pmatrix} \vec{u}_{ne\,k+1-s}^{(l)} + a_0^{-1} \sum_{s=1}^{p} a_s \vec{u}_{C\,k+1-s}^{(l)} - \vec{e}_{k+1}^{(l)} \\ 0 \end{pmatrix} \right) = 0; \\
&\begin{pmatrix} \vec{i}_{k+1}^{(l+1)} \\ \vec{\psi}_{k+1}^{(l+1)} \end{pmatrix} = \begin{pmatrix} \vec{i}_{k+1}^{(l)} \\ \vec{\psi}_{k+1}^{(l)} \end{pmatrix} - \begin{pmatrix} \Delta\vec{i}_{k+1}^{(l)} \\ \Delta\vec{\psi}_{k+1}^{(l)} \end{pmatrix}
\end{aligned} \tag{7}$$

In order to write a mathematical model in the contour coordinates (contour currents and contour mutual flux) the second equation of the model 6 will be replaced by vector of branch currents and mutual fluxes by the contour coordinates and their vectors use relationship:

$$\begin{aligned}
&\begin{pmatrix} \vec{i}_{k+1}^{(l)} \\ \vec{\psi}_{k+1}^{(l)} \end{pmatrix} = \mathbf{\Gamma}_t \begin{pmatrix} \vec{i}_{K,k+1}^{(l)} \\ \vec{\psi}_{K,k+1}^{(l)} \end{pmatrix}; \\
&\begin{pmatrix} \Delta\vec{i}_{k+1}^{(l)} \\ \Delta\vec{\psi}_{k+1}^{(l)} \end{pmatrix} = \mathbf{\Gamma}_t \begin{pmatrix} \Delta\vec{i}_{K,k+1}^{(l)} \\ \Delta\vec{\psi}_{K,k+1}^{(l)} \end{pmatrix}
\end{aligned} \tag{8}$$

After substitution we get:

$$\mathbf{\Gamma}\cdot\begin{Vmatrix}\mathbf{R}_{k+1}^{(l)}+a_0h^{-1}\mathbf{M}_{k+1}^{(l)}+a_0^{-1}h\mathbf{C}_{k+1}^{-1} & a_0h^{-1}\mathbf{K}_{12}\\ a_0h^{-1}\mathbf{K}_{21} & -a_0h^{-1}\mathbf{R}_{\mu k+1}^{(l)}\end{Vmatrix}\times\mathbf{\Gamma}_t\begin{pmatrix}\Delta\vec{i}_{k+1}^{(l)}\\ \Delta\vec{\psi}_{k+1}^{(l)}\end{pmatrix}=$$
$$\mathbf{\Gamma}\cdot\begin{Vmatrix}\mathbf{R}_{k+1}^{(l)}+a_0h^{-1}\mathbf{M}_{k+1}^{(l)}+a_0^{-1}h\mathbf{C}_{k+1}^{-1} & a_0h^{-1}\mathbf{K}_{12}\\ a_0h^{-1}\mathbf{K}_{21} & -a_0h^{-1}\mathbf{R}_{\mu k+1}^{(l)}\end{Vmatrix}\times\mathbf{\Gamma}_t\begin{pmatrix}\vec{i}_{k+1}^{(l)}\\ \vec{\psi}_{k+1}^{(l)}\end{pmatrix}+ \quad (9)$$
$$+\mathbf{\Gamma}\left(h^{-1}\cdot\begin{Vmatrix}\mathbf{M}_{k+1}^{(l)} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & -\mathbf{R}_{\mu k+1}^{(l)}\end{Vmatrix}\mathbf{\Gamma}_t\cdot\sum_{s=1}^{p}a_s\begin{pmatrix}\vec{i}_{k+1-s}^{(l)}\\ \vec{\psi}_{k+1-s}^{(l)}\end{pmatrix}+\begin{pmatrix}\vec{u}_{ne\,k+1-s}^{(l)}+a_0^{-1}\sum_{s=1}^{p}a_s\vec{u}_{C\,k+1-s}^{(l)}-\vec{e}_{k+1}^{(l)}\\ 0\end{pmatrix}\right)=0;$$

$$\begin{pmatrix}\vec{i}_{K,k+1}^{(l+1)}\\ \vec{\psi}_{K,k+1}^{(l+1)}\end{pmatrix}=\begin{pmatrix}\vec{i}_{K,k+1}^{(l)}\\ \vec{\psi}_{kK,+1}^{(l)}\end{pmatrix}-\begin{pmatrix}\Delta\vec{i}_{K,k+1}^{(l)}\\ \Delta\vec{\psi}_{K,k+1}^{(l)}\end{pmatrix} \quad (10)$$

where:

$R_{k+1}^{(l)}+a_0h^{-1}M_{k+1}^{(l)}+a_0^{-1}hC_{k+1}^{-1};a_0h_{-1}K_{21}-a_0h^{-1}R_{\mu k+1}^{(l)}$ – elements of Jacobian matrix for the iteration and ($k+1$) integration step,

h – numeric integration step,

$R_{k+1}^{(l)}$ – resistance matrix of linear and nonlinear elements of equivalent circuit for iteration and ($k+1$) integration step,

C_{k+1} – capacity matrix of electrical circuits for ($k+1$) integration step,

$\vec{e}_{k+1}^{(l)}$ – vector of electromotive force of schema branch for the l iteration and ($k+1$) integration step,

$\vec{u}_{C\,k+1-s}^{(l)}$ – vectors of voltage on the capacitors and mutual flux for l iteration and ($k+1-s$) integration step,

$\vec{u}_{ne\,k+1-s}^{(l)}$ – vector of voltage on the nonlinear resistive elements of schema (branch of EAF arcs) for l iteration and ($k+1-s$) integration step,

$\vec{i}_{K,k+1}^{(l+1)};\vec{\psi}_{K,k+1}^{(l+1)};\vec{i}_{K,k+1}^{(l)};\vec{\psi}_{K,k+1}^{(l)}$ – vectors of contours currents and contours mutual magnetic flux of transformers for ($l+1$) and l iteration and ($k+1$) integration step,

$\Delta\vec{i}_{K,k+1}^{(l)};\Delta\vec{\psi}_{K,k+1}^{(l)}$ – vectors of increments contours currents and contours mutual magnetic flux of transformers for l iteration and ($k+1$) integration step,

$M_{k+1}^{(l)}$ – matrix of own and mutual inductance of electrical circuit for iteration and ($k+1$) integration step.

Currents, mutual magnetic flux and voltage on the capacitors after their calculating for ($l+1$) iteration we define by the following formula:

$$\vec{u}_{C,\kappa+1}^{(l+1)}=a_o^{-1}\cdot\left(C_{\kappa+1}^{(l)}\right)^{-1}\cdot h\cdot\vec{u}_{C,\kappa+1}^{(l+1)}-a_o^{-1}\sum_{s=1}^{p}a_s\vec{u}_{c,\kappa+1-s}^{(l)} \quad (11)$$

where:

$\vec{i}_C^{(l+1)}$ – vectors of branch currents with capacitors for $(l + 1)$ iteration and $(k + 1)$ integration step.

The mathematical model has been implemented in the form of automated software for the analysis of electromagnetic processes in the instantaneous values of currents, voltages and magnetic fluxes in the programming environment DELPHI [7]. Developed software package based on the minimum input information (connection arrays of electromagnetic circuit graph, parameters of the branch of schema, characteristics of a voltage source and array of magnetic association between the branches of the schema) forming in an automatic cycle of the system of equations. The adequacy of the mathematical model is checked by comparing the instantaneous values of voltage and current of 35 kV power supply system of EAF with a capacity of 100 tons obtained by measurements (Fig. 5a) with simulation results (Fig. 5b).

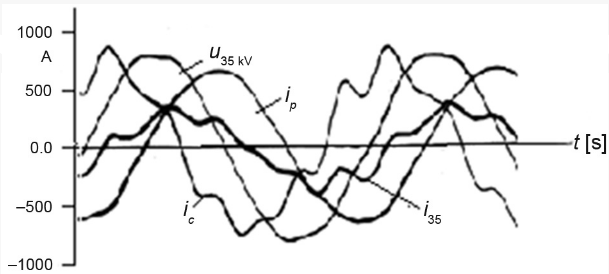

Fig. 5a. Instantaneous values of voltage and current in the power system of EAF (measurement)

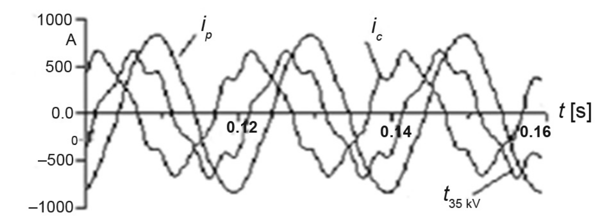

Fig. 5b. Instantaneous values of voltage and current in the power system of EAF (simulation)

From the comparison of the presented results we can see that the characteristic of these processes are very similar, which confirms the correctness of the mathematical model in the quasi steady state. Fig. 6 shows the waveforms of the instantaneous current after switching the filter (series connected capacitor banks with inductor) to the rails 35 kV received by measurement (Fig. 6a) and received in a simulation (Fig. 6b), which shape is similar, that confirms the correctness of the mathematical model in transient state.

The mathematical model was also verified by statistical comparing analysis of the second current harmonic when the EAF melting metal, which also confirms the enough adequacy at the time of melting. It should be noted that the model is useful to simulate the steady state and transient electrical power systems with a large selection of electrical devices (transformers, inductors, capacitors, cable and overhead lines, harmonic filters, semiconductors and others) and may be used to predict and estimate the various states in the design and as well as during operation such electrical devices. The software package, created based on a mathematical model and implemented in DELPHI environment, equipped with a graphical user interface (GUI) for entering and displaying information and process control of simulation, provides comfort and easiness of use.

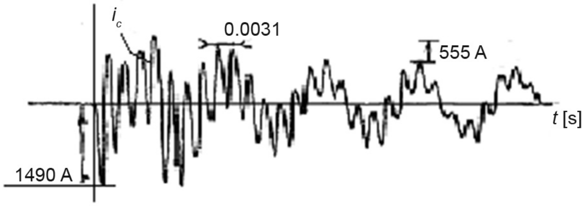

Fig. 6a. Instantaneous value of capacitor current after switching the filter (measurement)

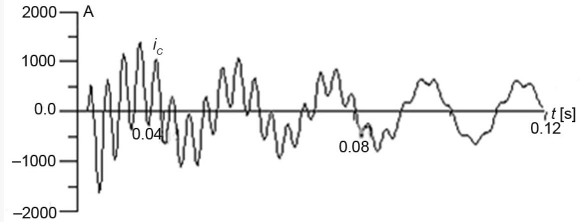

Fig. 6b. Instantaneous value of capacitor current after switching the filter (simulation)

3. Conclusions

1. Based on classical approach control vector formulation for the mathematical model of complex electromagnetic circuits is universal in the sense of creating a state space representation of electromagnetic state for any structure diagrams.
2. The correctness of the mathematical model was confirmed by comparing the results of measurements of real object and mathematical simulation in steady state and transient state for the same schemes with similar electrical parameters.
3. The mathematical model allows to model the electromagnetic processes in complex systems with electrical installations having non-linear characteristics, for the account choice of an effective method of numerical solution of non-linear stiff equation.

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