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DETERMINATION OF CHARACTERISTIC FLOW VALUES DOWNSTREAM OF THE RACIBÓRZ RESERVOIR USING MONTE CARLO METHODS BASED ON COPULA FUNCTIONS

OKREŚLENIE WARTOŚCI PRZEPŁYWÓW CHARAKTERYSTYCZNYCH PONIŻEJ ZBIORNIKA RACIBÓRZ Z ZASTOSOWANIEM METOD MONTE CARLO OPARTYCH NA FUNKCJACH COPULA

Abstract

This case study of the Racibórz reservoir discusses the application of a determination method of characteristic flow values below retention reservoirs. The study describes the selected theoretical foundations, the individual steps of the *Monte Carlo* (MC) method used to generate hypothetical flood wave hydrographs, and the results of simulations leading to the determination of characteristic flow values below retention reservoirs. The method is based on probability density functions for a multidimensional random variable obtained using copula link functions. The results of the analyses are represented in tables and graphs. The results of the simulation analysis are presented for comparison purposes, assuming a constant and a variable time for the hypothetical base flood elevation.

Keywords: copula functions, multivariate distributions of a random variable, flood wave characteristics, Monte Carlo method, random number generator, flows with a defined probability of exceedance, retention reservoirs

Streszczenie

W artykule zapresentowano aplikację określania wartości przepływów charakterystycznych poniżej zbiorników retencyjnych na przykładzie zbiornika Racibórz. Przedstawiono wybrane podstawy teoretyczne, poszczególne kroki metodyki, zastosowanie metody Monte Carlo (MC) do tworzenia hipotetycznych hydrogramów fal powodziowych oraz wyniki obliczeń symulacyjnych prowadzące do określenia wartości charakterystycznych przepływów poniżej zbiorników. U podstaw tej metody są funkcje gęstości prawdopodobieństwa wielowymiarowej zmiennej losowej budowanej z wykorzystaniem spinających funkcji copula. Wyniki analiz w artykule zostały przedstawione w postaci tabelarycznej i graficznej. Dla porównania przedstawione są wyniki analizy symulacyjnej przy założeniu stałego i zmiennego czasu podstawy hipotetycznej fali powodziowej.

Słowa kluczowe: funkcje copula, rozkłady wielowymiarowej zmiennej losowej, parametry fali powodziowej, metoda Monte Carlo, generowanie liczb losowych, przepływy o określonym prawdopodobieństwie przewyższenia, zbiorniki retencyjne

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1. Introduction

Monte Carlo methods are useful in determining characteristic flow values downstream of retention reservoirs due to their capability of generating (multidimensional) populations of elements with statistical characteristics consistent with experience. The experiment (generating characteristics of hypothetical flood waves) is unrelated to time, but the process of generation of those characteristics is based on marginal distribution values determined for extreme annual flood wave parameters. Consequently, it may be assumed that a population obtained using a Monte Carlo method corresponds to the population of extreme annual flood waves. This assumption also supports the statement that the proposed method of determining characteristic flows is reliable.

The events in 2010 emphasised deficiencies in the impounding structure controls. The control procedures applied in more than 80% of impounding structures during the flood of May 2010 failed to meet the conditions defined in the basic document binding on a reservoir manager – the Instructions for Water Management in the Reservoir. This study represents another step towards applying a stochastic approach to generating hypothetical hydrographs of flood waves and assessing characteristic flow values downstream of reservoirs.

The main stimulus underlying this study was the need to obtain clear and precise information about the flood control effectiveness of the Buków-Racibórz embankment and polder system. The varying level of detail in available information and substantial differences in the published values of reduction capacities cast doubt on the reliability of data used as a design basis for engineering structures located downstream of the polders in the Oder river valley that are governed by the Regulations contained in Journals of Laws Dz.U. No. 86, item 579, Dz.U. of 2015, item 329, Dz.U. No. 63, item 735, Dz.U. of 2013 item 104. The described status leads to erroneous decisions in the planning process of flood control measures and in designs of engineering structures such as roads, railways, bridges, wrong dimensions and locations of floodplains that have a significant effect on zoning and spatial planning, and on the amounts of premiums paid by insurance takers. The effects of the described status include external costs. If the reduction values assumed in economic analyses are too low, they may result in external benefits: the conditions are safer than estimated and excessively high bridges, embankments, large culverts, large dry detention basins are designed that are more expensive than necessary, insurance takers pay excessively high premiums, etc. On the other hand, if the reduction values assumed are too high, external costs appear, i.e. a portion of unreasonable costs of project implementation is paid by society, already overburdened with taxes (and in particular by potential flood victims), the conditions are less safe than assumed, and threats and losses are higher in the case of flood. Both cases may lead to the conclusion that a planned capital expenditure project does not meet social needs for flood protection level and thus entails an undesirable deadweight flood loss in the economic balance sheet.

The problem analysed consists in determination of maximum annual flow values with a defined probability of exceedance. A statistical analysis is impracticable in numerous cases due to insufficient data sets. This is particularly true for river valley stretches situated downstream of dam cross-sections of retention reservoirs. The difficulties described often cannot be overcome downstream of retention reservoirs. The problem is particularly grave because statistical values play a key role in the assessment of existing conditions and development of guidelines for engineering and hydraulic structure design. Moreover, if an assessment of such statistics downstream of reservoirs is impossible, experts are forced to take unreasonable steps, assuming no reduction of maximum flows while such reduction takes place. This approach has an obvious effect on the capital expenditure of projects implemented, increasing their costs and reducing their economic efficiency.

A known probability distribution is necessary to apply MC methods, so that the reservations described above also apply to parameter estimation of a multivariate distribution. In addition, known applications of multivariate normal distribution did not ensure consistency of experimental materials with the results of MC methods – marginal theoretical distribution functions with empirical ones. This inconvenience, or rather material deficiency, is remedied by the use of the copula function in generating a probability density function for a multidimensional random variable.

2. A brief history of the Racibórz reservoir

Following the disastrous flood of 1880, the idea was put forward to construct a retention reservoir in the Oder river valley upstream of Racibórz. The first concept of a suburban dam on the Oder River to be constructed as part of the planned Oder–Danube canal was developed as early as 1833. The capacity of the reservoir was estimated in 1905–1906 at 640 million m^3. Modifications introduced in the following years, principally due to plans to transform the canal into a large transport route connecting the Oder with the Danube Rivers resulted in a reduced total reservoir capacity of 390 million m^3. A navigable canal was planned, bypassing the reservoir along its eastern side. Due to the partition of the reservoir among Poland, Czechoslovakia and Germany after World War I, the idea of canal construction was shelved. In order to protect Racibórz against floods, the German authorities commenced in 1934 the construction of the Neue Oder canal (known as Ulga at present), planning to excavate about 4 million m^3 of earth. The work was completed in 1942. The canal was capable of transporting 2,000 m^3/s of water along its paved bed. Following the flood of July 1997, it was revealed that about 1 million m^3 of mud had deposited in the canal. The canal was cleaned after that disaster. When the first Polish concept of making the Oder navigable was proposed in 1954, the total planned capacity amounted to 695 million m^3. Czechoslovakia used its absolute veto against the construction of such a large reservoir and objected to all subsequent designs proposed.

A concept developed in 1957 assumed that water from the Oder would be supplied to the Warta drainage area and to the Upper Silesia conurbation using a reservoir with a total capacity of 326.4 million m^3. Four new versions of the reservoirs were developed in 1962. The first version provided for a capacity of 247 million m^3, the second – 290, the third – 302, and the fourth – 507 million m^3. The shape of the reservoir in its plan view was the same in the first three versions, the fourth version was completely different from previous ones.

All versions were submitted to the Czechoslovakian government that approved the first concept by way of a special resolution adopted in 1961. However, in time the solution turned out to be economically inefficient for Poland, the plans were abandoned, and the negotiations with Czechoslovakia discontinued. In 1975, the Czechs submitted a proposal to make the river navigable between Koźle and Ostrava, with prospective construction of an

Oder–Danube canal. The need emerged again to develop a comprehensive solution to water management problems. A compromise was proposed by *"Hydroprojekt"* of Wrocław – the construction of the reservoir was to be combined with aggregate mining from its prospective basin. Following the flood of July 1997, the flood control function of the planned reservoir rose in importance.

3. The Racibórz reservoir – structure parameters

The flood control reservoir Racibórz Dolny on the Oder River in the Śląskie province (a polder) is planned as a component of the flood control system in the Oder valley. It is designed as a polder collecting water only in periods of flood wave passage, and with no water management functions outside flood periods.

The basic parameters of the reservoir are as follows:

- dam crest elevation: 197.50 m above sea level,
- maximum impoundment level: 195.20 m above sea level,
- water volume at maximum impoundment: 185.0 Mm^3,
- maximum water surface area: 26.3 km^2,
- total length of earthen dams: 21.8 km,
- maximum height of earthen dams: 11.1 m.

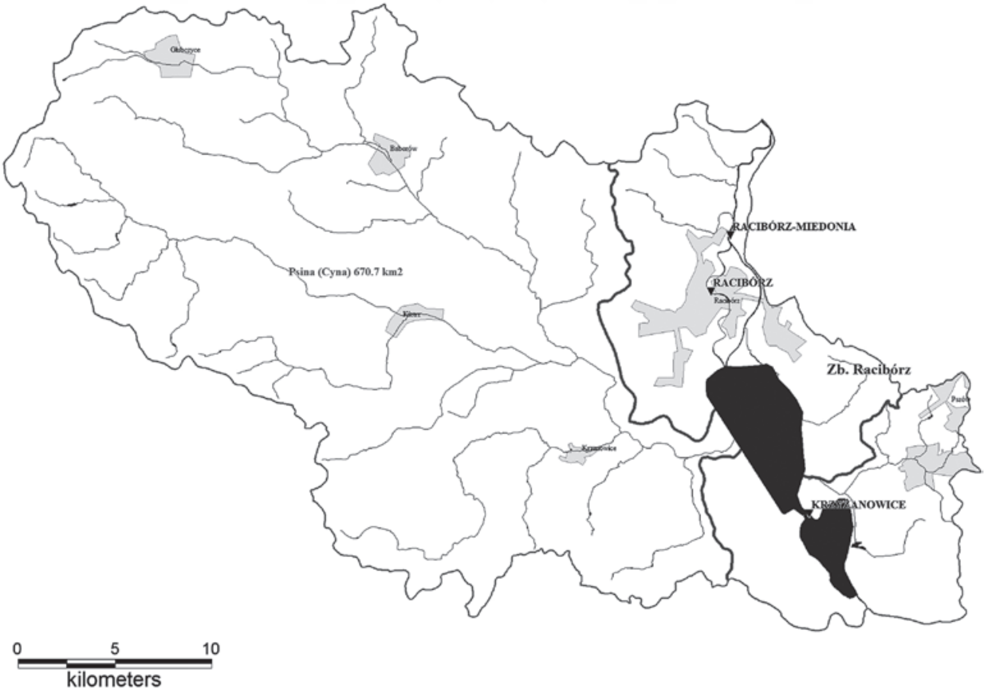

Fig. 1. Location of the Racibórz reservoir with indicated drainage area of the Psina River

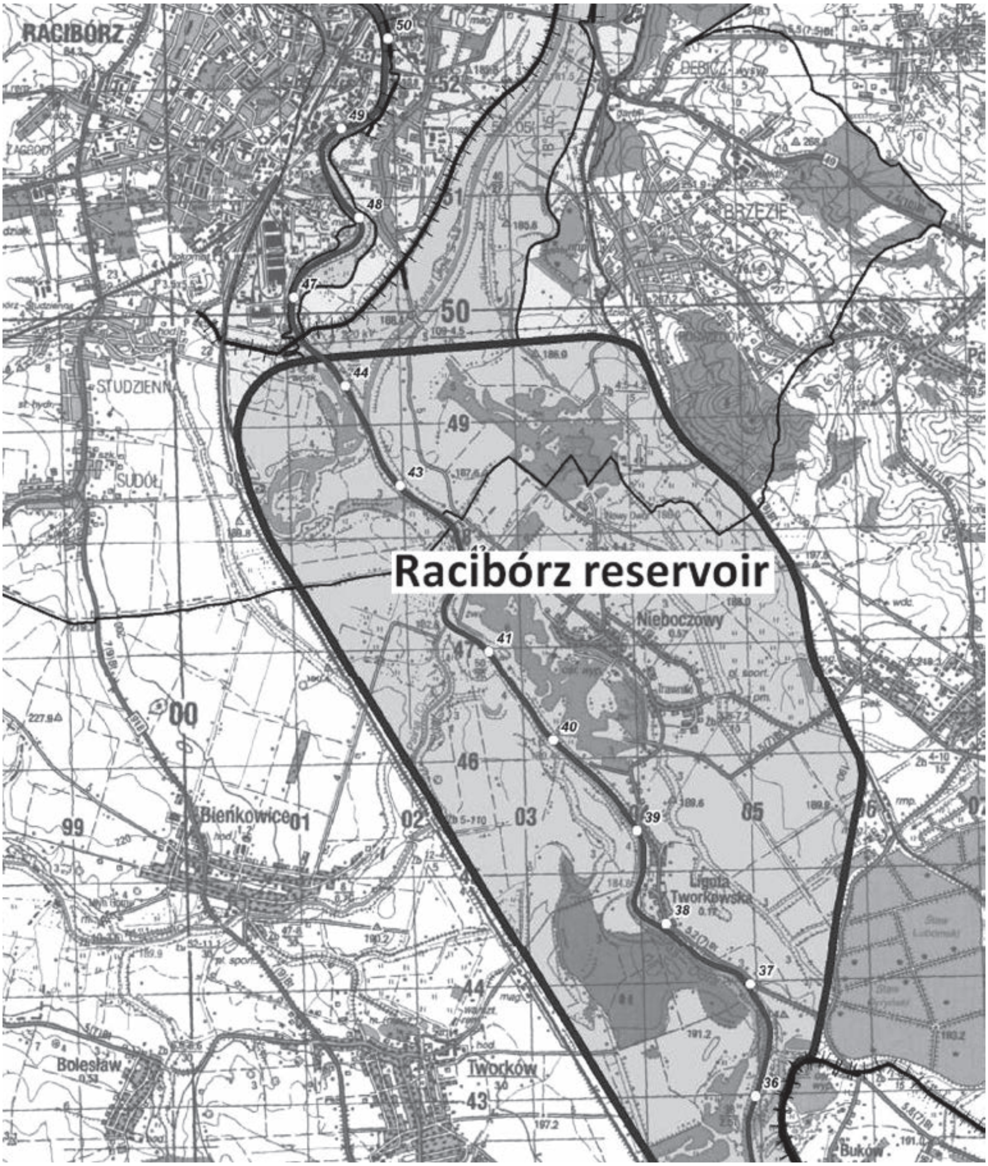

Fig. 2. The Racibórz reservoir [16]

4. Hydrological model used in the analysis

Table 1

Parameters of water-level gauges near the cross-section of the Racibórz–Buków polders

River	Name of water-level gauge	km of river course	Drainage area [km^2]
Oder	Krzyżanowice	712.85	5875.14
Oder	Racibórz-Miedonia	693.63	6728.90

The method used to define a random sample was described in other studies published by the author [1–4]. This study is focused on application of the proposed approach to using MC methods with certain modifications in order to generate hypothetical hydrographs of flood waves. Statistics of flood waves have been estimated on the basis of characteristics obtained at the water-level gauge cross-section in Racibórz–Miedonia, confirmed as reliable for an assessment of the Racibórz reservoir. It is the nearest water-level gauge downstream of the dam cross-section providing a sufficient set of hydrological materials collected (hydrographs of extreme flood waves).

Table 2

Sample characteristics of extreme waves used in analyses of flood control efficiency of polders [10]

Probability of exceedance	Peak value [m3/s]	Volume [Mm3]
0.100	1078	314.7
0.030	1781	519.9
0.020	2031	593.0
0.010	2364	690.1
0.005	2885	842.2
0.003	3199	934.0
0.002	3504	1,022.9

A log-normal distribution is used to describe both the peak wave form and the wave volume. The adopted parameters for the probability density function for maximum annual flows are those for the Racibórz-Miedonia water-level gauge, based on a study for the ISOK project [14]. The values of distribution parameters describing the second random variable, i.e. wave volume, are taken from analyses of the Buków polder performed for the Regional Water Management Board in Gliwice [9, 10].

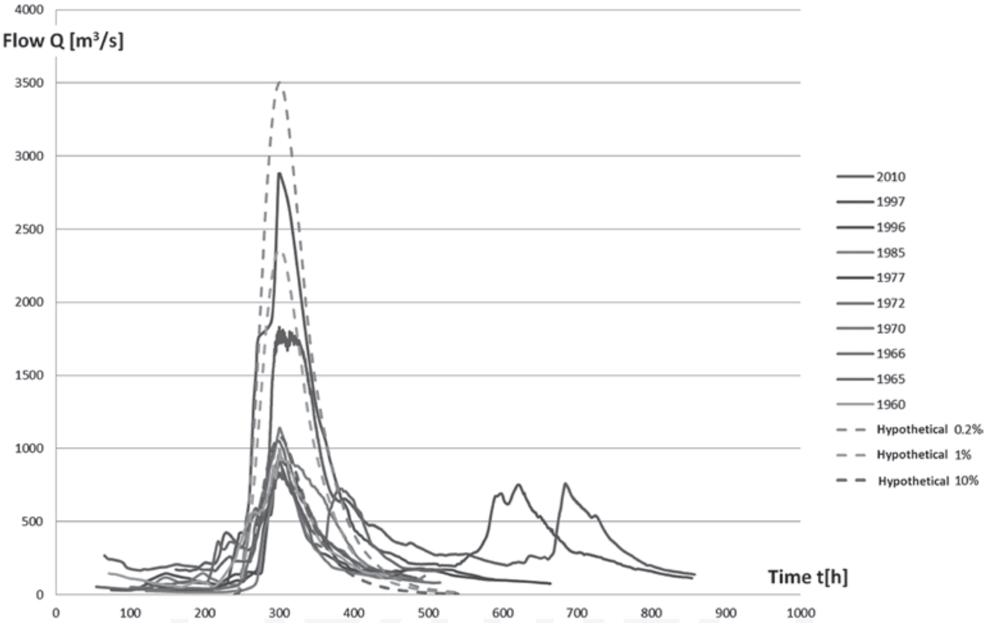

Fig. 3. Sample largest flood waves, the Krzyżanowice water-level gauge [10]

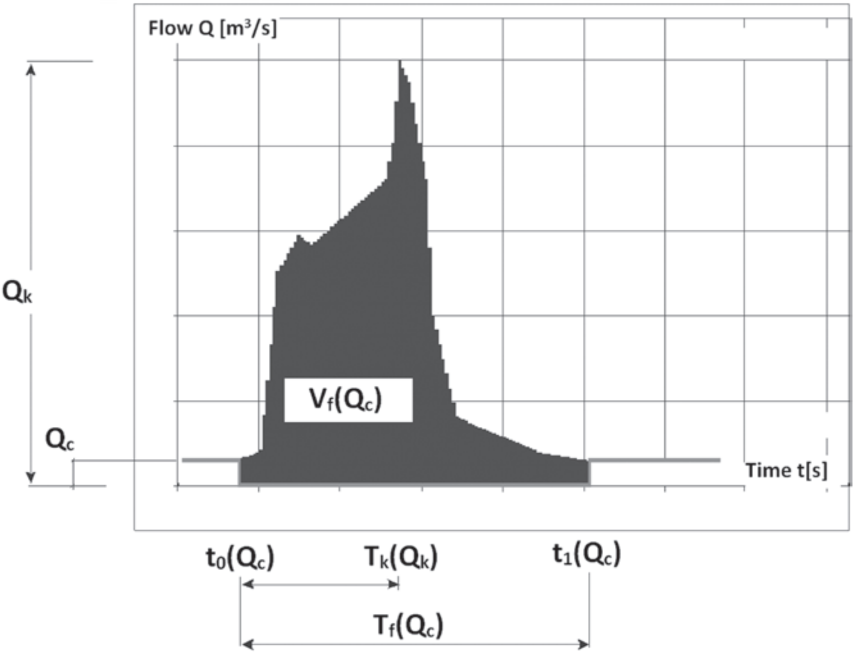

Fig. 4. Method used to assess flood wave characteristics

5. Assumptions underlying hypothetical inflow hydrographs

The use of non-deterministic formulas to generate flood wave hydrographs requires application of equations linking their parameters. The basic equation describes the volume of a flood wave. A variation problem may be defined to find a hydrograph function based on equations describing the hydrograph form [1–4], but such an equation will not always have a solution:

$$V_f - \int_{t_0}^{T_f} \left[Q(t, \Psi) \right] dt = 0 \tag{1}$$

where:

$\Psi(T_k, Q_k, T_f)$ – decision variable vector,
t [s] – time
V_f [m^3] – flood wave volume,
T_f [s] – flood wave duration,
T_k [s] – time of appearance of peak wave,
$Q(t)$ [m^3/s] – wave hydrograph,
Q_k [m^3/s] – flood wave peak.

The above section contains a brief discussion of the theoretical assumptions adopted in generating hypothetical flood wave hydrographs. A set of points in a two-dimensional space containing peak and volume values is generated using a Monte Carlo method [3, 4]. The following equations are then solved for all points in the generated set, depending on parameters a, b and thus forms of hypothetical flood wave hydrographs are obtained.

The analysis discussed includes formulas for two types of hypothetical flood wave populations. A population with a constant time of base flood elevation and a population with a variable time of base flood elevation were developed.

The relation between the flood wave volume and wave duration time is obtained using correlation analyses. The moment of appearance of the peak value equals to 25% of the wave duration time.

5.1. The case with a variable time of base flood elevation

For any:
$T_k \in (0, T_f)$, let us assume that:

$$Q(t) = \begin{cases} \dfrac{Q_k}{T_k^{1/a}} t^{1/a}, t \in \left[0, T_k\right] \\ \dfrac{Q_k}{(T_f - t)^{1/\mathrm{b}}} (T_f - t)^{1/\mathrm{b}}, t \in \left[T_k, T_f\right) \end{cases} \tag{2}$$

$$T_f = 0.5014 V_f$$

$$T_k = 0.25 T_f$$

where:
V_f [Mm^3] – flood wave volume,
T_f [h] – flood wave duration,
T_k [h] – time of appearance of peak wave,
$Q(t)$ [m^3/s] – wave hydrograph,
Q_k [m^3/s] – flood wave peak,
a, b – optimized parameters of a function describing the inflow hydrograph satisfying the equation of flood wave volume (coordinates of the decision variable vector).

5.2. The case with a constant time of base flood elevation

For any:
$T_k \in (0, T_f)$, let us assume that:

$$Q(t) = \begin{cases} \dfrac{Q_k}{T_k^{1/a}} t^{1/a}, t \in \left[0, T_k\right] \\ \dfrac{Q_k}{(T_f - t)^{1/b}} (T_f - t)^{1/b}, t \in \left[T_k, T_f\right) \end{cases} \tag{3}$$

$$T_f = 322$$
$$T_k = 0.25 T_f$$

Regardless of the population type, the basic equation to be satisfied by the equation of a hypothetical flood wave for the conditions indicated above is given below:

$$V = \frac{Q_k}{T_k^{1/a}} \int_0^{T_k} t^{1/a} dt + \frac{Q_k}{(T_f - T_k)^{1/b}} \int_{T_k}^{T_f} (T_f - t)^{1/b} dt \tag{4}$$

6. Application of a Monte Carlo method with the use of the copula function

Copula functions are used as a statistical tool in multi-dimensional modelling of random variable distributions. They link marginal unidimensional probability distributions for a single random variable into a complete multivariate distribution of a multidimensional random variable. Below the reader will find an example of generating a multivariate random variable distribution based on a function from the Archimedean copula family, known as the Gumbel-Hougaard copula. The form of this function is often used in generating extreme distributions describing characteristics of hypothetical flood waves [3].

$$F(u_1, u_2, \ldots, u_m) = \exp\left(-\left[\sum_{k=1}^{m}(-\ln u_k)^{\theta}\right]^{\frac{1}{\theta}}\right) \tag{5}$$

where:

F – cumulative distribution function,

$u_1 = F_1(x_1), u_2 = F_2(x_2), \ldots, u_m = F_m(x_m)$ – represent any marginal distribution function,

Θ – an optimum parameter for the selected copula function,

m – dimension of a multidimensional random variable.

7. Two-dimensional distribution of flood wave characteristics

A two-dimensional random variable distribution based on a function from the Archimedean copula family, known as the Gumbel-Hougaard copula, was used to generate flood wave characteristics.

$$F(u_1, u_2) = \exp\left(-\left[\sum_{k=1}^{2}(-\ln u_k)^{\theta}\right]^{\frac{1}{\theta}}\right) \tag{6}$$

where:

F – cumulative distribution function,

$\left.\begin{matrix} u_1 = F_1(x_1) \\ u_2 = F_2(x_2) \end{matrix}\right\}$ – are marginal distribution functions,

Θ – an optimum parameter for the selected copula function,

$u_k = \int_0^{x_k} \frac{1}{\zeta_k \sigma_{x_k} \sqrt{2\pi}} e^{\frac{-(\ln(\zeta_k) - \mu_{x_k})^2}{2\sigma_{x_k}^2}} d\zeta_k$ – marginal distribution function of random variable *k*,

σ_{x_k}, μ_{x_k} – parameters of marginal distribution of random variable *k*.

Table 3

Values of adopted statistical parameters

Parameters of estimated marginal distributions	Random variable	μ	σ
Log-normal marginal distribution	Q_k/ wave peak	**5.6819**	**0.8943**
Log-normal marginal distribution	V_f/ wave volume	**5.5349**	**0.5422**
Gumbel-Hougaard copula			
Theta	53.3039		

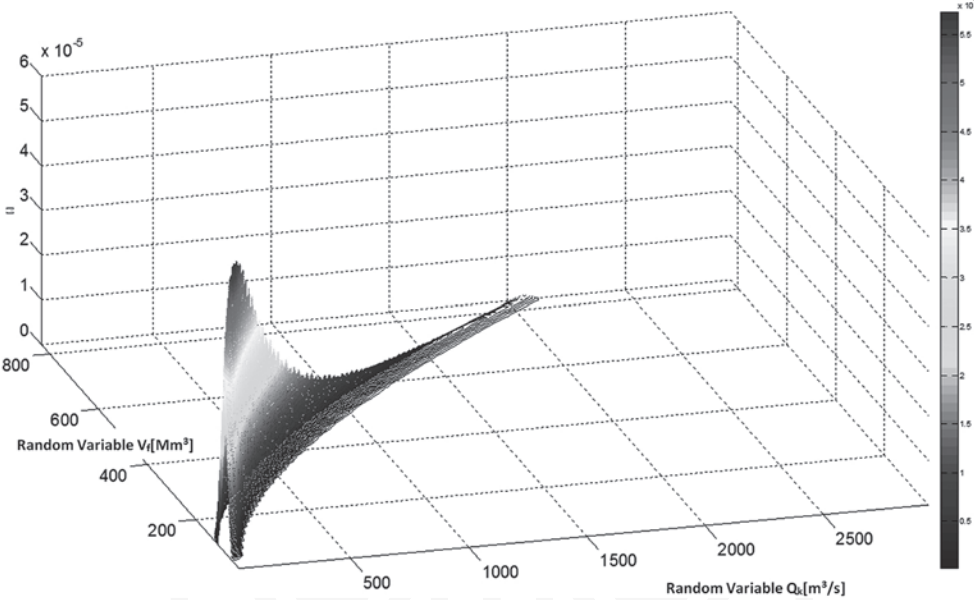

Fig. 5. Probability density function of the two-dimensional random variable (Q_k [m^3/s], V_f [Mm^3])

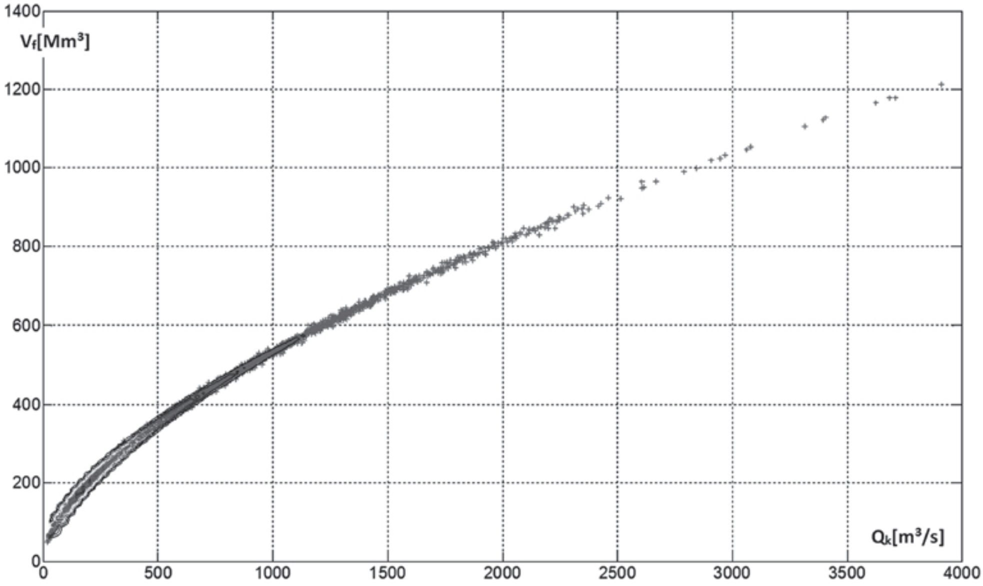

Fig. 6. The characteristics of flood waves generated using a Monte Carlo method
and a two-dimensional random variable distribution (Q_k [m^3/s], V_f [Mm^3]), 10 000 points

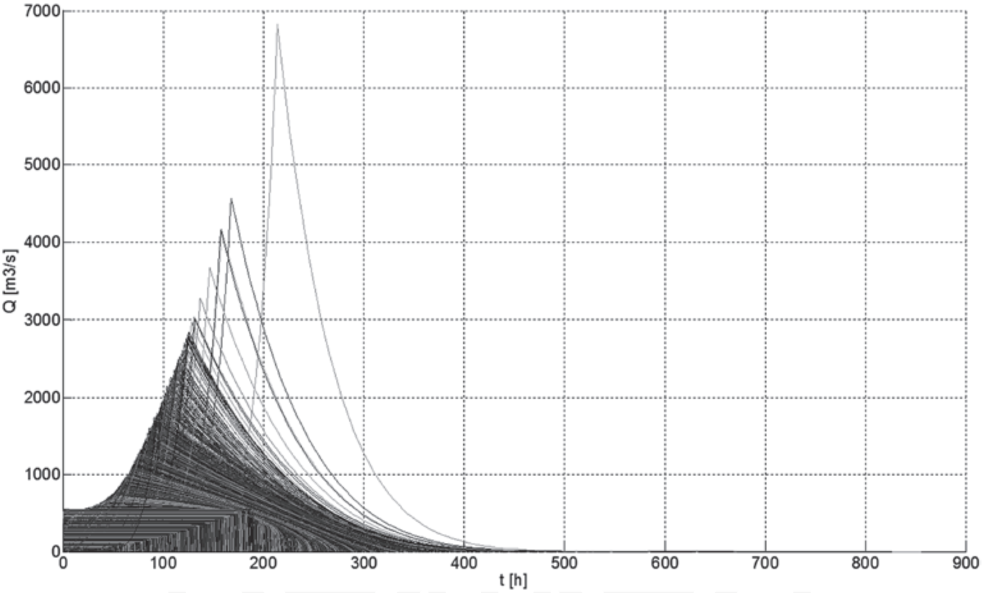

Fig. 7. Obtained hydrographs of flood waves based on the generated parameters, 10 000 waves

8. Control algorithms of the Racibórz reservoir adopted in the analysis

This study is based on the three most popular reservoir control algorithms. These are known as fixed outflow, semi-fixed outflow and step outflow control methods. The basic assumptions for the algorithms and rules for determining outflow from the reservoir are discussed below.

8.1. Fixed outflow control

One of the methods used to manage water volumes in reservoirs is management with a fixed outflow or discharge. This method ignores forecast inflows to the reservoir. If the fixed outflow method is used, control decisions are made depending on current circumstances.

The adopted flood control capacity equals 185 [Mm^3], and the value of allowable (safe) flow is $Q_{\mathrm{all}} = 470$ [m^3/s].

8.2. Semi-fixed outflow control

In the semi-fixed outflow control regime, the values of outflow U are determined as a function of inflow $U = f(Q)$ or the retention volume V at the beginning of the analysed period $U = f(V)$, or of both those factors simultaneously $U = f(Q, V)$. Both U and V are thus random variables. The functions must be defined so as to obtain desired changes in time of both outflow from the reservoir and storage volume.

An example of semi-fixed outflow control is provided by the rule of determining outflows in proportion to excessive inflows compared to an assumed balanced (e.g. safe) flow downstream of the reservoir. If the inflow value exceeds the flow Q_{WYR} downstream of the reservoir, the outflow volume is determined by the equation:

$$U = Q_{WYR} + K \cdot (Q - Q_{WYR}) \tag{7}$$

however, assuming that until Q_{WYR} is reached, the outflow and inflow volumes are equal, and consequently the retention volume is constant in the initial flood stage. After wave passage, i.e. when the inflow value is less than Q_{WYR}, the flood retention volume is discharged with an outflow equal to Q_{WYR}.

The flood control capacity adopted in the analysis equals 185 Mm^3, the value of allowable (safe) flow is $Q_{\mathrm{all}} = 470\ \mathrm{m}^3/\mathrm{s}$, and the proportionality constant $K = 0.5$.

Step outflow control

As the volume of water that may be discharged from the reservoir rises, the controlled outflow volume is increased stepwise.

$$U(t) = f(Q(t), V_{akt}(t), Q_{1WYR}, Q_{2WYR}, Q_{3WYR}, \ldots) \tag{8}$$

where:

$U(t)$ [m^3/s] – controlled discharge,
$Q(t)$ [m^3/s] – inflow to the reservoir,
$V_{akt}(t)$ – water retention in the reservoir,
Q_{1WYR}, $Q_{2\,WYR}$, $Q_{3\,WYR}$ [m^3/s] – characteristic outflow values.

The adopted flood control capacity equals 185 Mm^3, and the adopted values of flows equal: 470 m^3/s, 800 m^3/s, 1070 m^3/s and 1600 m^3/s.

9. Results of analyses

An analysis of the control rules applied in reservoir management under flood conditions requires that certain reservoir parameters be adopted, namely a total capacity and a flood control capacity. The control rule used is another indispensable component. One value of the Racibórz reservoir capacity and three control rules applied under flood conditions are assumed in the calculations discussed. The adopted value of safe outflow downstream of the reservoir, as proposed in the study [8, 9], equals to 470 m^3/s.

A set of parameters for 10 000 hypothetical hydrographs was obtained from a set of 10 000 points in a two-dimensional space, using a Monte Carlo method and assuming log-normal marginal distributions. Equations for 10 000 cases were subsequently solved to obtain 10 000 hydrographs of hypothetical flood waves. A simulation of the Racibórz reservoir operation was then conducted for the entire set. Outflow hydrographs were obtained from the simulation. Peak values of those hydrographs were used to assess the values of outflows downstream of the reservoir with a certain probability of exceedance.

The figures below illustrate selected simulation results in the form of outflow hydrographs, histograms of maximum outflow values and graphic comparisons of cumulative distribution functions of empirical maximum inflows to the reservoir and maximum outflows from the reservoir.

Analyses of the control rules were performed for two populations, each consisting of 10 000 hypothetical flood waves. The first population was solved for a variable duration time of the hypothetical flood wave. The second population was developed for a constant duration time of the hypothetical flood wave.

9.1. An analysis of Racibórz polder control for a population of hypothetical flood waves characterised by a variable time of base flood elevation

9.1.1. The case of fixed outflow control rule

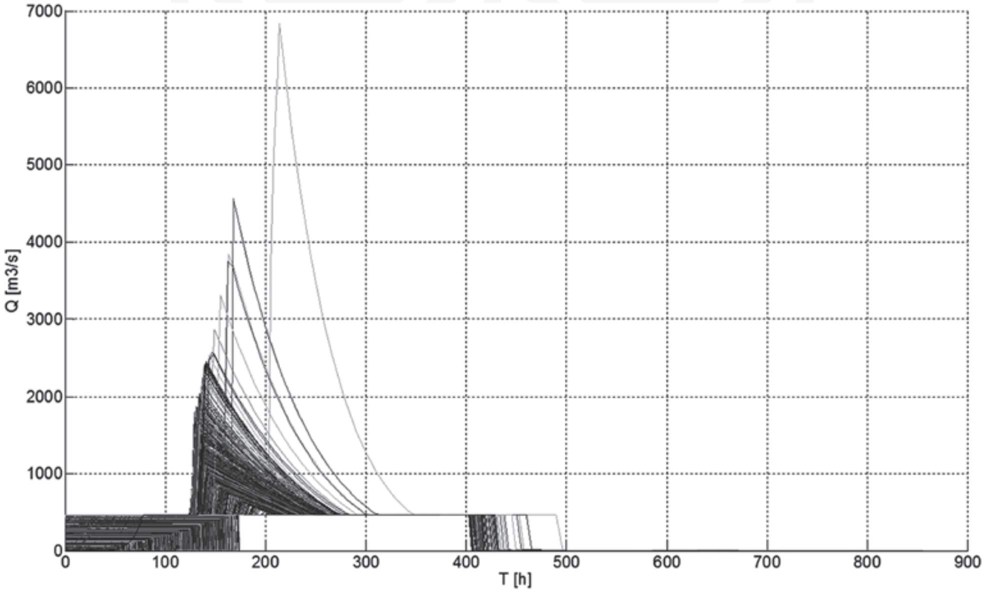

Fig. 8. Hydrographs of outflows from the Racibórz reservoir, 10 000 waves, a variable time of base flood elevation

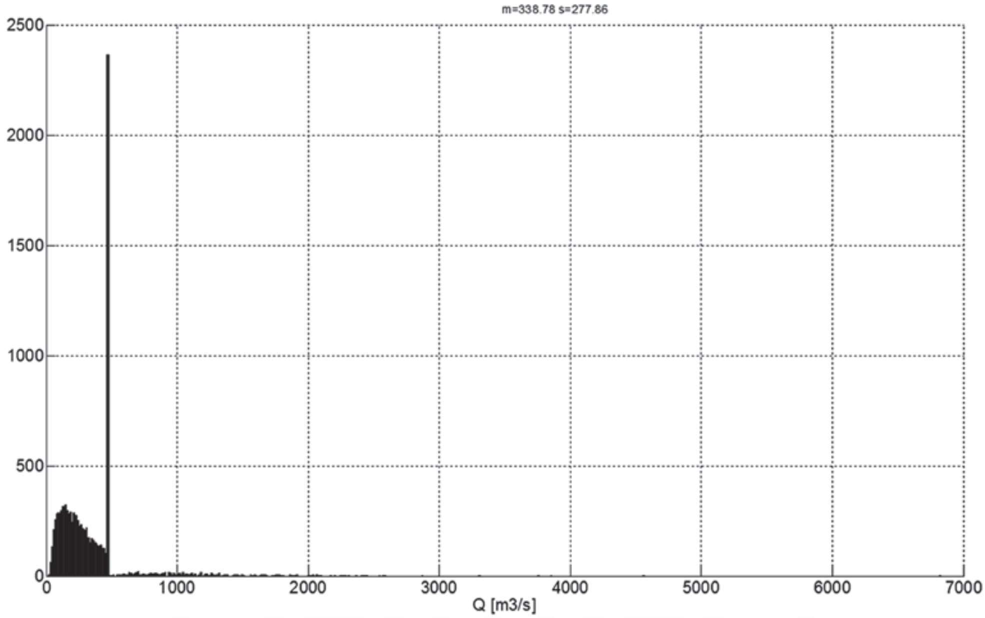

Fig. 9. A histogram of maximum outflow values, 10 000 waves, a variable time of base flood elevation

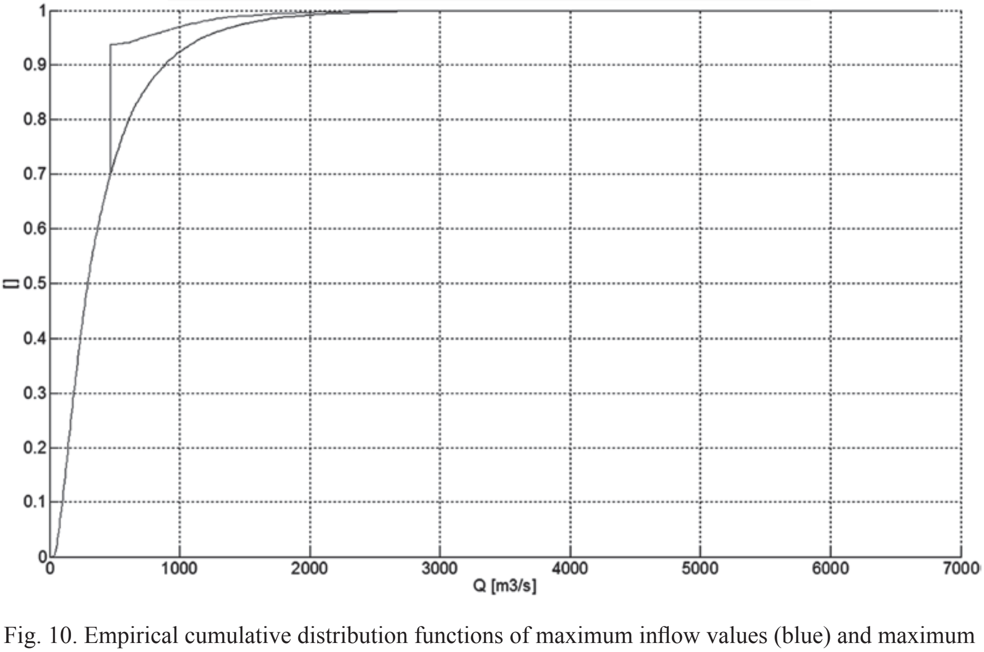

Fig. 10. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a variable time of base flood elevation

9.1.2. The case of semi-fixed outflow control rule

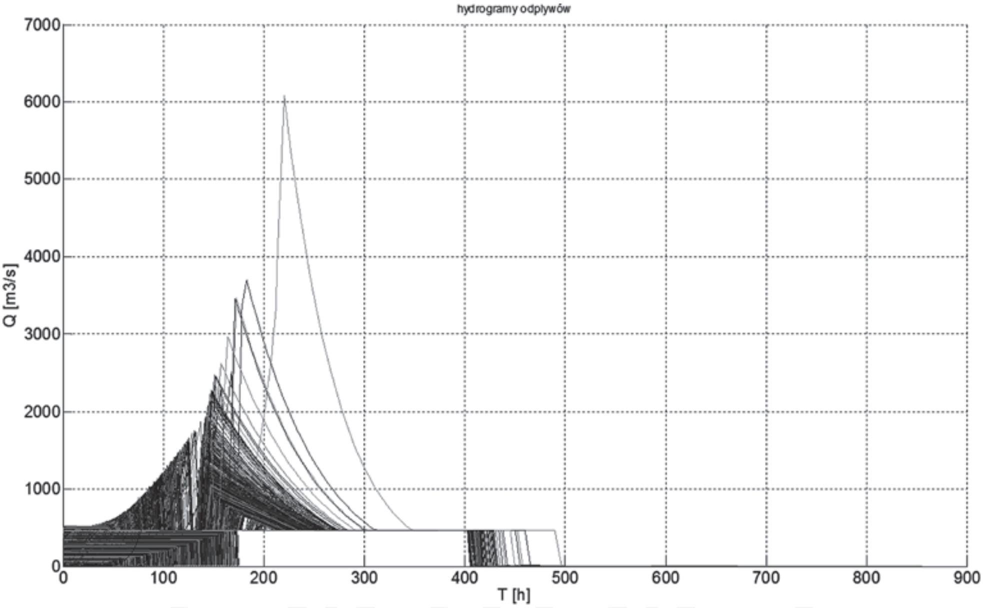

Fig. 11. Hydrographs of wave outflows from the Racibórz reservoir, 10 000 waves, a variable time of base flood elevation

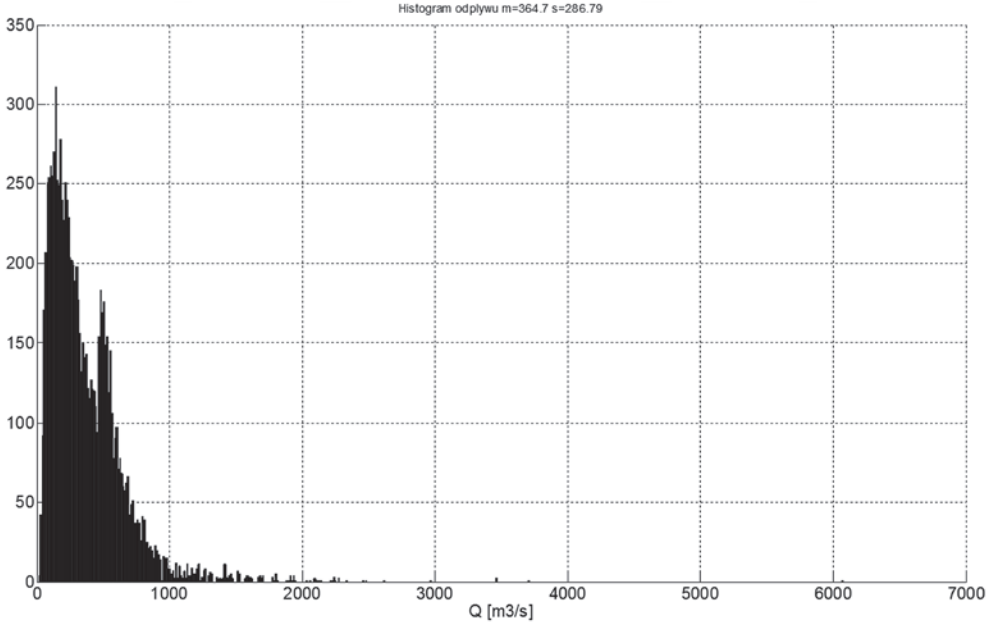

Fig. 12. A histogram of maximum outflow values, 10 000 waves, a variable time of base flood elevation

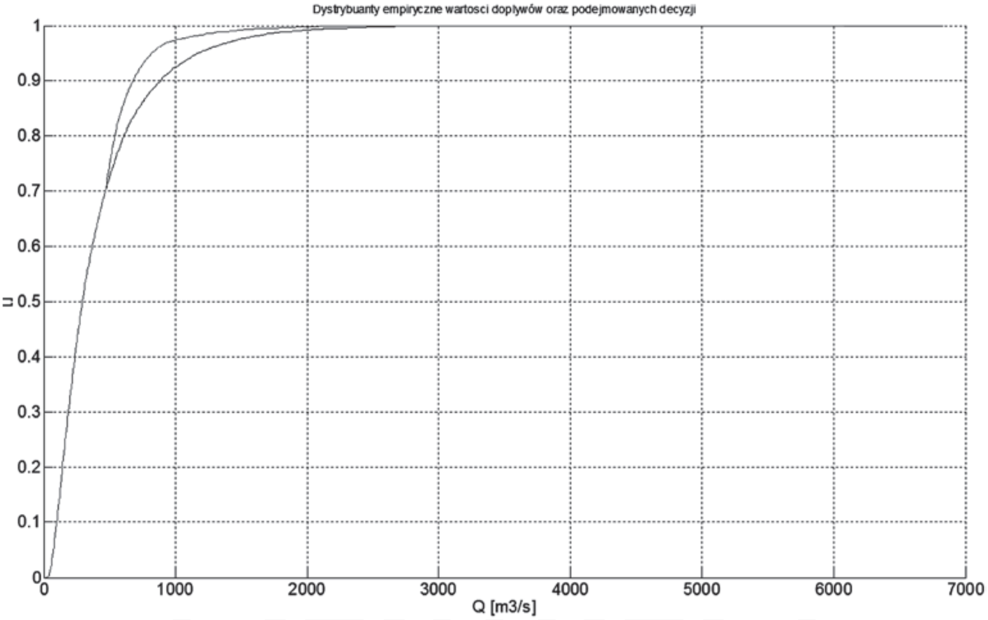

Fig. 13. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a variable time of base flood elevation

9.1.3. The case of step outflow control rule

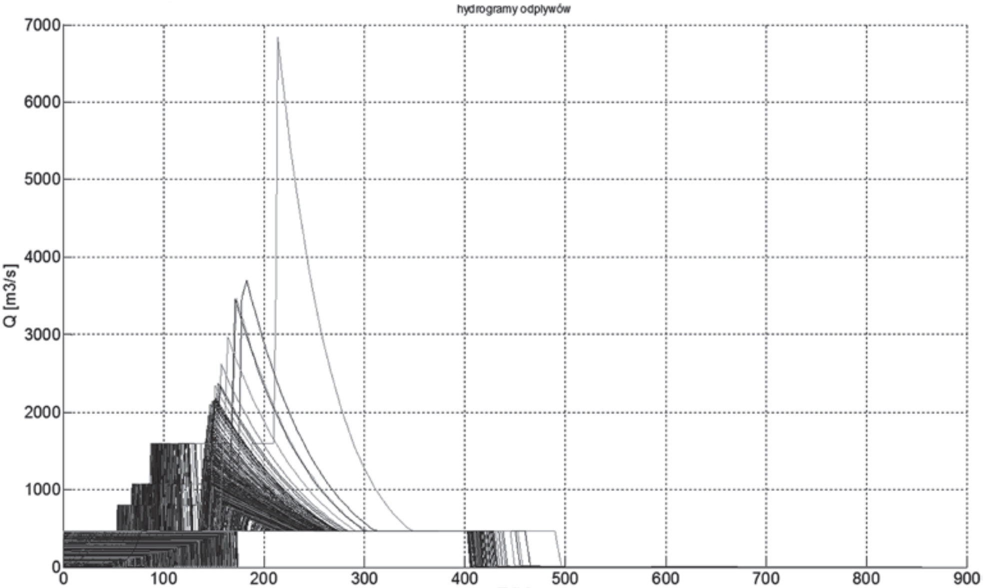

Fig. 14. Hydrographs of wave outflows from the Racibórz reservoir, 10 000 waves, a variable time of base flood elevation

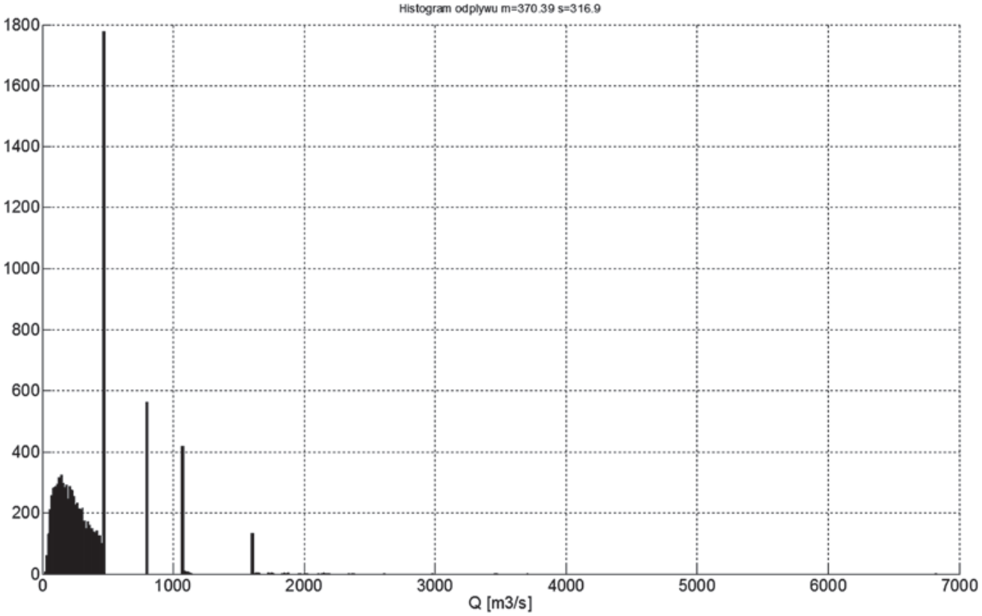

Fig. 15. A histogram of maximum outflow values, 10 000 waves, a variable time of base flood elevation

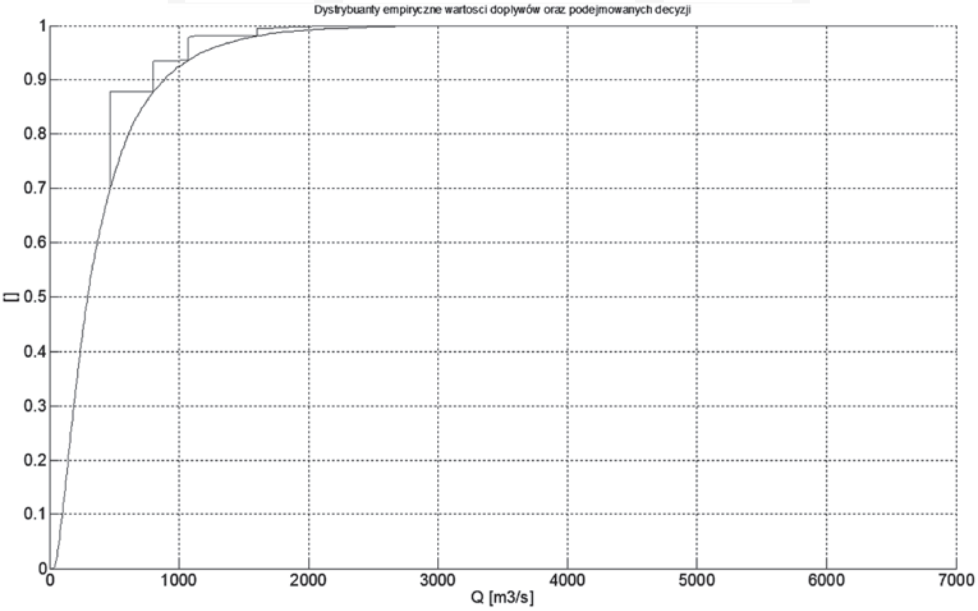

Fig. 16. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a variable time of base flood elevation

9.2. An analysis of Racibórz polder control for a population of hypothetical flood waves characterised by a constant time of base flood elevation

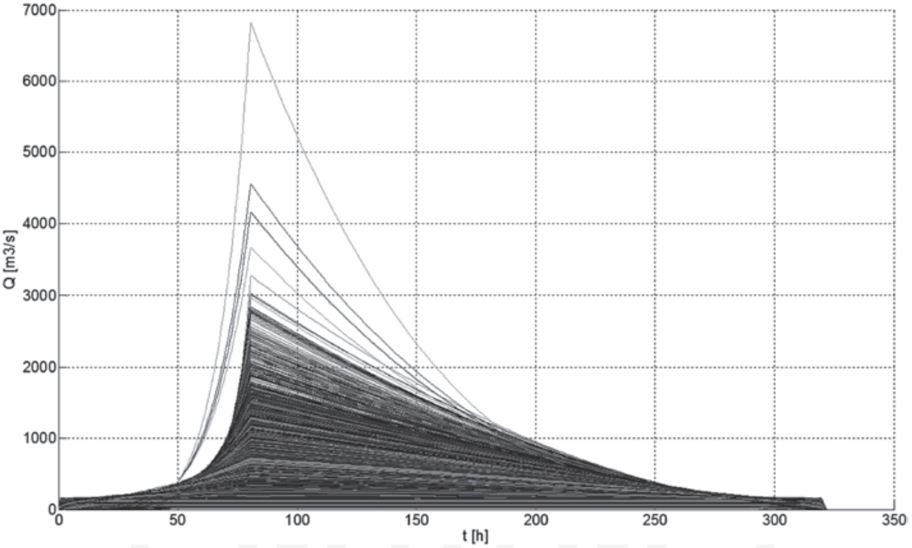

Fig. 17. Obtained hydrographs of flood waves based on the generated parameters, 10 000 waves, a constant time of base flood elevation

9.2.1. The case of fixed outflow control rule

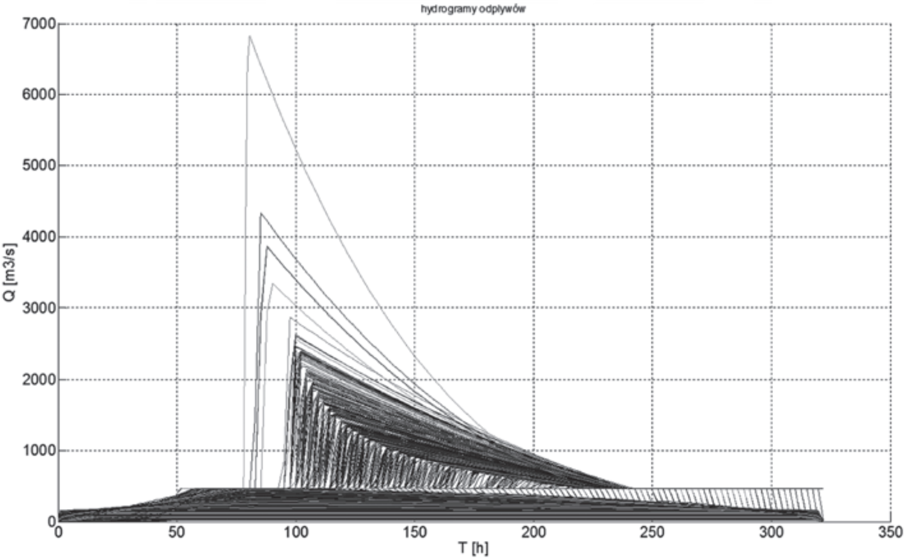

Fig. 18. Obtained hydrographs of outflows, 10 000 waves, a constant time of base flood elevation

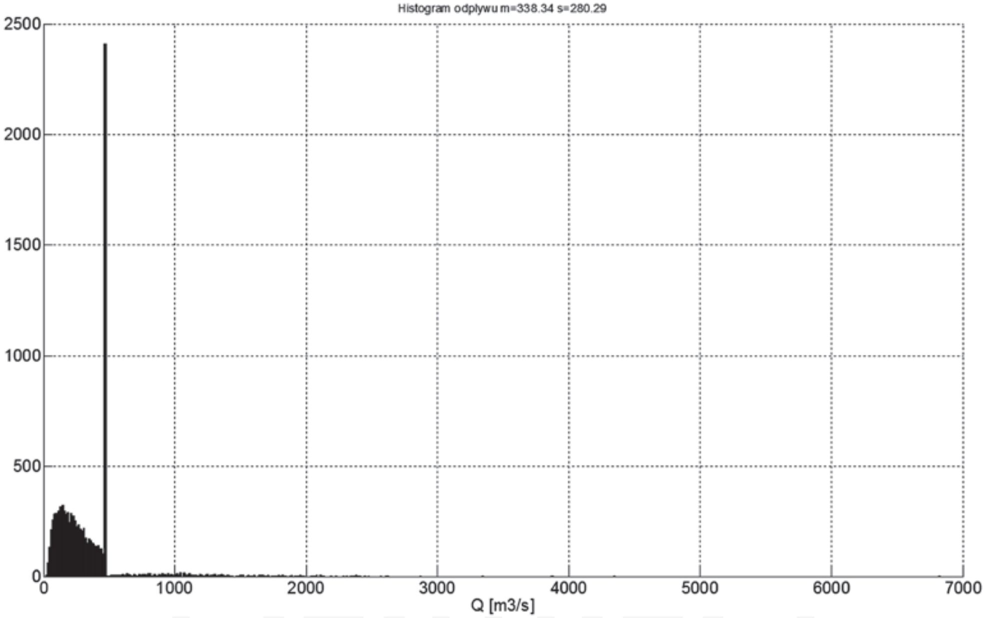

Fig. 19. A histogram of maximum outflow values, 10 000 waves, a constant time of base flood elevation

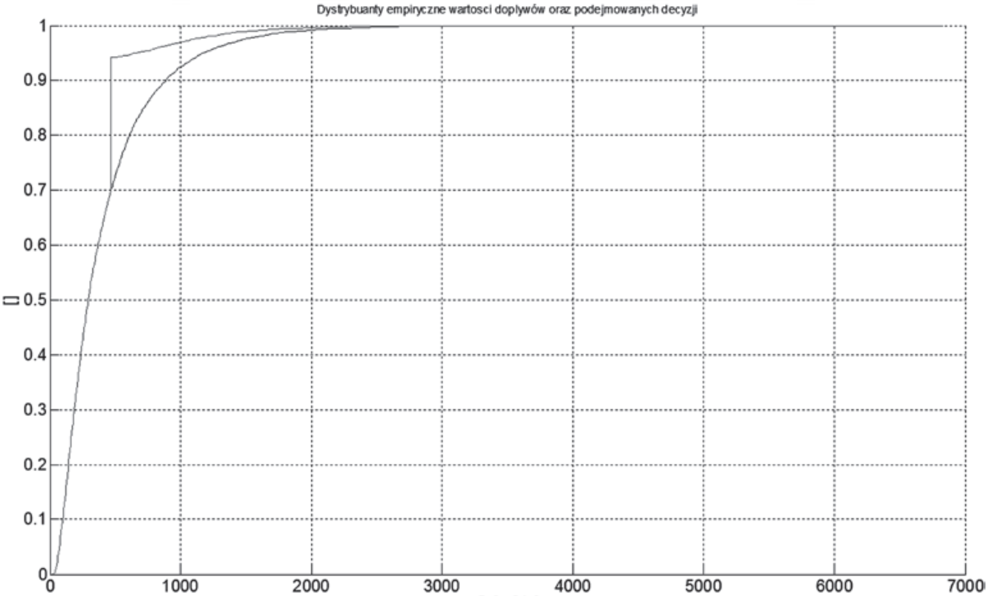

Fig. 20. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a constant time of base flood elevation

9.2.2. The case of semi-flexible outflow control rule

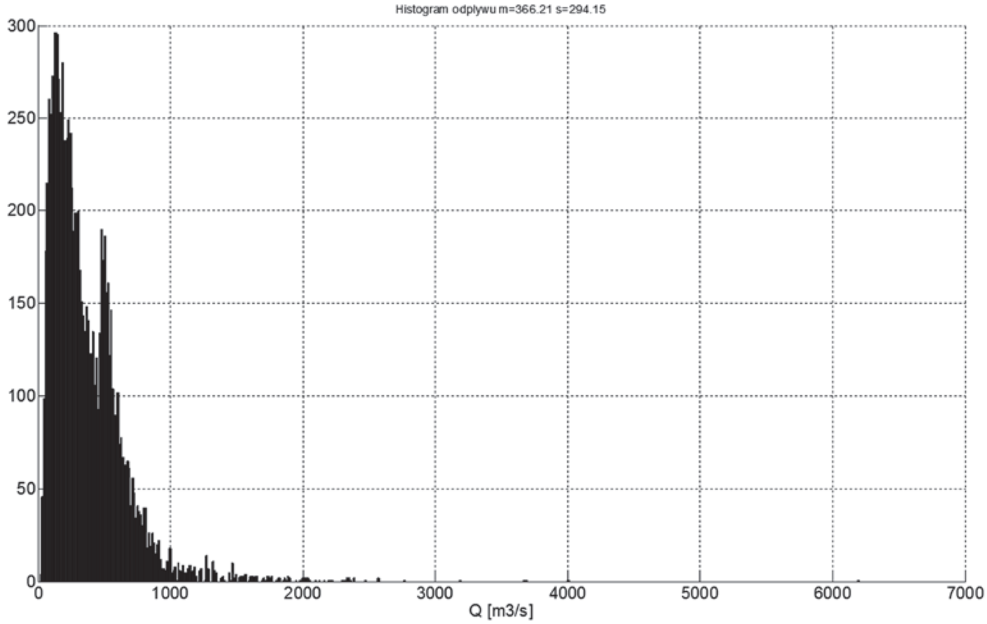

Fig. 21. A histogram of maximum outflow values, 10 000 waves, a constant time of base flood elevation

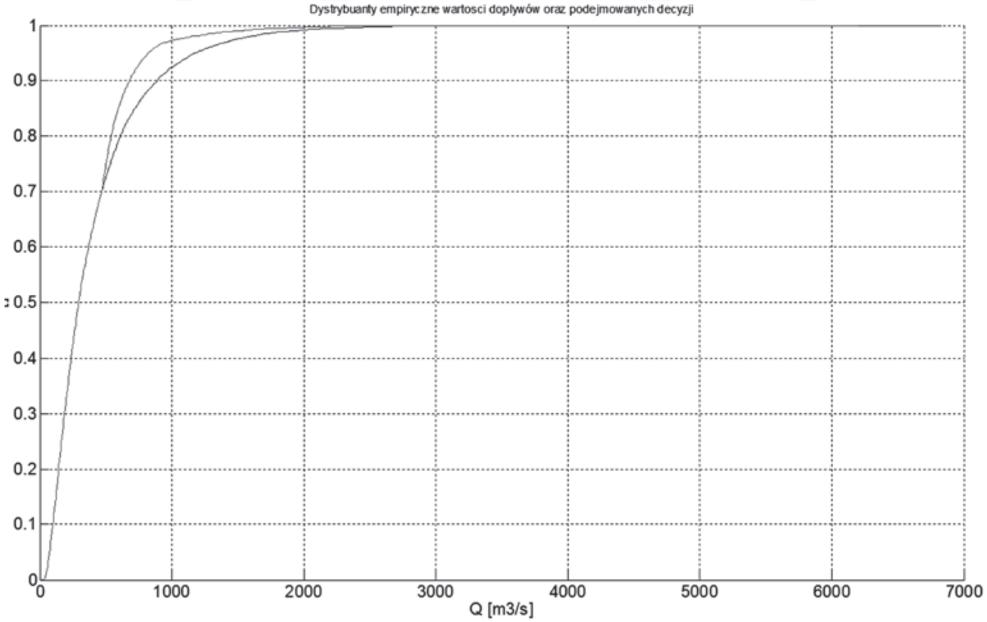

Fig. 22. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a constant time of base flood elevation

9.2.3. The case of step outflow control rule

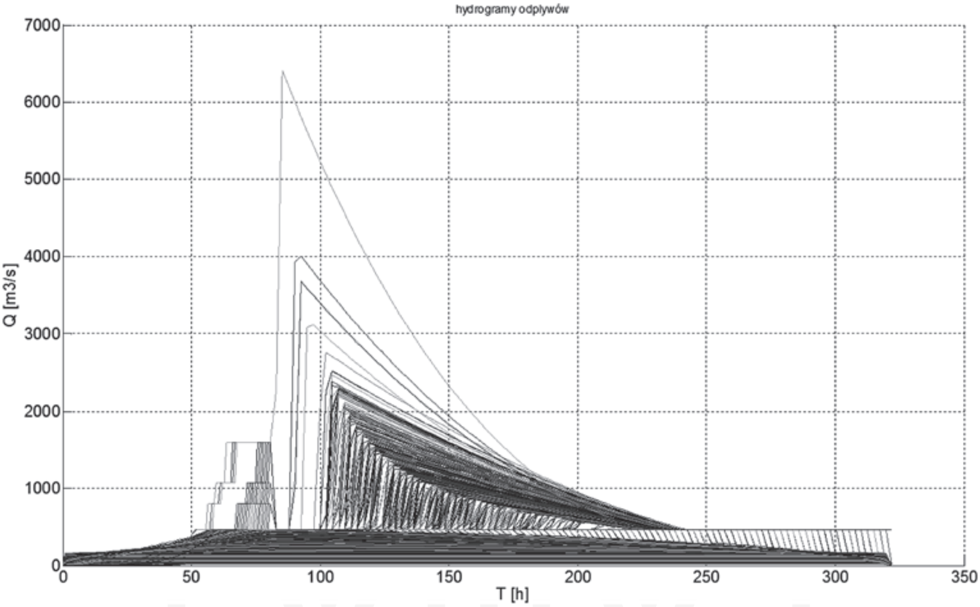

Fig. 23. Hydrographs of outflows, 10 000 waves, a constant time of base flood elevation

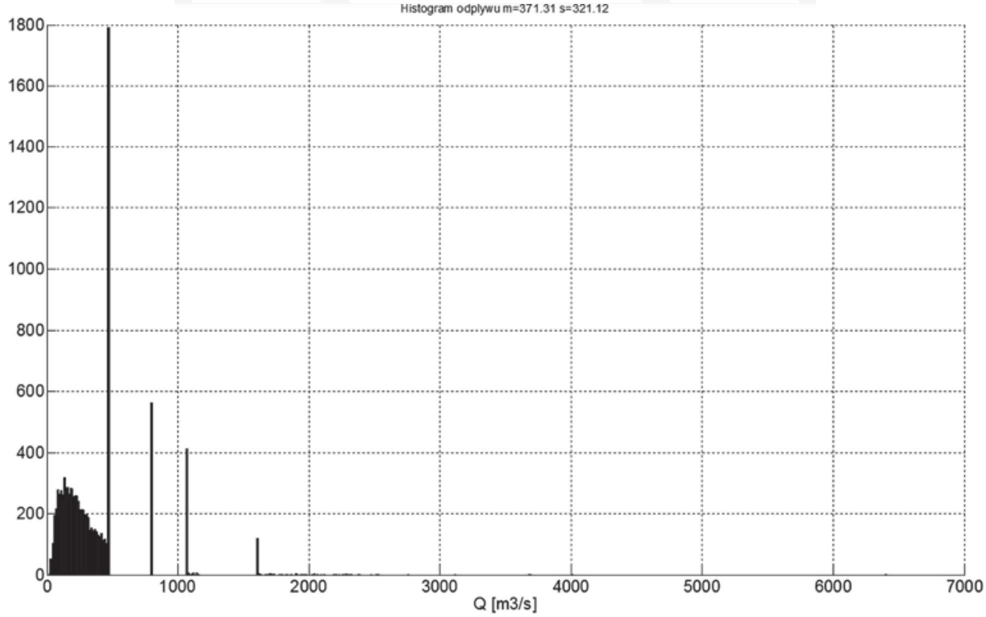

Fig. 24. A histogram of maximum outflow values, 10 000 waves, a constant time of base flood elevation

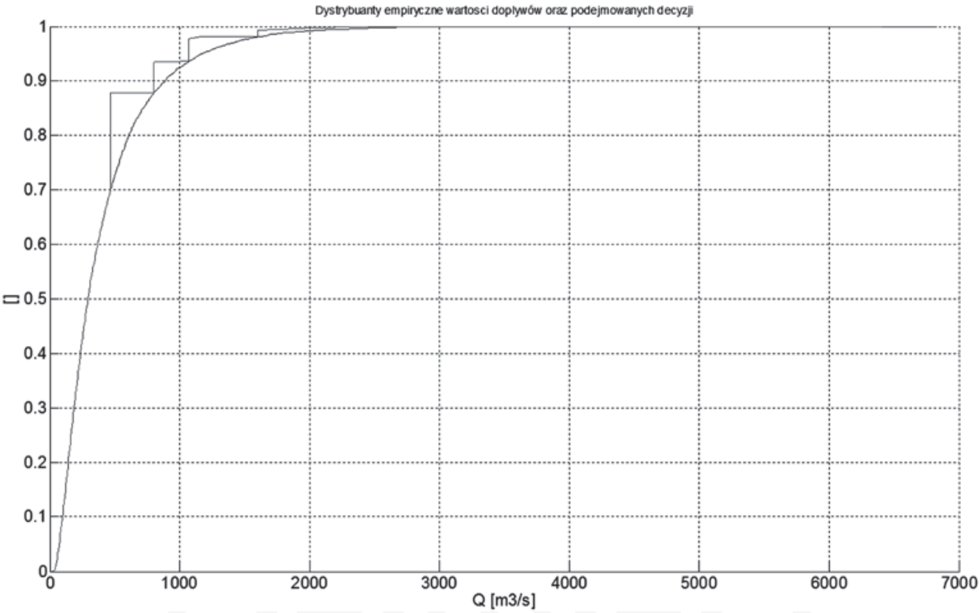

Fig. 25. Empirical cumulative distribution functions of maximum inflow values (blue) and maximum discharged outflows (red) in the reservoir, 10 000 waves, a constant time of base flood elevation

10. Conclusions

This study discusses the application of a method for determining characteristic flow values downstream of a retention reservoir and generating flood wave hydrographs using a Monte Carlo method, a two-dimensional copula function and power functions describing the hydrograph forms. The example discussed shows the interrelations between generated flood wave characteristics and hydrograph forms; the study also demonstrates the applicability of the results of this method in assessing the operation of a retention reservoir. A two-dimensional random number generator for peak values and volumes of flood waves was developed for the adopted Gumbel-Hougaard copula distribution. The forms of hydrographs of hypothetical flood waves were determined for a generated set of 10 000 two-element sets of flood wave characteristics, including the peak value and volume. The described operations were completed in order to perform an analysis of decisions made by staff managing a retention reservoir with the use of three control rules: fixed outflow, semi-flexible outflow and step outflow control. The analysis was performed for two types of populations of hypothetical flood waves, each consisting of 10 000 elements: with a variable and with a constant time of base flood elevation. All analyses were performed assuming a constant flood control capacity equal to 185 Mm^3.

The analyses performed clearly demonstrate that the reduction values of characteristic flows for a probability of exceedance of 1% reach a maximum value up to about 920 m^3/s in the control regimes analysed. Calculations were also performed for larger waves (a lower probability of exceedance, peak, volume), but conclusions concerning the value of reduction of larger waves are affected by a larger error due to the level of precision in the analyses performed. About 2% points per generated population represent a set without correct solutions of the proposed equations. This absence of solutions results from the relation between the random variable of volume and the product of random variables of time and peak value. The problem may be solved by introducing truncated distributions that will be discussed in a separate study. The indicated points are concentrated principally in the areas of relatively small peak values and very large volumes.

A comparison of results for the populations with a variable time and a constant time of base flood elevation represents another important component of the analyses performed. In the analysis with a constant time value, an average value of wave duration was adopted. It may be concluded that the results of both analyses are almost identical with an accuracy of 5% (Fig. 34). This results principally from the time of base flood elevation being expressed as a function of wave volume (or vice versa, the volume as a function of time, this is irrelevant for calculation results), as a natural consequence of the relation between time and volume, and from a correlation analysis. This consistency would not exist if time was expressed as a function of peak value. There are no obstacles (except the limited number of elements in historical sequences) that would prevent generation of three numbers (Q, V, T), but such an approach extends calculation time, and consequently it is more reasonable to use the time–volume relationship. Assumptions for the indicated alternative approach and examples of analyses were presented in another study [3]. It should also be emphasised that the statistical independence of random variables Q and V was assumed in the analyses which limits the upper values of reduction in a justified manner.

Table 4

A statement of characteristic flow values and calculated reduction values for the Racibórz reservoir, Racibórz cross-section, flood control capacity of 185 Mm^3, the method with a variable time of base flood elevation

Probability of exceedance	Racibórz-Miedonia water-level gauge	Fixed outflow rule	Reduction – fixed outflow rule	Semi-fixed outflow rule	Reduction – semi-fixed outflow rule	Step outflow rule	Reduction – step outflow rule
0.5	293.5	294.4	–0.9	294.4	-0.9	294.4	-0.9
0.4	368.1	368.6	–0.4	368.6	-0.4	368.6	-0.4
0.3	469.1	469.8	–0.6	469.8	-0.6	469.8	-0.6
0.2	623.0	470.0	153.0	540.5	82.5	470.0	153.0
0.1	923.3	470.0	453.3	680.3	243.0	800.0	123.3
0.05	1277.8	708.5	569.3	820.1	457.7	1070.0	207.8
0.03	1578.0	1017.6	560.3	953.9	624.1	1070.0	508.0
0.02	1841.9	1181.8	660.1	1139.5	702.4	1119.9	722.0

Table 4 cont.

0.019	1877.0	1213.8	663.3	1170.8	706.3	1600.0	277.0
0.018	1914.4	1225.1	689.4	1180.8	733.6	1600.0	314.4
0.017	1954.5	1260.7	693.8	1214.7	739.8	1600.0	354.5
0.016	1997.5	1270.5	727.0	1223.4	774.1	1600.0	397.5
0.015	2043.9	1305.9	737.9	1257.3	786.6	1600.0	443.9
0.014	2094.2	1318.1	776.1	1268.4	825.8	1600.0	494.2
0.013	2149.1	1357.7	791.4	1305.8	843.3	1600.0	549.1
0.012	2209.3	1381.6	827.7	1354.3	855.1	1600.0	609.3
0.011	2276.0	1445.4	830.6	1415.8	860.2	1600.0	676.0
0.01	2350.4	1489.6	860.8	1432.8	917.6	1600.0	750.4

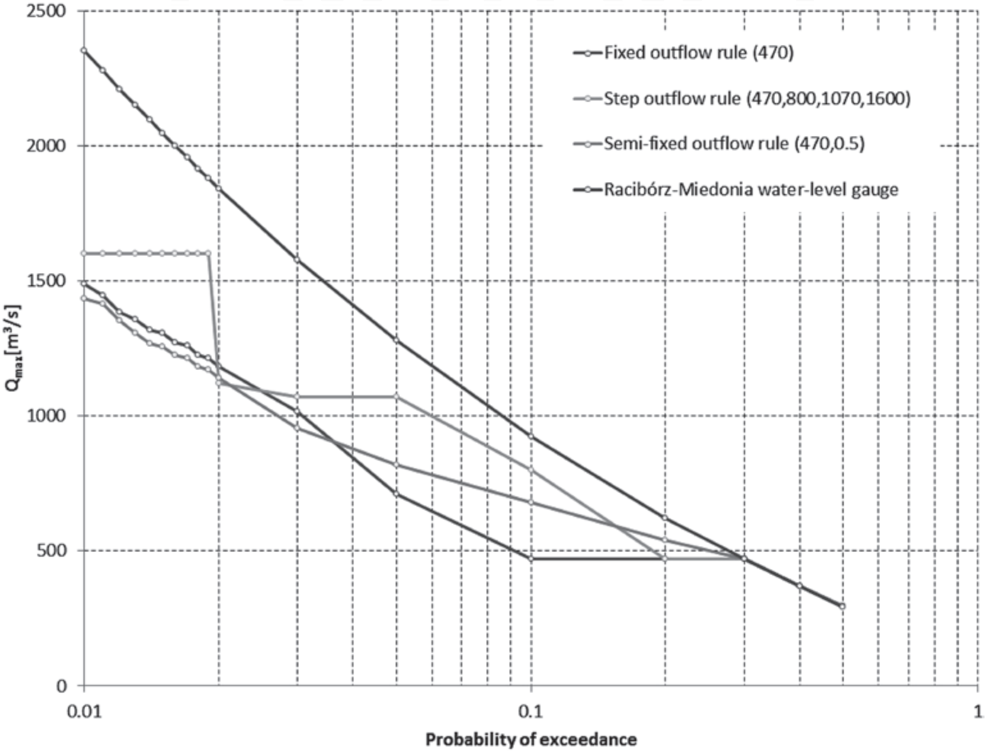

Fig. 26. Characteristic flow values downstream of the Racibórz reservoir for the analysed control rules, 10 000 waves, a variable time of base flood elevation

Table 5

A statement of characteristic flow values and calculated reduction values for the Racibórz reservoir, Racibórz cross-section, flood control capacity of 185 Mm³, the method with a constant time of base flood elevation

Probability of exceedance	Racibórz-Miedonia water-level gauge	Fixed outflow rule	Reduction – fixed outflow rule	Semi-fixed outflow rule	Reduction – semi-fixed outflow rule	Step outflow rule	Reduction – step outflow rule
0.5	293.5	294.4	–0.9	294.4	–0.9	294.4	–0.9
0.4	368.1	368.6	–0.4	368.6	–0.4	368.6	–0.4
0.3	469.1	469.8	–0.6	469.8	–0.6	469.8	–0.6
0.2	623.0	470.0	153.0	540.5	82.5	470.0	153.0
0.1	923.3	470.0	453.3	680.3	243.0	800.0	123.3
0.05	1277.8	653.1	624.7	820.1	457.7	1070.0	207.8
0.03	1578.0	1014.9	563.1	967.2	610.8	1070.0	508.0
0.02	1841.9	1206.4	635.5	1162.3	679.6	1152.4	689.5
0.019	1877.0	1242.5	634.5	1184.5	692.5	1600.0	277.0
0.018	1914.4	1277.3	637.2	1219.2	695.3	1600.0	314.4
0.017	1954.5	1299.8	654.7	1237.8	716.7	1600.0	354.5
0.016	1997.5	1323.9	673.6	1269.2	728.3	1600.0	397.5
0.015	2043.9	1347.0	696.9	1283.6	760.3	1600.0	443.9
0.014	2094.2	1364.2	730.0	1321.4	772.8	1600.0	494.2
0.013	2149.1	1404.4	744.7	1338.2	810.9	1600.0	549.1
0.012	2209.3	1456.0	753.3	1393.4	815.9	1600.0	609.3
0.011	2276.0	1512.1	763.9	1463.9	812.1	1600.0	676.0
0.01	2350.4	1554.9	795.6	1480.8	869.6	1600.0	750.4

Table 6

A statement of differences between characteristic flows according to the methods with a variable and with a constant time of base flood elevation for the Racibórz cross-section, flood control capacity of 185 Mm³

Probability of exceedance	Fixed outflow rule	Semi-fixed outflow rule	Step outflow rule
0.5	0	0	0
0.4	0	0	0
0.3	0	0	0
0.2	0	0	0
0.1	0	0	0

Table 6 cont.

0.05	55	0	0
0.03	3	–13	0
0.02	–25	–23	–33
0.019	–29	–14	0
0.018	–52	–38	0
0.017	–39	–23	0
0.016	–53	–46	0
0.015	–41	–26	0
0.014	–46	–53	0
0.013	–47	–32	0
0.012	–74	–39	0
0.011	–67	–48	0
0.01	–65	–48	0

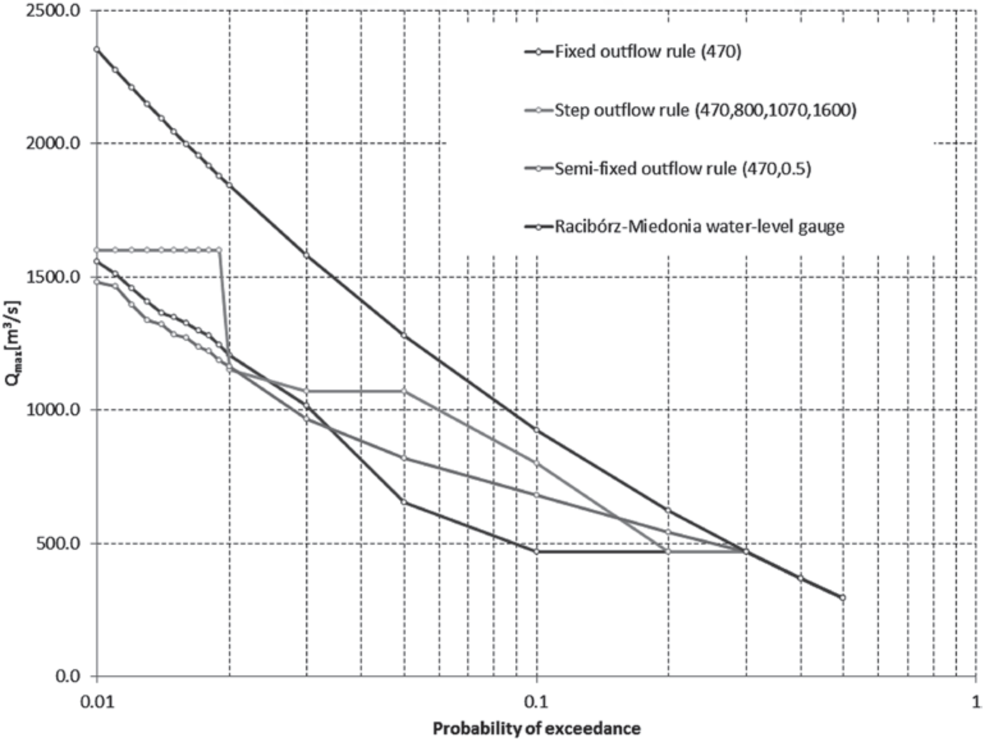

Fig. 27. Characteristic flow values downstream of the Racibórz reservoir for the analysed control rules, 10 000 waves, a constant time of base flood elevation

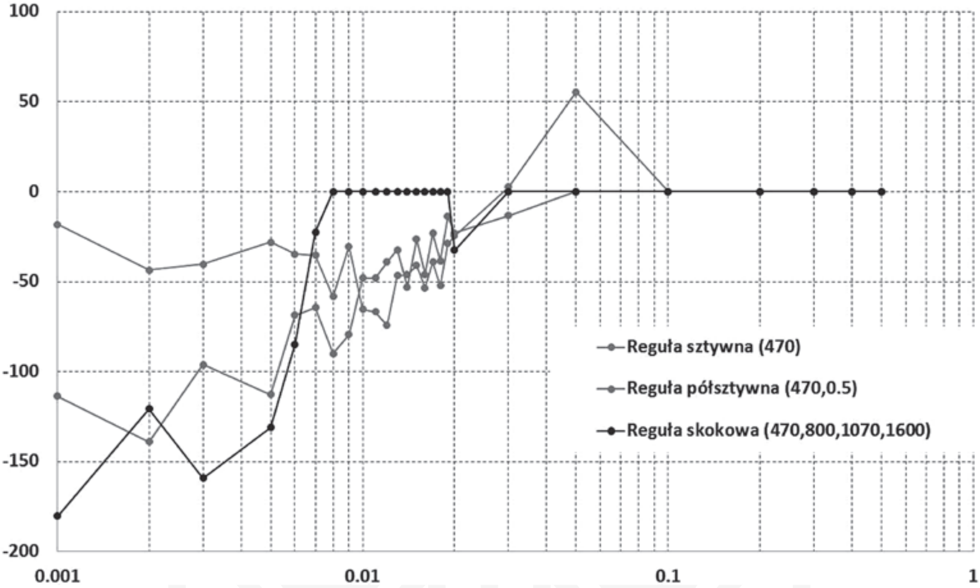

Fig. 28. A statement of differences between characteristic flows according to the methods with a variable and with a constant time of base flood elevation

The method proposed provides an alternative tool that may be used in testing or stochastic optimization of retention reservoir parameters and control rule parameters under flood conditions. Due to the possibilities offered by the copula function, as used in this study, the proposed method of generating hydrographs and testing reservoir parameters provides invaluable material for simulations, characterised by theoretical marginal distribution functions consistent with empirical marginal distribution functions.

The possibilities of the proposed use of MC methods are not limited to generating flood waves and analyses of retention reservoir operation. The method discussed gives freedom in designing and testing structure and equipment parameters that are dependent on certain random, statistical values. The effectiveness of MC methods in the area of application described is enhanced by the use of the copula function in generating multivariate distributions of a random variable with various, freely selected marginal distributions.

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