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THE DETERMINATION OF NON-STANDARD SEEPAGE PARAMETER VALUES BASED ON STANDARD PARAMETER VALUES

WYZNACZANIE WARTOŚCI NIESTANDARDOWYCH PARAMETRÓW FILTRACJI NA PODSTAWIE WARTOŚCI PARAMETRÓW STANDARDOWYCH

A b s t r a c t

The estimated characteristics of water seepage in a the vadose zone have been determined using analytical formulae based on the Carman-Kozeny capillary beam model. Only two standard soil parameters are needed to achieve this objective: hydraulic conductivity and porosity. The characteristics obtained find their application in a simplified description of infiltration known as the piston model.

Keywords: *seepage parameter*, *capillary rise*, *soil moisture-retention*, *soil conductivity*, *specific capacity*

S t r e s z c z e n i e

Przybliżone charakterystyki filtracji wody w strefie aeracji wyznaczone zostały ze wzorów analitycznych bazujących na modelu wiązki kapilarnej Carmana-Kozeny'ego. Wymagane są do tego celu jedynie dwa standardowe parametry gruntu – wodoprzepuszczalność i porowatość. Uzyskane charakterystyki mają zastosowanie w uproszczonym opisie infiltracji znanym jako model tłokowy

Słowa kluczowe: *parametry filtracji*, *wznios kapilarny*, *retencja wilgoci w gruncie*, *przewodność gruntu*, *współczynnik retencji*

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1. Introduction

Models of groundwater circulation incorporate parameters that are already recognised as standard in hydrogeology and have established procedures for their determination. Such parameters include the coefficients of hydraulic conductivity, porosity and specific yield as well as the geometrical parameters of the water-bearing horizon – the thickness, the ordinates of the floor and top and groundwater table. Models simulating flows in the subsurface stratum, and seepage through the vadose zone in particular, require that a series of additional values be defined which are not determined using the standard procedures. Their determination is associated with numerous problems, although a high degree of precision is not required. However, distinct and significant correlations between non-standard and standard parameters may be identified. The values obtained based on these correlations are sufficient for practical purposes in numerous instances.

2. The non-standard parameters

In the classical models of seepage in the vadose zone using the Richards equation, a series of complex functional relations have to be determined, such as soil moisture-retention and conductivity characteristics or the specific capacity. The characteristics derived from them are used in Buckingham's moisture diffusion model. The determination of the indicated interrelations is expensive and time consuming, even if the commonly occurring hysteresis is ignored – this results in the limited use of these models.

Due to the negligible water conductivity of soil, as compared to the saturated zone, vertical movements prevail in the vadose zone, usually in the form of the infiltration process. The application of the simplified piston model is an effective solution in these circumstances [8]. The model is based on the Green-Ampt model [4] in the case of saturated infiltration, on the Bouwer concept [1] in the case of unsaturated infiltration , and on the Morel--Seytoux concept [9] in the case of moisture redistribution. The model gives approximate results, but this feature is insignificant due to the considerable spatial diversification of soil types – it uses characteristics that incorporate only a few parameters, and are simplified to the maximum extent. The soil moisture retention curve may be described using the Corey-Brooks equation [2, 3], the conductivity curve may be described using the Irmay-Averianov equation [5], and the specific capacity and the hysteresis are replaced with the water balance. Only a few constant parameters of these characteristics need to be determined – the exponent of the effective conductivity curve m_k and its residual moisture θ_k, the exponent of the moisture-retention curve m_θ, and the height of capillary rise h_k. These values are determined in this study using an analytical approach based on the Carman-Kozeny capillary beam model [7, 6].

2.1. The Carman-Kozeny capillary beam model

The Carman-Kozeny capillary beam model (and equation) [7] is used to determine the hydraulic conductivity coefficient k_o:

$$k_o = \frac{\rho g n^3}{5\eta s^2} = \frac{1}{180}\frac{n^3}{(1-n)^2}\frac{\rho g}{\eta} d_m \tag{1}$$

where:

k_o – hydraulic conductivity coefficient [cm s^{-1}],
ρ – density of water (1.0 g cm^{-3}),
g – standard gravity (981 cm s^{-2}),
n – porosity coefficient [–],
η – water dynamic viscosity coefficient
(for 10ºC $\eta = 1.3 \cdot 10^{-3}$ Pa·s $= 1.3 \cdot 10^{-2}$ $g \cdot cm^{-1} \cdot s^{-1}$),
s – active area of grains[cm^{-1}],
d_m – effective grain size [cm].

2.2. Capillary rise height h_k

The height of capillary rise can also be computed from the Carman-Kozeny model [11]:

$$h_k = \frac{\sigma s}{n \rho g}, \tag{2}$$

where:

h_k – capillary rise height (capillary uplift) [cm],
σ – plane stress (for 10ºC $\sigma = 74.2\ 10^{-3}$ $N \cdot m^{-1} = 74.2$ $g \cdot s^{-2}$).

The existing active area may be determined using the Carman-Kozeny equation:

$$s = \sqrt{\frac{\rho g n^3}{5\eta k_o}}, \tag{3}$$

and this enables one the calculation of the capillary height, using only the hydraulic conductivity and porosity determined as standard parameters:

$$h_k = \sigma \sqrt{\frac{n}{5\eta k_o \rho g}} = 9.26 \cdot 10^{-3} \sqrt{\frac{n}{k_o}}. \tag{4}$$

2.3. Power exponent m_k

Widely used the Irmay-Averianov equation – simple and relatively precise at the same time, i.e.:

$$k = k_o \left(\frac{\theta - \theta_k}{n - \theta_k} \right)^{m_k} \tag{5}$$

where:

θ – moisture content,
θ_k – residual moisture content,

contains two additional parameters that need to be determined. It may be assumed that the residual moisture content, inhibiting water flow at very low saturation values, corresponds to the pellicular moisture content θ_b. At small values of this moisture, water is so strongly bound with the soil skeleton that its movement practically does not occur. As regards the exponent, Mualem [10] has demonstrated that it is a function of the energy needed to dry the cavities:

$$m_k = 1.5 \cdot 10^{-4} p_s + 3.0, \tag{6}$$

where:

p_s – pressure needed to dry cavities [Pa]:

$$p_s = \gamma \int_{\theta_a}^{n} h_s d\theta \cong 1.5\gamma h_k (n - \theta_a), \tag{7}$$

γ – water specific weight [N m^{-3}],

θ_a – minimal adhesive moisture [N m^{-3}].

And thus finally:

$$m_k = 3 + 0.15 \cdot 1.5 h_k (n - \theta_a) \cong 3 + 0.225 h_k n \tag{8}$$

The results obtained using this equation are characterised by a relatively high consistency with empirical results, in particular for fine-grained soils in which pellicular moisture plays a significant role.

2.4. Power exponent m_θ

The moisture characteristic is not used in the piston model, but is indispensable in determining the evapotranspiration value that plays an important role in an infiltration model. The correlation between water pressure and moisture is necessary to define the threshold moisture values causing a reduction in growth, and consequently, in transpiration in plants. These moisture values are very low; therefore, we can use the simplest Corey--Brooks equation to describe this characteristic [2, 3]:

$$\theta = (\theta_n - \theta_b)\left(\frac{h_k}{h_s}\right)^{m_\theta} - \theta_b \tag{9}$$

where:

θ_b – pellicular moisture [6]:

$$\theta_b = 0.163 n \sqrt{h_k} \sqrt[3]{\frac{1-n}{n}} \sqrt[6]{\frac{h_k}{h_s}}. \tag{10}$$

The m_θ parameter used in this equation is correlated with the m_k parameter used in the formula for the conductivity characteristic as follows [2]:

$$m_k = 2\frac{1+m_\theta}{m_\theta} \tag{11}$$

thus:

$$m_\theta = \frac{2}{m_k - 2}. \tag{12}$$

3. Conclusions

The formulae described in this paper enable the researcher to easily determine soil moisture-retention and conductivity characteristics using data on soil conductivity and porosity only. The correlations obtained using this approach are undoubtedly approximate but sufficient and suitable for numerous applications. Expensive and laborious laboratory tests of the described characteristics apply only to specific soil samples and are not usually representative enough for the larger area of the vadose zone.

References

[1] Bouwer H., *Infiltration into increasingly permeable soils*, J. Irrig. Drain. Div. Am. Soc. Civ. Eng. '76/IR1, 127-136
[2] Brooks R.H., Corey A.T., *Hydraulic properties of porous media*, Hydrology Paper 3, Colorado State University, Fort Collins, 1964.
[3] Brooks R.H., Corey A.T., *Properties of porous media affecting fluid flow*, J. Irrigation and Drainage Div. ASCE 1966/IR2, 1966, 61-88.
[4] Green W.H., Ampt G.A., *Studies on soil physics: 1. The flow of air and water through soils*, J. Agr. Sci. 1911/4, 1-24.
[5] Irmay S., *On the hydraulic conductivity of unsaturated soils*, Trans. Am. Geophys. Union 1954/1, 1954.
[6] Kovács G., *Seepage hydraulics*, Akadémiai Kiadó, Budapest 1981
[7] Koženy J., *Hydraulik*, Springer, Wien 1953.
[8] Książyński K. W., *Tłokowy model filtracji w strefie niepełnego nasycenia*, Monography No. 353, Cracow University of Technology, Kraków 2007.
[9] Morel-Seytoux H.J., *Some recent developments in physically based rainfall-runoff modeling*, Frontiers in Hydrology, Water Resources Publications: Littleton/Colorado 1984.
[10] Mualem Y., *Hydraulic conductivity of unsaturated porous media: Generalized macroscopic approach*, Water Resour. Res. 1978/2, 1978, 325-334.
[11] *Instrukcja oznaczania właściwości hydrogeologicznych skał na podstawie analizy uziarnienia*, http://www.ig.pwr.wroc.pl/zaklady/ZGiWM/pliki/Cw2_wlasc.pdf.