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ON SOME ASPECTS OF THE SET THEORY AND TOPOLOGY IN J. PUZYNA'S MONUMENTAL WORK

O NIEKTÓRYCH APSEKTACH TEORII MNOGOŚCI I TOPOLOGII W MONUMENTALNYM DZIELE PUZYNY

A b s t r a c t

The article highlights certain aspects of the set theory and topology in Puzyna's work *Theory of analytic functions* (1899, 1900). In particular, the following notions are considered: derivative of a set, cardinality, connectedness, accumulation point, surface, genus of surface.

Keywords: *set theory*, *point-set topology*, *surface topology*, *mathematics at the edge of XIX and XX centuries*, *history of complex analysis*, *University of Lvov, Józef Puzyna*

S t r e s z c z e n i e

W artykule uwypuklono wybrane aspekty dotyczące teorii mnogości i topologii w dziele Puzyny *Teorya funkcyj analitycznych* (1899, 1900). Odniesiono się m.in. do następujących pojęć: pochodna zbioru, moc zbioru, spójność, punkt skupienia zbioru, powierzchnia, rodzaj powierzchni.

Słowa kluczowe: *teoria mnogości*, *topologia teoretyko-mnogościowa*, *topologia powierzchni*, *matematyka na przełomie XIX i XX w.*, *historia analizy zespolonej*, *Uniwersytet we Lwowie, Józef Puzyna*

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1. Introduction

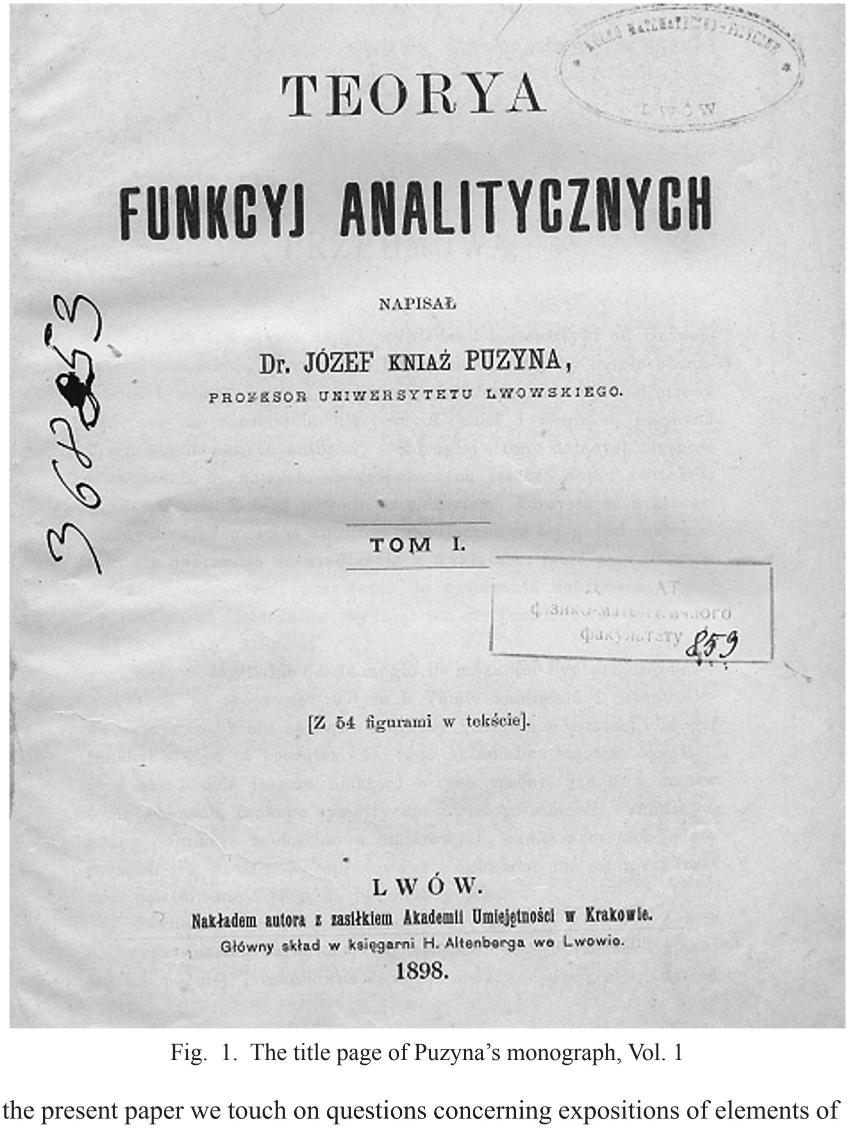
Józef Puzyna was a Polish mathematician. He is recognized as a precursor of the Lvov School of Mathematics. He was born on the 18th of April, 1856 in Nowy Martynów, a place near Rohatyń (now Ukraine). Let us recall the most important events from his biography (see, e.g. [4, 6, 15] for details). After studying at the Lvov University in 1875–1882 as W. Żmurko's student, and at the University of Berlin as K. Weierstrass' student, and, after finishing the doctorate degree in 1883 on the basis of the dissertation *O pozornie dwuwartościowych określonych całkach podwójnych* (*On seemingly bivalent definite double integrals*) at the Lvov University, he associated his scientific and teaching activities only with the Lvov University. In 1885 he got habilitation and taught mathematics as an assistant professor. He headed the Department of Mathematics as an associate professor in the period of 1889–1892 and since 1892 as a professor until his death. He was a very good lecturer and lectured on many branches of mathematics. He also held positions of responsibility at the university: he was the rector in 1904/5 academic year, and vice-rector in 1905/6, Dean of the Faculty of Philosophy in 1894–1895. In 1907 Puzyna participated in the work related to a survey conducted among all university professors of mathematics of the Monarchy. The purpose of this survey was to develop a memorandum which was submitted to the Minister of Religious Affairs and Education in Vienna. The memorandum showed the necessity of increasing the number of chairs of mathematics at universities of the Monarchy. Since 1917, Puzyna served as the President of the Mathematical Society in Lvov. Among his scientific descendants there were Franciszek Leja, Hugo Steinhaus, Antoni Łomnicki, Wacław Sierpiński, Stanisław Ruziewicz.

Puzyna died in 1919 in Stryj.

Józef Puzyna

Józef Puzyna was extremely devoted to the issues of teaching mathematics, both in schools and at the universities. From his numerous reviews one can learn that he paid a lot of attention to the contents of textbooks emphasizing the role of general ideas in exposition of the material. As Puzyna wrote, "a student of mathematics should know about those who for centuries made it possible for us to get that knowledge in a general and comfortable form that we can enjoy today".

One of Puzyna's main achievements was his monograph *The theory of analytic functions.* When J. Puzyna asked the Ministry of Religion and Enlightenment in Vienna for a grant to publish *The theory of analytic functions*, he received a negative response as there was a shortage of resources for research (as well as positions). The book was published in two volumes [16, 17]. Both volumes were self-published by the author, with some support by Academy of Sciences and Arts in Kraków.



TEORYA

FUNKCYJ ANALITYCZNYCH

NAPISAŁ

Dr. JÓZEF KNIAŻ PUZYNA,

PROFESOR UNIWERSYTETU LWOWSKIEGO.

TOM I.

[Z 54 figurami w tekście].

LWÓW.

Nakładem autora z zasiłkiem Akademii Umiejętności w Krakowie.

Główny skład w księgarni H. Altenberga we Lwowie.

1898.

Fig. 1. The title page of Puzyna's monograph, Vol. 1

In the present paper we touch on questions concerning expositions of elements of the set theory as well as topology in Puzyna's monograph. We use some materials already published by the first author.

2. Elements of the set theory

The set theory was created by Georg Cantor in 1874. Its foundations were expounded in Cantor's paper [3]. Immediately after its inception, the new theory actually divided the mathematical world. Some mathematicians (Frege, Dedekind, Hilbert) fully accepted it while others, including Poincaré and Hermann Schwarz, categorically rejected it.

A new wave of interest in the set theory emerged in the early 20th century, when the famous paradoxes of the set theory were discovered. In particular, it became clear that the concept of the set of all sets led to contradiction.

The history of the set theory, or rather part of it related to Georg Cantor, as well as penetration of the ideas of set theory in Polish mathematics is described in detail in the book [19]. However, we have to remark that not much is said about Puzyna, although his significant achievements in this area are emphasized.

The history of the development of the set theory in Poland usually begins with the name of W. Sierpiński, who became interested in this theory in 1908 and gave the first lecture on the set theory at the Lvov University in 1909. Sierpiński drew attention of his students to this subject. Several of his works on the general set theory and theory of functions of a real variable were published in the "Wektor" magazine in 1912–1913. He wrote a book *Zarys teoryi mnogości* (*The outline of the set theory*) in 1919. But it was Puzyna who was the pioneer in introducing the language of set theory, and used the language of intuitive topology in teaching mathematics. Note that *Studya topologiczne* (*Topological studies*) appear in the list of courses taught at the Jan Kazimierz University in Lvov already in 1898.

The book was immediately noticed in Poland as well as abroad. In particular, Placyd Dziwiński wrote in "Kosmos" (XXIV, 1900): "Already the first volume drew attention of the world by the richness of its content and independent treatment of the subject". Here Dziwiński also cited a report from "Naturae Novitates", where its author criticizes that the book was written in an "incomprehensible" language. Nevertheless, the reviewer in "Naturae Novitates" emphasizes that the monograph is "an original work from the beginning to the end".

Decades later, Puzyna's book was characterized by Saks and Zygmund in the monograph [18] as follows: "This work is a veritable encyclopedia of Analysis: in addition to the "Theory of Analytical Functions – partially in beautiful Weierstrass presentation – includes knowledge of Set Theory and Topology (Analysis Situs), Group Theory, Algebra, Differential Equations, Harmonic Functions. If it appeared in any of the more prevalent foreign languages, it would have a further, increasingly sophisticated editions, with all the makings for becoming a classic textbook. Today, after 40 years since the year of the original, a new development of the comprehensive work by Puzyna and adapting it to modern forms of treatment of the subject is beyond capabilities the authors of this book. (...)".

We can assume that the exposition of the material, based on set theory, seemed quite revolutionary. Puzyna's book was published before the invention of well-known paradoxes of the set theory.

The third part of the monograph by Puzyna is called "the theory of sets". The material begins with a definition of finite and infinite field of real and imaginary (complex) variable.

Literatura podręcznikowa w obcych językach z zakresu Teorii Funkcyj Analitycznych jest bardzo obfita i podawanie jej na tym miejscu byłoby nawet kłopotliwe. Natomiast w języku polskim obok jednego przekładu (E. Goursat, *Kurs Analizy Matematycznej*, t. II, z francuskiego przełożył T. J. Łazowski, Warszawa 1919) mamy jedno tylko dzieło oryginalne, mianowicie: J. Puzyna, *Teoria Funkcyj Analitycznych*, t. I (str. XVIII+549), Lwów 1898, t. II (str. XIV+693) Lwów 1900. Dzieło to jest prawdziwą encyklopedią Analizy: obok właściwej Teorii Funkcyj Analitycznych — wyłożonej częściowo w pięknym Weierstrassowskim ujęciu — zawiera wiadomości z zakresu Teorii Mnogości i Topologii (Analysis Situs), Teorii Grup, Algebry, Równań Różniczkowych, Funkcyj Harmonicznych. Gdyby ukazało się w którymkolwiek z bardziej rozpowszechnionych języków obcych, doczekałoby się dalszych, coraz doskonalszych wydań, mając wszelkie dane, by stać się podręcznikiem klasycznym. Dziś, po 40 latach od chwili ukazania się oryginału, nowe opracowanie wszechstronnego dzieła Puzyny i przystosowanie go do współczesnych form ujmowania przedmiotu przerasta możliwości autorów tej książki. Wybrali przeto drogę opracowania książki nowej, jakkolwiek o niewspółmiernie węższym zakresie i skromniejszych ambicjach.

Fig. 2. A fragment from the Introduction to the Saks' and Zygmund's monograph

The boundary of a domain is defined rather informally. The author remarks without precise definition that the boundary can be formed by (parts of) curves and points. One of the most important notions here is that of neighborhood. Neighborhoods at infinity are also considered in the book. It is proved that any infinite countable set of points contains an accumulation point (which may be infinity).

The notion of a derived set was introduced by Cantor in 1872. The (first) derivative of an infinite set P is denoted by P'. If P' is infinite, then one similarly defines the second derivative P'' etc. If the set $P^{(\nu)}$ is finite, then Puzyna writes that there is no derivative of the $(\nu + 1)$-st order i.e., $P^{(\nu+1)} = 0$ (this means that this derivative is the empty set).

By the definition, the first order sets are those whose some finite derivative is empty. Otherwise, they are called the second order sets. Puzyna provides an example from Mittag-Lefler's paper [10] of a set of reals P such that $P^{(\nu-1)}$ is countable and $P^{(\nu)}$ is empty (i.e. the degree of P is ν). The set of rationals on the segment (0,1) is an example of the second order set.

If the points of the derivative of a set P do not belong to P (i.e. if $PP' = 0$), then P is called an isolated set (the set of isolated points).

According to Cantor, the sets P such that $PP' = P'$ are said to be closed. The everywhere dense sets are also defined.

The intersection of all finite derivatives of a set P is denoted by $P^{(\omega)}$. The equation $P^{(\omega)} = 0$ characterizes the first order sets.

The derivatives of transfinite order are also defined. Puzyna neither provides the definition of a transfinite (ordinal) number nor cites Cantor's paper [2] in which the transfinite numbers are introduced. Puzyna does not strive to be precise in these considerations and proceeds by using rather informal description. He first defines the derivatives

$$P^{(\omega+1)}, P^{(\omega+2)}, P^{(\omega+3)}, \ldots \qquad (*)$$

Similarly as in the case of $P^{(\omega)}$ he defines the derivative $P^{(2\omega)}$ as the common part of the derivatives (*). Without further explanations (and without exposition of the theory of well-ordered sets), the author provides the following table for all the countable ordinal numbers as the degrees of the derivatives:

I.		1	2	...	ν...
	ω	$\omega+1$	$\omega+2$	...	$\omega+\nu$...
	2ω	$2\omega+1$	$2\omega+2$	...	$2\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_1\omega$	$\nu_1\omega+1$	$\nu_1\omega+2$	...	$\nu_1\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	ω^2	ω^2+1	ω^2+2	...	$\omega^2+\nu$...
	$2\omega^2$	$2\omega^2+1$	$2\omega^2+2$	...	$2\omega^2+\nu$...
II.	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_2\omega^2$	$\nu_2\omega^2+1$	$\nu_2\omega^2+2$	...	$\nu_2\omega^2+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_2\omega^2+\nu_1\omega$	$\nu_2\omega^2+\nu_1\omega+1$	$\nu_2\omega^2+\nu_1\omega+2$	...	$\nu_2\omega^2+\nu_1\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_k\omega^k+\ldots+\nu_1\omega$	...	...	...	$\nu_k\omega^k+\ldots+\nu_1\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	ω^ω	$\omega^\omega+1$	...	...	...

Fig. 3. A fragment from Puzyna's monograph: ordinal numbers

The numbers in this table are called transfinite numbers. Without formally defining the notion of well-ordering Puzyna however notices its fundamental property, namely that for every element of such a (well ordered) set there exists a well defined immediate subsequent element of this set. The finite (resp. infinite countable) ordinal numbers are called the numbers of class I (resp. of class II).

Note that the first uncountable ordinal is usually denoted by ω_1 not ω^ω and it will be seen later that the latter notation leads to a confusion. Note also that it was hardly possible to provide in the monograph a strict exposition of the theory of well-ordered sets.

Then Puzyna provides Mittag-Leffler's examples of set of reals P such that:

a) $P^{(\omega)}$ = point zero, $P^{(\omega+1)} = 0$,
b) $P^{(\omega+\nu)}$ = point zero, $P^{(\omega+\nu+1)} = 0$,
c) $P^{(2\omega)}$ = point zero, $P^{(2\omega+1)} = 0$.

Puzyna does not define the notion of cardinality. The countable sets are defined as the sets that can be exhausted by means of a process of successive elimination of their elements. He uses the term the sets of the first cardinality for the finite and countable infinite sets.

Some fundamental properties of these sets are established, in particular:

a) any subset of a set of the first cardinality is of the first cardinality,
b) the union of any countable family of sets of the first cardinality is also of the first cardinality.

Without formal proofs it is explained later that the well-ordered sets of the second class are of the first cardinality.

Next, Puzyna considers the cardinality of the segment (0,1). He denotes this cardinality by ω^ω. The explanation uses the expansion of real numbers into continued fractions. Finite fractions are in one-to-one correspondence with the set of all maps of n into itself, i.e., n^n. Similarly, the set of all irrational numbers in (0,1) is in one-to-one correspondence with the set of all maps of ω into itself, i.e., the set ω^ω. Note that the latter is the upper bound of n^n, where n is natural. This is in a sense similar to some of Euler's arguments or to the proofs in the style of non-standard analysis. The notation ω^ω appears already on page 97.

It denotes the ordinal number which is the least upper bound of all countable cardinals. One can hardly find an explanation of this notation, but a few pages later it leads to an erroneous conclusion concerning the continuum hypothesis.

Returning to the set (0,1), the author shows its cardinality is that of cardinality of the mentioned set of all irrational numbers in (0,1). Therefore, the cardinality of the set of all real numbers is ω^ω. Then, using the completeness of the set of reals, he proves that the cardinality of the set (0,1) is uncountable.

Page 108 contains the (clearly wrong) conclusion that the cardinality ω^ω immediately follows the countable cardinality. Puzyna calls such sets to be of class II.

Then the following question is considered: what is the cardinality of a subset in the n-dimensional real domain? At the very beginning, the author considers the (closed) n-dimensional cube. It is interesting to note that the notation for this set rather differs from the modern style and is the following:

$$(x_1, \ldots, x_n) = (0 \ldots 1, 0 \ldots 1, \ldots, 0 \ldots 1)$$

(Here we see that Puzyna does not use the symbol $\in$ (or the script epsilon) for the set membership, despite the fact that Peano used this notation already in 1889.)

In order to prove that the n-dimensional cube has the same cardinality as the unit segment, Puzyna first passes to the set of points with all irrational coordinates (earlier, it is established that the setter set is of the same cardinality). Then he uses the trick of forming one number out of n using the decimal expansions.

Note that this question was later asked by W. Sierpiński.

In the footnotes, Puzyna mentions G. Peano's article *Sur une courbe, qui remplit toute un aire plaine*, "Mathematische Annalen", T. 36, (1890), p. 157. In this article Peano discusses the considerably more complicated problem of existence of continuous maps from the unit segment onto the square.

It is proved that, for any countable set in the unit cube, there exists a point in the cube such that every its neighborhood contains a point of the set (the so-called accumulation point). In modern terminology, this is precisely the proof of compactness of the cube in the Euclidean space. The method used in the proof is that of dividing of the square into four equal parts. The required accumulation point is that of intersection of the family of descending squares containing the infinite set of points of the countable set under consideration. It is remarked that similar arguments work also in the n-dimensional domain.

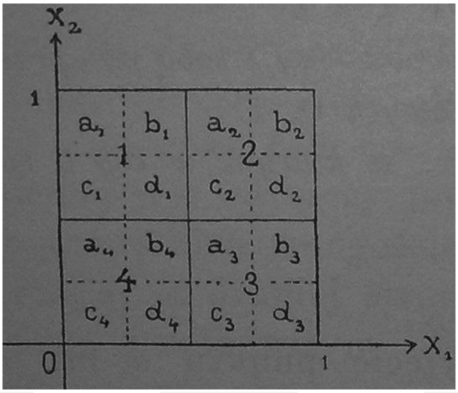


Fig. 4. A figure from Puzyna's monograph: division of the square

Since no precise definition of set is given, the author explains that "the *entire*, bounded or unbounded, domain should be regarded as a set". These sets are called continua or continuous domains. The definition of a continuum is, however, that of an open set. Also, Puzyna introduces the notion of the boundary of an open set (continuous domain).

The author does not define the notion of compactness explicitly. In the subsequent sections, the property of compactness is needed in the proof of the fundamental theorem of algebra, therefore the proof enclosed in the monograph seems to be incomplete.

Later the sets of the first and the second cardinality are considered in the *n*-dimensional domains. Puzyna proves that these domains are of the same cardinality as the set of reals.

The notion of continuum is defined as the set of points satisfying the following property: all the points in a neighborhood of any of its points belong to the set as well. The notion of connected domain („obszar zwarty" in Polish; note that in the modern Polish mathematical terminology „zwarty" means "compact") is rather informal, it sounds as follows: a "continuum" is a set such that from any of its point one can pass to any other its point through points only belonging to this „continuum". It is proved that the complement to any countable set in a continuum is also a continuum. Note that the modern form of the notion of connectedness was hardly known to Puzyna.

It is remarked that the notions of upper bound and lower bound are derived from the set theory.

Section III is concluded with the notion of stereographic projection. This map is a homeomorphism between the plane and the punctured unit sphere. Puzyna uses the term "pokrewieństwo" ("kinship") for this map and speaks of a "circumference kinship" (circumference-preserving homeomorphism) or "isogonal kinship" (conformal map).

3. Topology of surfaces

Let us turn our attention to Part V of the monograph that deals with Riemann surfaces. We already remarked that not all mathematicians preferred using the language of the set theory in their research.

The exposition here starts with the definition of a closed surface. However, this definition is necessarily not rigorous as the author avoids using charts, i.e. homeomorphisms onto domains of Euclidean spaces. Actually, we find here an informal description of surfaces.

The simply connected surfaces are introduced by means of intuitive definition. These are the surfaces that satisfy the following properties:

1) Every curve connecting two points of the surface can be transformed into another one so that it does not leave the surface in the process of transformation. The endpoints of the curve either are the same or can change.
2) Every connected curve contained in the surface can be shrunk to an arbitrary point, while remaining on the surface in the process of shrinking.
3) If the surface possesses the boundary, then every simple (non-self-intersecting) curve that connects two distinct points of the boundary divides the surface into two separate parts.

In modern terms, the author implicitly uses the notion of homotopy (isotopy) of continuous maps in this definition.

It is remarked that the boundary of any simply connected surface cannot contain two closed separate pieces and that any cut of a simply connected surface yields two simply connected surfaces.

Then n-connected surfaces are also introduced. These are the surfaces in which one can make n–1 cuts such that the result of cutting is a simply connected surface.

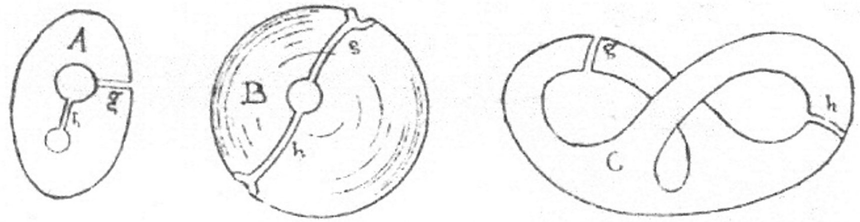


Fig. 5. Examples of 2-connected surfaces are: a planar annulus, sphere with two holes etc. The third figure in the above picture is an example of a 3-connected surface. The proofs of statements on the surfaces are based on intuitive approach

A closed curve on a surface (either having self-intersections or not) is called a *circumference* on this surface. A circumference is reducible (contractible, in modern terminology) if it can be deformed within the surface to a point, otherwise it is called irreducible. The notion of a complete system of irreducible circumferences is introduced and it is shown that every irreducible circumference can be uniquely represented as an equivalent one to a combination of circumferences from a chosen complete system. Actually, the homotopy classes of circumferences form the fundamental group of a surface and the mentioned complete system of irreducible circumferences is precisely a set of generators of this group. The notion of fundamental group was introduced by H. Poincaré in [14]; this article is cited by Puzyna despite the fact that he avoids using here the language of the group theory.

Next, the notion of genus of a surface is defined. The genus is the half of the number of cuts needed to obtain a simply connected surface.

All the above considerations work for oriented surfaces and it was implicitly assumed earlier that the surfaces under consideration possess this property. Perhaps the simplest example of a non-oriented surface is the Möbius band. Puzyna cites V. 2 of *Werke* by Möbius [11].

When describing the notion of deformation of surfaces Puzyna uses the informal terms:

1) points that are infinitely close remain so in the mapped surface,
2) finitely distanced points are also finitely distanced in the mapped surface.

He formulates the following statement: Given two surfaces, one can deform one of them into the other whenever they are of the same connectedness and have the same number of the circles on the boundary. The arguments given in the monograph are informal and cannot be considered as a proof of this fact.

In the case of closed surfaces, the following statement is formulated: every closed (oriented) surface can be deformed into a sphere with finite number of handles. This is the classification theorem for oriented surfaces, which is known to be a powerful and complicated result. The theorem was stated in various forms by different authors.

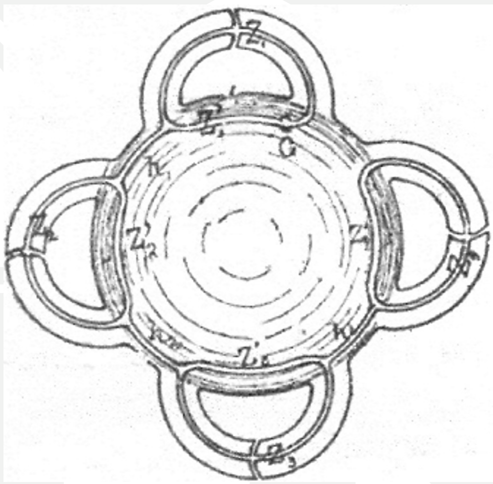


Fig. 6. A figure from Puzyna's monograph: Sphere with handles

The exposition of this proof is again based on an intuitive approach.

A generalization of Euler's theorem to (triangulable) surfaces is also given. This allows the author to consider the Euler characteristic of a surface.

The following section contains a description of the construction of the Riemann surfaces, first, at a neighborhood of a branching point. This construction is illustrated by the following pictures.

It is proved that the algebraic notion of genus of any Riemann surface can be also described in topological terms. Actually, the genus is a topological invariant of a surface.

The material also contains various information on algebraic curves. In particular, an analysis of singularities of the algebraic curves by means of the quadratic maps is given.

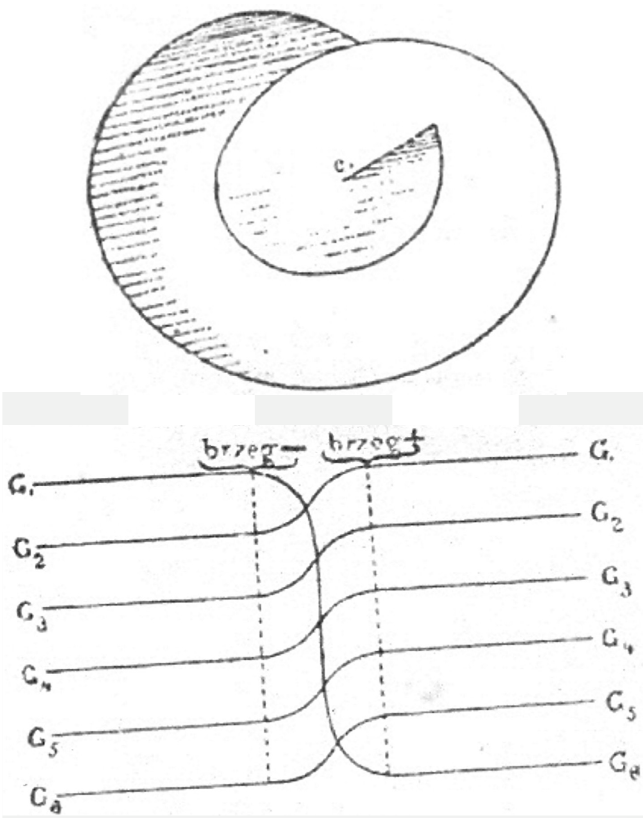


Fig. 7. Neighborhood of a branching point

It is interesting to look on Puzyna's book from the point of view of the unity of mathematics. The introductory parts contain material from set theory and geometry, as well as algebra, in particular group theory.

The exposition of the material is rigorous throughout the book. However, in some places the style becomes rather narrative when the author deals, e.g., with topology of the plane.

Note that even simply formulated and intuitively evident statement of the planar topology can have complicated proofs, and the famous Jordan curve theorem is a good example supporting this statement.

4. Conclusions

The material of Puzyna's book demonstrates that the author belonged to the part of mathematical community that accepted the most fundamental ideas of set theory. One can hardly overestimate the importance of the monograph for the further development of the set theory in Poland.

At the same time, in the monograph one can find an approach to exposition of topological notions which is not based on set-theoretical language. Describing the topological properties of (Riemann) surfaces Puzyna prefers the intuitive and visual arguments, rather in the spirit of Poincaré. This combination of styles is somewhat eclectic, but can be justified from a didactic point of view.

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