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PHILOSOPHY OF EXACT SCIENCES (LOGIC AND MATHEMATICS) IN POLAND IN 1918–1939

FILOZOFIA NAUK ŚCISŁYCH (LOGIKI I MATEMATYKI) W POLSCE W LATACH 1918–1939

A b s t r a c t

This paper describes the philosophy of logic and mathematics in Poland in the years 1918–1939. The special attention is attributed to the views developed in the Polish Mathematical School and the Warsaw School of Logic. The paper indicates various differences between mathematical circles in Warszawa, Lvov and Kraków.

Keywords: *set theory*, *the foundations of mathematics*, *pluralism*

S t r e s z c z e n i e

Artykuł opisuje filozofię logiki i matematyki w Polsce w latach 1918–1939. Szczególną uwagę zwrócono na poglądy rozwinięte w Polskiej Szkole Matematycznej oraz Warszawskiej Szkole Logicznej. Artykuł wskazuje na rozmaite różnice pomiędzy środowiskami matematycznymi w Warszawie, Lwowie i Krakowie.

Słowa kluczowe: *teoria mnogości*, *podstawy matematyki*, *pluralizm*

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Poland recovered its independence in 1918 and the building of education and science became one of the main tasks of new Polish state. Due to partition of the country into three zones, Russian, German (formerly, Prussian) and Austro-Hungarian (formerly, Austrian) a the end of 18th century, it was a very challenging aim. The policy of particular states occupying Poland toward cultural ambitions of Poles was different. Restricting attention to the period after 1864, relatively best situation occurred in Galicia, that is, Polish territory belonging to the Habsburg empire. A considerable liberalization of Austro-Hungary in the 1870s resulted in a partial autonomy of Galicia and considerable freedom in cultivating Polish culture. As far as the issue concerns science and higher education, Galicia had two universities and the Polytechnic School.The Jagiellonian University in Kraków continued its character as fully Polish university after a short period (after 1849) in which German language served as official teaching. The University of Lvov became polonized around 1870; the same concerns the Lvov Polytechnic School. Academy of Arts and Sciences was established in Kraków in 1872. These institutions managed normal scientific and/or educational activities in Polish intellectual circles.

The situation in the Russian zone was complex and essentially dependent on the actual policy of Tsarist authorities, which was sometimes relatively liberal or sometimes took a completely opposite course. In 1862, the Main School, practically a university was established in Warsaw. However, it was closed in 1869 and replaced by the Imperial University with Russian as the official language. On the other hand, Poles developed in the Russian zone, mostly in Warsaw, a half-official, but principally tolerated by Russians, system of education, which played an important role. In 1907, the authorities agreed for the rise of the Warsaw Scientific Society. The Mianowski Kasa, the Foundation for Supporting Polish Science, established in 1881, was another significant institution. The worst situation occurred in the German zone. The policy of Germanization, called *Kulturkampf*, was consequently executed by German authorities. To exclude Polish scientific and educational life (on higher level than secondary school) was one of the principles of *Kulturkampf*. The Poznań Society of the Friends of Science existed since 1857, but its real role was not particularly great. Although both Galician universities attracted many students from German and Russian zones, many Poles decided to study abroad, mostly in Germany, Austria, Russia or France.

The above (very) brief historical account explains why the unification of Polish educational system linked with creation of new universities became one of priorities after 1918. In the years 1915–1920, four universities (I omit other types of high schools – note that the term "high school" refers, according to Polish tradition, to any school at the university level) were established or renewed, namely in Warszawa (renewed), Poznań (new), Vilna (renewed) and Lublin (new; it was a catholic university and it did not play any role in exact sciences). In 1916, the Mianowski Kasa invited leading Polish scientists working in Poland and abroad to answer the question "What should be done in independent Poland in order to develop national science?" The very numerous answers outlined the state of art and needs of Polish science. This material was included in the two first volumes of the journal "Nauka Polska. Jej potrzeby, organizacja i rozwój" (Polish Science. Its Needs, Organization and Development)), published in 1918–1919 (presumably, it was the first professional journal devoted to the science of science). This enterprise shows that

the building the foundations of science in Poland was considered very seriously by Polish scholars. Zygmunt Janiszewski's famous programmatic essay on the needs of mathematics in Poland was one of papers published in this volume.

Speaking about remarkable development of mathematics and logic in Poland in 1918–1939, one should not ignore Polish (or Warsaw) positivism. This movement, influenced by French and English positivism, was closely connected with the mentioned Main School. Polish positivism was a reaction to romanticism and its unsuccessful attempts to recover Polish independence by military fighting, directed mostly against Russia considered in Poland as the main invader. Two uprisings, namely in 1830–1931 (the November Uprising) and in 1963–1964 (the January Uprising) ended with total defeats as well as with subsequent political and cultural repressions. Polish positivists proposed another solution, namely so-called organic (or basic) work consisting in improving society (particularly, its lower classes) by increasing its economic prosperity and education. This second point was especially important for the rise of interests in the philosophy of science in Poland. Since Polish tradition was not very strong in this field, many translations of works of foreign leading philosophers and scientists appeared in 1865–1918. The Mianowski Kasa was very instrumental in this enterprise. Books and papers on the foundations of exact sciences written by Dedekind, Enriques, Helmholtz, Jevons, Maxwell, Mill, Bain, Poincaré, Riemann, Pieri, Russell, Young and Whitehead were published in Polish. They allowed Polish intellectuals to follow the world development of science and its methodology. In this environment, first Polish scientists, like mathematicians Samuel Dickstein, Władysław Gosiewski and Edward Stamm (he was also a philosopher) began their activity as professional scholars.

Looking at the academic level, the philosophy of exact sciences in the interwar period did not start from the scratch around 1918. In Lvov, Kazimierz Twardowski, professor of philosophy since 1895, created a very good atmosphere for the development of logic. His metaphilosophical program of scientific philosophy, insisting on clarity of thinking and expressing thoughts in a language, demanding that philosophers should correctly justify the proposed statements and opposing metaphysical speculations, considered logic as a natural instrument of achieving cognitive goals. Twardowski did not work in logic, except a very few contributions to general logic, mostly semantics and the methodology of sciences. However, he recommended mathematical logic s worthy to a serious study to his students. Alfred Tarski summarized the role of Twardowski in the development of logic in Poland in the following words [18, p. 20].

Almost all researchers, who pursue the philosophy of exact sciences in Poland, are indirectly or directly the disciples of Twardowski, although his own works could be hardly be counted within this domain.

Twardowski trained a number of students who began to work in formal logic and the methology of sciences. This group included Jan Łukasiewicz, Stanisław Leśniewski, Kazimierz Ajdukiewicz, Tadeusz Czeżowski Tadeusz Kotarbiński and Zygmunt Zawirski. The mentioned scholars formed the first generation of the so-called Lvov–Warsaw School, a powerful group of analytic philosophy (in fact, it functioned as the Lvov School until 1915). Logic and the foundations of mathematics became popular among Lvov mathematicians still before 1914. Wacław Sierpiński lectured on set theory at the end of the first decade pf the 20th century. He was joined by Zygmunt Janiszewski. Both, Sierpiński and Janiszewski,

decided to devote their habilitation lectures to the problems of foundations of mathematics (the former spoke about the concept of mathematical correspondence and the latter – about realism and idealism in mathematics). However, Krakow, not Lvov, became the first serious centre of mathematical logic in Poland. Stanisław Zaremba a distinguished Polish mathematician, had strong interests in logic and the foundations of mathematics. He considered logic as a peripheral branch of mathematics, having only a secondary importance, mainly in teaching mathematicians. He was influenced in this attitude by the French style of doing mathematics. Zaremba's views prevailed among mathematicians in Krakow. It is interesting that the Jagiellonian University had a special professorship in mathematical logic, occupied by Jan Śleszyński. Other logicians working in Krakow included Leon Chwistek and Witold Wilkosz. Warszawa appeared on the stage of logic in 1915, when German authorities (the city was occupied by Germans since the summer end of 1915) allowed to re-open the (Polish) university. Łukasiewicz was appointed as the professor of philosophy and began systematic lectures in logic and the foundations of mathematics which attracted many young mathematicians. He was joined by Leśniewski, who became the professor of the philosophy of mathematics in 1919. Also Kotarbiński moved to Warsaw as professor of philosophy in 1918.

According to the mentioned Janiszewski program, Polish mathematicians should concentrate on carefully chosen mathematical fields which could be promising from the point of view achieving new results. Set theory, topology and their applications to classical mathematics were identified as such domains. This choice made mathematical logic and the foundations of mathematics as located in the very centre of mathematics. Janiszewski also postulated that Poland should have a special mathematical journal published in international languages. This idea found its realization in *Fundamenta Mathematicae* (the first volume appeared in 1920). The first idea of *Fundamenta Mathematicae* particularly stressed the significance of logic and the foundations of mathematics by projecting two series of the journal, namely one devoted to set theory, topology and their applications, and second to logic and the foundations. However, this idea was abandoned and the *Fundamenta* was published as the unified journal, but with a considerable amount of papers on foundational problems. Lvov became the second main centre of Polish mathematical school. Krakow remained more traditional in the spirit of Zaremba. Sierpiński, Janiszewski and Stefan Mazurkiewicz (all move to Warsaw after 1918) played the main role in Warszawa, while Stefan Banach and Hugo Steinhaus became the leaders in Lvov. Yet one important difference between the two centres of modern mathematics in Poland must be noted. Although mathematicians in Lvov worked mainly on functional analysis, the operator theory and similar problems, eventually with using of set theory and topology, the mathematical circle in Warszawa focused more on abstract matters.

The University of Warsaw organized the separate Faculty of Mathematics and Natural Sciences. Łukasiewicz and Leśniewski, two philosophers with rather a limited mathematical education were appointed as professors just at this specialized faculty. Incidentally, it was a brave sociological move, not practised in other countries. Łukasiewicz and Leśniewski, supported by leading mathematicians working at the Warsaw Univwersity, particularly Sierpiński, Stefan Mazurkiewicz and Kazimierz Kuratowski (Janiszewski prematurely died in 1920) began very intensive teaching in mathematical logic and very soon found many gifted

students. Due to these activities, the Warsaw School of Logic was established. It included such logicians like Tarski, Adolf Lindenbaum, Mordchaj Wajsberg, Moses Presburger, Stanisław Jaśkowski, Bolesław Sobociński, Jerzy Słupecki and Andrzej Mostowski. This school had two parents, mathematics and philosophy, more explicitly the Warsaw School of Logic can be regarded as a part of Polish Mathematical School and the group originated with Twardowski since 1895, extended to the Lvov-Warsaw School after 1918. Last but not least, Twardowski idea of the development of Polish philosophy and the Janiszewski program shared similar points. Both great organizers of Polish science were convinced that Polish scholars have to have a very close contacts with novelties and tendencies executed in leading scientific investigations. By a coincidence, mathematical logic became a common focus for Twardowski and Janiszewski. On the other hand, although we have no written historical evidence, it is quite possible that Twardowski, Sierpiński and Janiszewski discussed the organization of science in Poland after recovering independence and reached similar conclusions, also those articulated by Janiszewski in his program. To complete personal issues, Ajdukiewicz stayed in Lvov (with exception of the years 1926-1928, when he taught in Warszawa) and lectured on logic for philosophers and mathematicians; he was appointed as professor of philosophy. The University of Lvov established the chair for mathematical logic in 1928. Chwistek won the competition for this postand he created a small school working in logic.

The double, mathematical and philosophical genesis, of the Warsaw School of Logic, immediately provokes the question about the philosophy of exact sciences in this circle. A similar problem concerns the entire Polish Mathematical School. Due to the mentioned differences between Warsaw and Lvov mathematicians, the first group treated the foundational the the second. Mathematicians from Kraków are not counted among members of Polish Mathematical School. As I already noticed, Zaremba had strong interests in logic and the foundations of mathematics, but he considered logic as a fairly marginal branch of mathematics, having only a secondary and auxiliary importance as a preparation for doing hard mathematics. The already indicated French influence on Zaremba resulted in his even hostile attitude to logic, similar to Poincaré's view. Zaremba's opinions prevailed among mathematicians in Krakow. Śleszyński's position was cancelled when he became retired. More importantly, there was a great controversy between Zaremba and Polish Mathematical School concerning the foundations of mathematics. Zaremba entirely rejected the view that set theory constitutes the fundament of mathematics. In fact, Wilkosz was the only exception and conducted investigations in abstract (or pure) set theory. Chwistek, as I already noticed, moved to Lvov. The above explanations justify the further schematization of my report about the philosophy of exact sciences in Poland in the period 1918–1939. It is reasonable to concentrate on the Warsaw branch of Polish Mathematical School, particularly the Warsaw School of Logic and on Chwistek who developed own approach to logic and the foundations of mathematics.

It is convenient to start with Leśniewski and Chwistek. Both developed general schemes, *grand logics*, so to speak, for grounding logic and mathematics best on very explicit philosophical premises. Leśniewski proposed a comprehensive system consisting of three parts, protothetic (a generalized propositional calculus), ontology (a logic of terms) and mereology (the theory of parts and wholes). Two first parts constitute pure logic, but the third one, mereological theory of classes, functions as a substitute of set theory, although

Leśniewski himself considered mereology as a part of logic. Leśniewski hoped to base the entire mathematics on his system. Leśniewski's formal systems are radically nominalistic (no abstract objects are admitted), fully formalized (he accepted so-called intuitionistic formalism consisting in the view that formal languages are always interpreted; note that this view has nothing in common with intuitionism as a program in the foundations of mathematics) and realistic (logic and mathematics describe reality; the term "ontology" was chosen by Leśniewski, because he considered the logic of terms as the general theory of objects. Chwistek began his logical investigations by attempts to improve Russell's ramified theory of types. More specifically, Chwistek tried to combine Poincaré's constructivism (predicativism) and Russell's approach by eliminating the axiom of reducibility. As a result he obtained a version of the simple theory of types based on strong nominalistic presuppositions. However, limitations of this solution in capturing the entire mathematics by the modified simple theory of type pushed Chwistek to a different conception consisting in constructing the hierarchy of semantic systems. Semantics in Chwistek's sense is a general formal theory of expressions, but not a theory of relations between languages and what they refer to. Thus, semantics is rather similar to syntax in it standard understanding. Chwistek in his new logical construction also preserved nominalism. The foundational proposals of Leśniewski and Chwistek, although different in essential points, can be considered as versions of logicism, that is, grounding mathematics as a part of logic, provided that mereology and semantics (in Chwistek's sense) are considered as parts of logic. Both, Leśniewski and Chwistek took this position.

Chwistek and Leśniewski were exceptions in Poland, because other Polish logicians and mathematicians did not develop general conceptual schemes as capturing mathematics and mathematical knowledge. Consequently, the Polish Mathematical School and the Warsaw Logical School had no official philosophy of exact sciences. More specifically, Polish logicians and mathematicians did not ascribed themselves to one of the dominant foundational projects of the first half of the 20th century, namely logicism, formalism or intuitionism. Using a label, popular nowadays (see [7]) the philosophy of logic and mathematics was considered in Poland as the second philosophy, according to the slogan (it is a paraphrase of the saying beginning with *primum vivere*) *primum mathematicari*, *deinde philosophari.* Putting this otherwise, mathematical practise is first, but mathematical philosophy – the second one. This background allowed to employ simultaneously various influences stemming from reading books and papers written by foreign scholars and having personal contacts with them. Due to the fame of *Principia Mathematica*, Russell was extremely popular in Poland, but nobody, even Chwistek or Leśniewski, accepted logicism as the only correct foundational scheme. In fact, Łukasiewicz began his serious logical investigations after reading Russell's, *The Principles of Mathematics*, pubhlished in 1903. Traces of formalism are clearly present in Chwistek, Ajdukiewicz, Leśniewski and Tarski; two first studied in Göttingen, but the rest knew formalism from the literature only. I already noticed, Poincaré's influence on Chwistek, but the latter was inspired by constructivism (predicativism) of the former, but not by his conventionalism. It interesting that even in the case of Polish authors (Chwistek, Leśniewski) proposing own foundation al schemes, we easily recognize influences coming from different, often mutually rival, philosophical programs. Thus, pluralism as a general standpoint in the philosophy of logic and mathematics

can be regarded as one of the most characteristic features of the worldview of the Polish school of logic and mathematics. Incidentally, pluralism was also a characteristic view in the entire philosophical Lvov–Warsaw School.

On the other hand, some Polish authors strongly influenced the rise of a quite new foundational paradigm. The programs of logicism, formalism and intuitionism were replaced in the 1930s by new projects, namely set theoretical, constructivism and proof-theoretical (see [10]). Due to the role of set theory in the reconstructing the whole edifice of classical mathematics, stressed by the Polish Mathematical School, it was quite natural that Polish mathematicians and logicians contributed to the set theoretical paradigm. Excluding set theory from logic, contrary to logicism, resulting in favouring first-order logic as the logic by most Polish logicians; higher-order logic was considered as a part of set theory. Although formalism did not attract Poles as a general foundational programs, several Tarski's works on consequence operations and general deductive systems can be viewed as contributions to the proof-theoretical foundational scheme, although Tarski himself was never committed to the view that formalism is the best proposal in grounding the foundations of logic and mathematics. In fact, most Polish mathematicians and logicians abstain from explicit declarations what is the best in science.

The attitude of Polish logicians and mathematicians to intuitionism and constructivism as grand projects in the foundations of mathematics provides perhaps a particularly instructive illustration of Polish pluralism in doing the philosophy of logic and mathematics. As it is well-known, constructivism (intuitionism is its very radical version) considers non--constructive mathematical methods as defective in a way which cannot be improved. This radical verdict also concerns infinitistic procedures exceeding those mathematical methods which can be formalized within arithmetic of natural numbers (in this respect, constructivism is somehow more liberal than Hilbert's original formalism admitting finitely performable constructions as the only admissible methods in the so called real mathematics). Tarski [16, p. 713] said once:

> "As an essential contribution of the Polish school to the development of metamathematics one can regard the fact that from the very beginning it admitted into metamathematical research all fruitful methods, whether finitary or not".

The message coming from this fragment is a good example of application the dictum *primum mathematicari*, *deinde philosophari.* Tarski's view can be characterized as methodological Platonism. Paradoxically, Tarski himself frequently expressed his sympathy to nominalism. However, if someone sharply distinguishes mathematics and its philosophical interpretations, such a discrepancy unnecessarily must be regarded as a lack of coherence.

Sierpiński commenting the axiom of choice, controversial as it is commonly admitted, applying the same ideology ([13, p. 95]; this opinion appeared in Sierpiński's writings much earlier, in fact, since early 1920s) as Tarski did in the following words:

> Still, apart from our personal inclinations to accept the axiom of choice, we must take into consideration, in any case, its role in the Set Theory and in the

> Calculus. On the other hand, since the axiom of choice has been questioned by some mathematicians, it is important to know which theorems are poved with its aid, and to realize the exact point at which the proof been based on ther axiom of choice, for it has frequently happened that various authors have made use of the axiom of choice in their proofs without being aware of it. And after all, even if no one questioned the axiom of choice, it would bnot be without interest to investigate which proofs are based on it and which theorems are proved without its aid – this as we know , id also done with regard to other axioms.

This attitude produced the entire program of investigations concerning the status of the axiom of choice and the continuum hypothesis in the entire body of mathematics. The deep results obtained by Polish mathematicians working on this issue (Sierpiński, Kuratowski, Tarski, Lindenbaum, Mostowski) belong to the most remarkable achievements of the Polish Mathematical School. They very essentially contributed to the development of the set--theoretical project of the foundations of mathematics.

Although Polish logicians did not belong to the constructivism school in the foundations of mathematics, they worked on topics suggested by intuitionism. Let me mention resuts of Łukasiewicz (axiomatization of intuitionistic propositional calculus, investigations of relations between classical and intutionistic logic), Tarski (intuitionistic logic and topology), Wajsberg (the separation theorem for intutionistic propositional calculus) and Jaśkowski (the construction of adequate matrix for intuitionistic propositional calculus). Mostowski remarked [9, p. VII]:

> "I am inclined to think that a satisfactory solution of the problem of the foundations of mathematics will follow the path pointed out by constructivism or in a direction close to it. However, it would be impossible to write a textbook of logic on this base at the moment".

He preceded these words by remarking that a deeper discussion of the philosophical foundations of logic does not belong to the scope of formal logic.

Generally speaking, the principles of extensionality and compositionally were axiomatically adopted as regulative ideas governing logic. In propositional calculus, according to this principle, every operation must be extensional, that acting as a truth--function of its constituents. Consequently, many-valued logic has to be equally extensional as classical (bivalent) logic. This view motivated Łukasiewicz to interpreted modalities within many-valued logic. The modal extensions of classical logic, like Lewis-style systems, were considered as somehow defective as not fully extensional. The principle of extensionality was extended to interpretations of quantifiers and semantic constructions. In particular, Tarski's famous semantic definition of truth was extensional in this sense, that he defined the set of true sentences, not the intensional concept of truth. Of course, one can, and Tarski did that, consider the definition in question as explaining the meaning of the term "true', but this move requires the assumption that extension and intension strictly coincide in this case.

Polish logicians generally accepted realism as a view concerning the relation of logic and reality. Roughly speaking, logic was conceived as a description of reality. This view was, as I already noticed, advanced by Leśniewski, was also shared by Łukasiewicz in the interwar period (he took a more conventionalist position after 1945). However, both leaders of the Warsaw School of Logic had considerably different answers to the question which logic is correct. Leśniewski decisively preferred classical logic and considered many-valued logics as formal algebraic constructions. On the contrary, Łukasiewicz maintained that one of the variety of rival systems is satisfied in the real world (he hoped that infinitely many-valued logic is correct). Tarski also preferred classical logic. He did not justify this view by ontological arguments, but appealed to simplicity, universality and productivity of rthe classical system. A consequence of realism (Tarski presumably shared this philosophy of logic) consisted in the rejection of the distinction between logical and extra-logical concepts as well as between logical and empirical, at least if both discriminations are taken as absolute. Tarski tried to explain the essence of logical notions by regarding them as invariant under all one-to-one transformations of the universe into itself. It was a generalization of the Erlangen program in the foundations geometry, proposed by Felix Klein.

Although Polish logicians had no official philosophy of logic, several important investigations were strongly motivated by philosophical considerations. Perhaps the most important examples of how philosophy influenced logic are provided by many-valued logic and the semantic definition of truth. Łukasiewicz's main (initial, because he changed his mind later) motivation in proposing many-valued system of logic explicitly focused on attempts to reject determinism for its inconsistency with postulates of freedom, creativity and responsibility. Tarski in his semantic truth-definition explicitly followed Aristotle's tradition seeing truth as saying as things are. On the other hand, one should definitely observe that motivation does not mean justification. In particular, Łukasiwicz regarded the principle of bivalence as metalogical, not purely logical and Tarski opted for the classical truth-definition for its intuitive plausibility and being subjected to a rigorous mathematical treatment. Consequently, we must try to justify bivalence or its denial by separate investigations, ontological or scientific, because pure logic is insufficient in this respect.

Finally, let me say something about the social significance of logic according to the Polish school. This goes back to Twardowski. He wrote [19, p. 71]:

> "The lack of logical education not only decreases the intellectual level from the theoretical point of view, but also brings ignorance and obscurity in practical applications of our thoughts. And the whole our life is this practical application".

Similar thoughts are to be found in Łukasiewicz [5, p. 615; 6, p. 5]:

> "I believe that only just mathematical logic will teach us strict thinking. In my opinion, it is its greatest significance and task, even its social mission.
>
> My entire scientific activity was guided by the remote thought that we will come at one time to more correct views about the world and life by improving logical thinking".

And Tarski said [15, p. XV]:

> "I shall be happy if this book contributes to the wider diffusion of logical knowledge. The course of historical events has assembled in this country [USA – J.W.] the most eminent representatives of contemporary logic, and has thus created here especially favourable conditions for the development of logical thought. These favourable conditions can, of course, be easily overbalanced by other and more powerful factors. It is obvious that the future of logic, as well as of all theoretical science, depends essentially upon normalizing the political and social relations of mankind, and thus upon a factor which is beyond the control of professional scholars. I have no illusions that the development of logical thought, in particular, will have an essential effect upon the process of normalization of human relationships; but I believe that the wider diffusion of the knowledge of logic may contribute positively to the acceleration of this process. For, one the one hand, by making the meaning of concepts precise and uniform in its own field and by stressing the necessity of such a precision and uniformiazation in any other domain, logic leads to the possibility of better understanding between those who have the will to do so. And, on the other hand, by perfecting and sharpening the tools of thought, it makes men more critical – and thus makes less likely their being misled by all the pseudo-reasonings to which they are in various parts of the world incessantly exposed today".

Perhaps we can sum up the last quotations by repeating Łukasiewicz's and Tarski's sayings (unpublished, but preserved in the oral Polish tradition): "Logic is morality of speech and thought"; "Religion (you can say "ideology") divides people, logic brings them together".

Bibliographical Appendix

For general accounts of Polish Mathematical School see [3, 4]. The Lvov branch of Polish Mathematical School is extensively presented in [2]. For account of the Lvov–Warsaw School, see [2, 14, 20]. The book [12] contains a detailed exposition of the philosophy of logic and mathematics in the interwar period as well as provides an extensive bibliography of sources.

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