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DEFORMATIONAL CALCULATION METHOD OF BEARING CAPABILITY OF FIBER-CONCRETE STEEL BENDING ELEMENTS

ODKSZTAŁCENIOWA METODA OBLICZANIA NOŚNOŚCI ZGINANYCH ELEMENTÓW FIBROBETONOWYCH Z WŁÓKNAMI STALOWYMI

Abstract

This article proposes to use parabolic-linear diagram of deformation of steel fiber concrete under the conditions of compression. General principles of calculation method of steel fiber concrete elements. with the use of deformation diagrams of steel fiber concrete, have been submitted. The aim of this study was to present a method for calculating the deformation and deformation capacity steel fiber concrete bending elements based on the results of experimental studies. Results of tensile testing of samples allowed to put forward an analytical description of the time dependence of the stress-strain of steel fiber concrete in tension and compression, and accurately calculate the load and deflection bending elements. The experimental significance of the bearing capacity of steel fiber concrete bending elements on the fiber from the sheet and theoretical implications calculated by the proposed method have shown quite a good convergence.

Keywords: steel fiber concrete, diagram, design, deformations

Streszczenie

W artykule przedstawiono podstawowe zasady i metody obliczania zginanych elementów stalefibrobetonowych za pomocą paraboliczno-prostokątnych wykresów odkształceń stalefibrobetonu. Celem niniejszej pracy było zaproponowanie odkształceniowej metody obliczania nośności i odkształcenia stalefibrobetonowyh elementów zginanych opartej na wynikach badań doświadczalnych. Wyniki badania próbek na rozciąganie pozwoliły na zasugerowanie analitycznego opisu wykresów zależności „naprężenie–odkształcenie" stalefibrobetonu przy rozciąganiu i ściskaniu oraz dokładniejsze obliczenie nośności i ugięcia elementów zginanych. Doświadczalne wartości nośności stalefibrobetonowyh elementów zginanych z włóknami stalowymi i wartościami teoretycznymi obliczonymi według proponowanej metody wykazały dobrą zbieżność.

Słowa kluczowe: stalefibrobeton, wykres, obliczenie, odkształcenie

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1. Introduction

Exertion of fiber reinforcement to reinforce concrete allows us to enhance its exploitation qualities. These can include: lifted comparing to regular concrete resistance to cracking, firmness to stretching and pressure, frost resistance, shockproof, toughness to destruction etc. However, in case of changing diametrical reinforcement of beams and nets, which are used to reinforce the slabs, it is possible to reduce the labor intensity of the production of constructions.

So the calculation method of the bearing capacity of steel fiber elements is crucial [1, 2] and has been elaborated by the analogy of reinforcement cement constructions. Thus at the stage of utmost balance, rectangular distribution diagrams of the strain of steel fiber concrete have been accepted in compact and stretch zones. The research, which has been conducted recently, has shown that such an approach is not fully reasoned and can lead to the situation when the constructions which were calculated according to the norms [1, 2], won't meet the reliability requirements. Therefore, the improvement of the calculation method for steel fiber concrete bending elements on the basis of using whole diagrams of deformation of steel fiber concrete and reinforcement is a problem, the solving of which is explained below.

2. Analysis of latest research and publications

In this work [3] it is proposed to use parabolic-linear diagram of deformation of steel fiber concrete under the conditions of compression. By the equation of a parabola, the ascending branch is described up to the level of strains, which are equal to temporary resistance f_{fcc}, and to linear equations $\sigma_{fc} = f_{fcc}$, in the rest of the diagram. The diagram, under the conditions of distension in the form of a parabola, the ascending branch and hyperbola or bilinear function, is at the descending. Experimentally received results concerning the diagram in assignment [3] haven't been provided.

The specialists of the German committee on concrete, offer an idealized diagram under the extension of steel fiber concrete which is accepted as three-cornered [4]. The ascending branch of this diagram is continuing as a horizontal line to deformations which equal $\varepsilon_{fct} = 3.5 \cdot 10^{-3}$, descending branch is finishing under the deformations $\varepsilon_{fctu} = 25 \cdot 10^{-3}$.

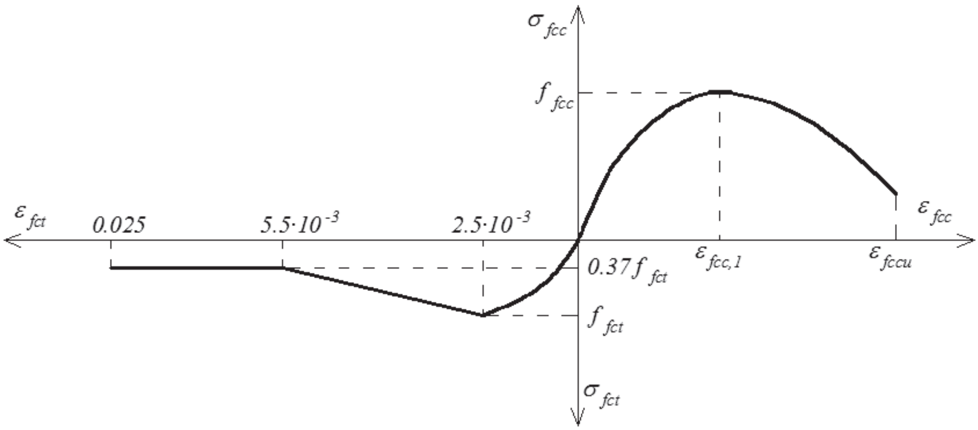
The work [5] has used the diagrams under extension and pressing, in a way of non-linear functions without descending branches.

Therefore, there is a list of propositions regarding the description of idealized diagrams, the parameters of which have not yet enough experimental explanation. Deformation diagrams are recommended to be used only at the stage of destruction, thus its criteria is to reach utmost deformation in concrete or reinforcement [3–5]. The aforementioned works didn't consider such criteria as being upper bound of bearing capacity at the diagram "bending moment – flection", which means the loss of balance abilities at the stretched and compressed zones of a bending element.

The aim of this work is to elaborate the deformation method of calculation of steel fiber concrete bending elements on the fiber of the sheet. The objectives of the research are the following: analytically describe whole diagrams of deformation of steel fiber concrete which were received in the assignment [6], elaborate mathematical apparatus to calculate bearing capacity of steel fiber concrete bending elements, compare the results of the calculations and experimentally received data trying out steel fiber concrete bending samples of beam slabs.

3. The method of calculating the deformation capacity of bending elements

The proposed parameters of idealized diagrams when they are stretched or compressed (Ill. 1) are well matched with the results of experimental research received in work [6] and they have a lot in common with the appropriate parameters of hard constructional concrete [7].



Ill. 1. Idealized diagrams of deformation of steel fiber concrete on the fiber of the sheet under stress and stretching

Here is compression stress:

$$\sigma_{fcc} = f_{fcc} \sum_{k=1}^{5} a_k \eta_c^k \tag{1}$$

where:

$\eta_c = \dfrac{\varepsilon_{fcc}}{\varepsilon_{fccu,1}}$,

a_k – polynomial coefficients;

– under the extension of ascending branch:

$$\sigma_{fct} = f_{fct} \sum_{k=1}^{5} a_k \eta_t^k \tag{2}$$

where:

$$\eta_t = \frac{\varepsilon_{fct}}{2.5 \cdot 10^{-3}};$$

– at the descending branch:

$$\sigma_{fcc} = f_{fct}(1.525 - 210\varepsilon_{fct}) \tag{3}$$

In the formulas (1), (2) the coefficients a_k are the polynomial coefficients which were calculated by the method accepted at work [7] and takes into consideration the experimental data [6]:

− at pressing:

$$a_1 = \frac{1.1E_{fc}\varepsilon_{fcc,1}}{f_{fcc}}$$

− at stretching for ascending branch:

$$a_2 = 1 - a_1 - a_3 - a_4 - a_5, \quad a_3 = a_1 - 2a_4 - 3a_5 - 2$$

$$a_4 = \left\{\left[k - 2a_1\left(3\gamma - 2\right) + 12\gamma - 6\right] - 2a_5\left(10\gamma^3 - 9\gamma + 2\right)\right\} / \left[2\left(6\gamma^2 - 6\gamma + 1\right)\right]$$

$$a_5 = \left\{\left[k - 2a_1\left(2\gamma - 3\gamma\right) + 12\gamma - 6\right]\left(\gamma - 1\right)^2\gamma^2 - \left[\beta + a_1\gamma\left(2\gamma - \gamma^2 - 1\right) + \gamma^2(2\gamma - 3)\right]\left(6\gamma^2 - 6\gamma + 1\right)2\right\} /$$
$$\left\{2\gamma^2\left[\left(10\gamma^3 - 9\gamma + 2\right)\left(\gamma - 1\right)^2 - \left(\gamma^3 - 3\gamma + 2\right)\left(6\gamma^2 - 6\gamma + 1\right)\right]\right\}$$

where:

− at pressing:

$$\gamma = \frac{\varepsilon_{fccu,1}}{\varepsilon_{fcc,1}}$$

− average value at the stretching for ascending branch:

$$\gamma = \frac{\varepsilon_{fctu,1}}{\varepsilon_{fct,1}} = \frac{2.5 \cdot 10^{-3}}{5.5 \cdot 10^{-3}} = 0.455$$

− at pressing:

$$\beta = \frac{1.1\varepsilon_{fccu,1}}{\varepsilon_{fcc,1}}$$

– at stretching for ascending branch:

$$\beta = \frac{5.5 \cdot 10^{-3}}{2.5 \cdot 10^{-3}} = 2.2$$

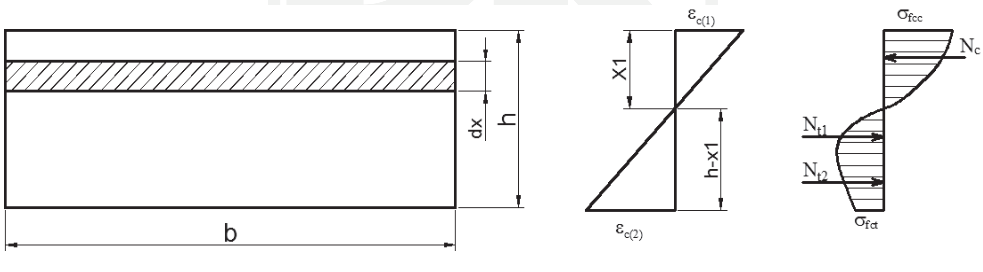
– at pressing:

$$k = 2.7 \cdot \left(\frac{\varepsilon_{fccu,1}}{\varepsilon_{fcc,1}} \right) - 6.1 - \frac{0.05}{\left(\frac{\varepsilon_{fccu,1}}{\varepsilon_{fcc,1}} - 1 \right)^2}$$

– at stretching:

$$k = 2.7 \cdot \left(\frac{5.5 \cdot 10^{-3}}{2.5 \cdot 10^{-3}} \right) - 6.1 - \frac{0.05}{\left(\frac{5.5 \cdot 10^{-3}}{2.5 \cdot 10^{-3}} - 1 \right)^2} = -0.195$$

The formulas (1)–(3) describe quite precisely the diagrams of pressing and stretching of steel fiber concrete up to the level of relative deformations which are $3 \cdot 10^{-3}...4 \cdot 10^{-3}$ at the stretching and pressing. Theoretically received meaning of appropriate tensions, only at certain dots, exceed the experimental ones at 0,3...3,5%, but in most cases they are less, thus the equation (1)–(3) at first approximation can be used to evaluate the bearing capacity of bending elements.



Ill. 2. Distribution diagram of deformation and compression cross cut of bending steel fiber concrete element

In Ill. 2 the distribution diagram of deformation and compression of bending element, which is reinforced by fiber, is shown. Bending element which can accept the element equals:

$$M = M_c + M_{t1} + M_{t2} \tag{4}$$

where: M_c, M_{t1}, M_{t2} – moments which are accepted by pressed and stretched zones according to the diagram relative to the neutral axis. The results of the experiments and calculations have proved as will be shown below that in the stretching zone at the stage of utmost balance, the third part of the work of fiber concrete for stretching due to the diagram (Ill. 1) can not be realized.

The balance of inner stresses is provided under the implementation of the conditions:

$$N_c = N_{t1} + N_{t2} \tag{5}$$

At $\varepsilon_{c(2)} \leq 0.002$, $N_{t2} = 0$.

Using the similarity of the triangles of the distribution diagram of deformation (Ill. 2) and known correlations of material resistance, it is possible to write for the compressed zone:

$$N_c = \int \sigma_{fc} dF = \int_0^{x_1} f_{fcc} \sum_{k=1}^{5} a_k \left(\frac{\varepsilon_c}{\varepsilon_{fcc,1}} \right)^k bdx =$$
$$= \int_0^{x_1} f_{fcc} \sum_{k=1}^{5} a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x}{h\varepsilon_{fcc,1}} \right)^k bdx = f_{fcc} b \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{fcc,1}} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+1} \tag{6}$$

where

$$x_1 = \frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}}$$

$$M_c = \int \sigma_{fc} x dF =$$
$$= \int_0^{x_1} f_{fcc} \sum_{k=1}^{5} a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x}{h\varepsilon_{fcc,1}} \right)^k xbdx = f_{fcc} b \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h\varepsilon_{fcc,1}} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+2} \tag{7}$$

Similarly by integrating for the stretching zone we receive:

$$N_{t1} = f_{fct} b \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h 2.5 \cdot 10^{-3}} \right)^k \left(\frac{2.5 \cdot 10^{-3} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+1} \tag{8}$$

$$M_{t1} = f_{fct} b \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h 2.5 \cdot 10^{-3}} \right)^k \left(\frac{2.5 \cdot 10^{-3} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+2} \tag{9}$$

$$N_{t2} = f_{fct} bh \left(1.525 \frac{(\varepsilon_{c(2)} - 0.0025)h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} - 105 \frac{\varepsilon_{c(2)}^2 - 0.0025^2}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right) \tag{10}$$

$$M_{t2} = \frac{f_{fct} bh^2 \left(0.7625(\varepsilon_{c(2)}^2 - 0.0025^2) - 70(\varepsilon_{C(2)}^3 - 0.0025^3) \right)}{(\varepsilon_{C(1)} + \varepsilon_{C(2)})^2} \tag{11}$$

To reach $\varepsilon_{c(2)} = 0.0025$, exertions $N_{t2} = 0$. The exertions at the stretched zone at this level:

$$N_{t1} = \int \sigma_{fc} dF = \int_0^{x_1} f_{fct} \sum_{k=1}^{5} a_k \left(\frac{\varepsilon_c}{0.0025} \right)^k bdx =$$
$$= \int_0^{x_1} f_{fct} \sum_{k=1}^{5} a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x}{h \cdot 0.0025} \right)^k bdx = f_{fct} b \sum_{k=1}^{5} \frac{a_k}{k+1} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h \cdot 0.0025} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+1} \quad (12)$$

$$M_{t1} = \int \sigma_{fct} x dF =$$
$$= \int_0^{x_1} f_{fct} \sum_{k=1}^{5} a_k \left(\frac{(\varepsilon_{c(1)} + \varepsilon_{c(2)})x}{h \cdot 0.0025} \right)^k xbdx = f_{fct} b \sum_{k=1}^{5} \frac{a_k}{k+2} \left(\frac{\varepsilon_{c(1)} + \varepsilon_{c(2)}}{h \cdot 0.0025} \right)^k \left(\frac{\varepsilon_{c(1)} h}{\varepsilon_{c(1)} + \varepsilon_{c(2)}} \right)^{k+2} \quad (13)$$

4. Comparative analysis of test results

This analytical apparatus has been used to assess bearing capacity of steel fiber concrete elements of slab type within the sizes l = 700 mm, b = 200 mm, h = 30 mm. The samples have been tested at operating execution 600 mm with the application of concentrated strengths at its thirds. The variations of percentage of fiber reinforcement of the volume ρ_{fv} and firmness of concrete-matrix f_c have been anticipated in the testing program. The results of the testing (table 1) have proved that the norms [1] in which the rectangular distribution diagrams of tension in compressed and stretched zones were accepted, don't allow to estimate precisely the bearing capacity of bending steel fiber concrete elements. Destructive bending moments $M_{\exp}$ have appeared to be essentially less than calculated $M_{[1]}$ by the Norms [1]:

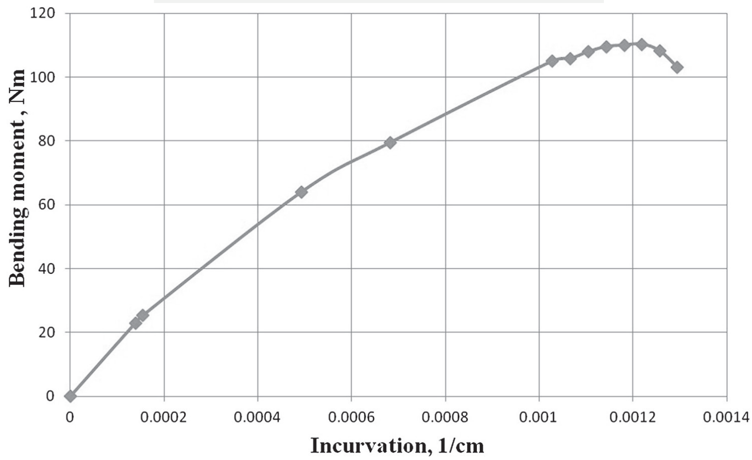
$$M_{[1]} = \frac{f_{fcc} \cdot f_{fct}}{f_{fcc} + f_{fct}} \times \frac{bh^2}{2} \quad (14)$$

Bending moments M_{theor} calculated under the proposed deformational method at 1...12% less than experimental ones. It is necessary to make a remark that the proposed deformational method allows to receive data about the pressed deformational state of the element during its work from the beginning of loading and up to its destruction. This way. for example. for the samples of first series SFB−1.1 (samples SFB−1.1.1. SFB−1.1.2 and SFB−1.1.3) have been received the following results which are given in the table 2 and in the Ill. 3. For the samples of these series as it is seen in the table 2. the maximum bending moment M_{theor} = 110.31 Nm. $\varepsilon_{c(1)}$ = 0.000657. $\varepsilon_{c(2)}$ = 0.003. Thus. as we see. at the utmost stage of destruction. deformations appear in the almost resilient pressed fibers. At the level of stretched fibers. the deformations agree with the descending branch of the diagram. The criterion of the destruction of a particular sample is the loss of balance of inner efforts (maximum at the diagram 'bending moment – flexures').

Table 1

Bearing capability of normal cuts of elements of slab type

Sample name	ρ_{fv} [%]	f_{fcc} [MPa]	f_{fct} [MPa]	$M_{\exp}$ [Nm]	$M_{[1]}$ [Nm]	M_{theor} [Nm]	$\frac{M_{theor}}{M_{\exp}}$
SFB−1.1.1	0.7	25.4	2.05	112.17	170.72	110.31	0.98
SFB−1.1.2				110.5			0.99
SFB−1.1.3				113.5			0.97
SFB−1.2.1	1.25	26.35	2.33	143.0	192.66	139.25	0.97
SFB−1.2.2				146.5			0.95
SFB−1.2.3				151.5			0.92
SFB−1.3.1	1.8	28.00	2.62	169.0	215.62	158.16	0.94
SFB−1.3.2				176.5			0.90
SFB−1.3.3				173.0			0.91
SFB−2.1.1	0.7	36.33	2.36	130.0	199.44	126.18	0.97
SFB−2.1.2				129.5			0.97
SFB−2.1.3				129.0			0.98
SFB−2.2.1	1.25	37.54	2.68	173.0	225.12	162.22	0.94
SFB−2.2.2				169.0			0.96
SFB−2.2.3				166.0			0.98
SFB−2.3.1	1.8	38.51	3.02	198.0	252.04	181.11	0.91
SFB−2.3.2				209.0			0.88
SFB−2.3.3				187.0			0.97



Ill. 3. The correlation of theoretical meanings of incurvation from bending moment for the pattern of series SFB−1.1

Table 2

Calculation results of the pattern of series SFB−1.1

$\varepsilon_{c(1)}$	$\varepsilon_{c(2)}$	$\chi = (\varepsilon_{c(1)} + \varepsilon_{c(2)})/h$	$\Sigma X = 0$	M_{theor} [Nm]
0	0	0	0	0
0.00009	0.00032645	0.0001388	4.33008E-05	22.9838465
0.0001	0.0003638	0.0001546	0.001746577	25.349335
0.0003	0.0011796	0.0004932	0.001398572	64.027182
0.0004	0.0016458	0.0006819	0.000767925	79.4477685
0.000584	0.0025	0.001028	0.000299537	104.988299
0.0006	0.0026	0.0010667	1.31868E-05	105.790229
0.000615	0.0027	0.0011051	0.000173637	107.967217
0.00063	0.0028	0.0011433	0.000410186	109.523891
0.000644	0.0029	0.0011813	0.000983105	110.00438
0.000657	0.003	0.0012192	7.21837E-05	110.312265
0.000671	0.0031	0.0012568	0.00035582	108.217099
0.000683	0.0032	0.0012943	0.000317555	103.160216

It is necessary to mention that calculations were being done with the use of the tabular processor Excel. Received data can also be used to calculate flexures. To accomplish this the use of Excel is needed and the dot chart “flexure – bending element” must be approximated by the equation of a straight line or parabola and then the flexures must be determined by Mohr’s integral.

5. Conclusions

1. Deformational calculation method of steel fiber concrete elements has been proposed and appropriate mathematical apparatus have been elaborated.
2. Experimental meaning of bearing capacity of steel fiber concrete bending elements on the fiber from the sheet and theoretical significance calculated by the proposed method have shown quite a good convergence (Table 1).
3. Bearing capacity of steel fiber concrete bending elements calculated by the method [1] provided to the formula of which the experimental meanings of the firmness of steel fiber concrete at the extension and pressing is 28...54% higher than the experimentally received one. It is reasonable to make changes to the standard [1] to take the deformation method as the main method of calculation.

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