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THE ORIGINS OF THE MOSCOW SCHOOL OF THE THEORY OF FUNCTIONS

GENEZA MOSKIEWSKIEJ SZKOŁY TEORII FUNKCJI

A b s t r a c t

The school known as the Moscow school of the theory of functions or the school of D.F. Egorov – N.N. Luzin, originated in 1910s within the framework of the Moscow philosophical-mathematical school. As a matter of fact, its birth was transplanting into the Moscow soil of the French studies on set theory and the theory of functions (E. Borel, H. Lebesgue, R. Baire). This school appeared as an attempt of Muscovites to reach the front line of modern mathematical studies in an area alien to interests of mathematicians from St.- Petersbourg. The attempt has turned successful: its result was creation (in a very short period) of one of the most effective European schools with its own subjects of studies (analytic sets etc.). As a result of the activity of this school Moscow became one of the leading world mathematical centers. Already in the late 1920s, the research done in this school (through the works of P.S. Aleksandrov, A.O. Gelfond, M.V. Keldysh, A.Ya. Khinchin, A.N. Kolmogorov, M.A. Lavrent'ev, L.A. Lyusternik, P.S. Novikov, L.S. Pontryagin, A.N. Tikhonov, P.S. Urysohn etc.) went out very far from the problems which marked the beginning of the Moscow school of the functions theory.

Keywords: *Moscow mathematical school of the theory of functions*, *set theory*, *theory of functions of a real variable*, *analytic sets*, *D.F. Egorov*, *N.N. Luzin, W. Sierpiński*

S t r e s z c z e n i e

Szkoła, znana jako Moskiewska Szkoła teorii funkcji lub Szkoła D. F. Jegorowa – N.N. Łuzina, powstała w drugiej dekadzie XX w. w ramach Moskiewskiej Szkoły Filozoficzno-Matematycznej. W rzeczywistości jej powstanie było przeniesieniem na grunt moskiewski francuskiej szkoły teorii mnogości i teorii funkcji (wyniki E. Borela, H. Lebesgue'a, R. Baire'a). Szkoła w Moskwie była próbą wejścia matematyków moskiewskich do światowej czołówki matematycznej w dziedzinie, która nie była przedmiotem badań matematyków z Sankt Petersburga. Tak pomyślana szkoła okazała się sukcesem: w bardzo krótkim czasie powstał tam jeden ze światowych ośrodków matematycznych z własną tematyką badań. Badania prowadzone w tej szkole już w końcu lat 20. XX w. (dzięki pracom P.S. Aleksandrowa, A.O. Gelfonda, M.W. Kiełdysza, A. J. Chinczyna, A.N. Kołmogorowa, M.A. Ławrentiewa, L.A. Lusternika, P.S. Nowikowa, L.S. Pontriagina, A.N. Tichonowa, P.S. Urysohna i innych), odeszły bardzo daleko od problemów, które zapoczątkowała Moskiewska Szkoła teorii funkcji.

Słowa luczowe: *Moskiewska szkoła teorii funkcji*, *teoria mnogości*, *teoria funkcji zmiennej rzeczywistej*, *zbiory analityczne*, *D.F. Jegorow*, *N.N. Łuzin*, *W. Sierpiński*

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1. Mathematics in Moscow in the early twentieth centary

By the early twentieth century, in the mathematical Moscow there was the following situation. A kind of school was formed around the Moscow Imperial University and the Moscow Mathematical Society, known in history as the Moscow philosophical and mathematical school [1]. One of the main characteristic features of this school were deep philosophical interests of its members, who wished to understand the mathematics – its subject and the methods used in it – in the broad philosophical context. Muscovites were in opposition to positivism, then highly fashionable in academic circles. Their propensity to the idealistic philosophy (including religious one), in particular, to the ideas of Leibniz, extremely popular at that time in Moscow philosophical circles, is well-known. The most influential mathematician in Moscow at that time, a professor of the Moscow University and a corresponding member of the Russian Academy of Sciences N.V. Bugaev (1837––1903), was an original philosopher, the author of a philosophical system of "evolutionary monadology", which had an evident impact on the very subject of Muscovites' mathematical research. One of the consequences of Bugaev's philosophical views was distinguishing the phenomenon of "discontinuous" in his teachings about the nature and the society. Contrasting his worldview with the analytical world outlook which dominated hitherto, whose core was Laplace's determinism and whose mathematical expression was the analysis of extremely smooth (analytic!) functions completely determined by being prescribed in an arbitrarily small neighborhood of any point of the area of their definition (it is a mathematical expression of the idea of total determinism!), Bugaev preached the idea of building a new mathematics, whose center should be the theory of discontinuous functions [2]. He began to build such a theory, which he called arithmology, together with his disciples. The starting point in this direction for Bugaev was the theory of functions of the number theory for the study of which considerable analytical apparatus has been created in mathematics. Thus the class of functions studied by Bugaev consisted of the piecewise smooth functions and the limits of sequences of such functions. Despite all the efforts made by his school, such theory turned out quite poor. One of the latest and most gifted of his students, D.F. Egorov (1869–1931), started his scientific career with arithmology (he devoted to arithmology his first paper [3] published in 1892), immediately dropped the subject, considering it hopeless (as a gifted mathematician, he had a remarkable intuition).

He chose differential geometry as his future field of studies – one of the main lines of research of the Moscow philosophical and mathematical school, which has grown from the work of K.M. Peterson (1828–1881). These studies (by Peterson, B.K. Mlodzeevskii (1858–1923), D.F. Egorov) became widely known and turned Moscow into an important European geometrical center [4, 5].

Another important area of research of the Moscow School was applied mathematics.

This direction, which began in Moscow with N.D. Brashman (1796–1866), received in this period a remarkable development in the works of N.E. Zhukovskii (1847–1921) and his students (S.A. Chaplygin, etc.). Zhukovskii's results (in particular, his work on the water hammer, which allows one to solve the persistent problem of failures in pipelines) made his name well-known in Europe and brought his school a prominent place among the contemporary schools of applied mathematics [5].

Studies in other branches of geometry (K.A. Andreev, A.A. Vlasov), results in number theory (Bugaev), in complex analysis (P.A. Nekrasov), in probability theory (Nekrasov), and in other areas of mathematics turned Moscow into an important mathematical center. However, this was not sufficient for the young ambitious Muscovites: they were not satisfied with the position of mathematicians who, although recognized abroad, were neglected by academic Petersburg. In the capital of the Empire P.L. Chebyshev school reigned, which tried to spread its decisive influence on the entire Russian mathematical community.

2. Mathematics in St. Petersburg and the conflict of mathematical communities of the two capitals

The mathematicians from Petersburg became famous in the world by their research in the areas which were developed in the outstanding studies of their common teacher P.L. Chebyshev (1821–1894). These were: number theory (E.I. Zolotariov, A.N. Korkin), probability theory (A.A. Markov, A.M. Lyapunov), constructive theory of functions (A.A. Markov, V.A. Markov), applied mathematics (Lyapunov), and mathematical physics (Lyapunov, V.A. Steklov). These studies were (and still are) highly appreciated in the world mathematical community and (what is especially important for us!) served as the basis for an even higher self-esteem of the St. Petersburg mathematicians. From their point of view (of course we are talking about prevailing opinions among them) it was necessary to develop only those parts of mathematics that have applications. This is evidenced by the list of the main areas of their research, in which number theory looks like odd man out. This section made it to the list, in a certain sense, by accident. Just arrived from Moscow to St. Petersburg, a young ambitious Chebyshev could not reject an offer of an influential academician V.Ya. Bunyakovskii to assist him with the preparation of a volume of Euler's works on the number theory for publication. Having plunged into the world of Euler's ideas, Chebyshev immersed himself into it and took so much interest in it that he grew into one of the classics of the theory of numbers. For St. Petersburg mathematicians who did not recognize the subjects without an applied orientation (hence their cold attitude toward the geometry of Lobachevskii, even when his ideas gained worldwide recognition, their opposition to Riemann's "decadent constructions", to S. Lie's ideas etc.), it became necessary to search for "excuses" in order to engage in research in the field of number theory. One of these "excuses" was the fact that the methods originating in number theory turned out to be applicable in other parts of mathematics, in particular, in mathematical analysis (number theory as "a forge" for the new methods of mathematical analysis!).

Positivism, the rejection of any idealistic philosophy and the militant atheism, were dominant in the worldview in the mathematical community of St. Petersburg. They became the basis of their negative attitude to the religion and the religious philosophy and caused their strongly negative view of Moscow mathematicians. Such attitudes also determined their rejection of Cantor's works on the set theory, which were often provided with theological introductions.

The studies of Muscovites on differential geometry were not supported by the mathematicians of the northern capital either, because these studies did not lead to applications, which were then considered rather important. As a result, the two mathematical communities were in confrontation (which should be considered in the context of a cultural confrontation between the two capitals [6]).

Since these communities determined the climate in the country (almost all professors in each Russian university graduated either from Moscow or from St. Petersburg University), this conflict gave rise to tensions in Russian mathematics as a whole. The acuteness of the conflict was tempered largely by Chebyshev himself – a native of Moscow, he maintained good relations with many Moscow mathematicians, supporting them in various undertakings (such as in the creation and activities of the Moscow Mathematical Society), and in the election to the Academy. But after his death, when A.A. Markov became the leader of the St. Petersburg mathematical community, the conflict escalated rapidly. We have already talked about the fact that mathematicians from Petersburg did not have any special respect for the results of Muscovites on differential geometry. Their attitude to Zhukovskii (a "Moscow celebrity", as V.A. Steklov sarcastically called him in his letters) was contemptuous. This tension was constantly manifested in the mathematical public life of the country and led to open clashes, which often ended with the scandals at the meetings of the Moscow Mathematical Society.

So criticism from St. Petersburg mathematicians against the works of academician V.G. Imshenetskii on the theory of integration of linear differential equations, supported by Muscovites, turned into a very hot battle at a meeting of the society, after which Imshenetsky returned to the hotel and died. Another well-known case of conflict, which also happened at a meeting of the Moscow Mathematical Society, were the attacks of St. Petersburg mathematicians on S.V. Kovalevskaya due to gaps in the demonstrations in her famous studies on the motion of a rigid body around a fixed point. Muscovites rose to defense of Kovalevskaya against A.A. Markov's aggressive attacks. And although, as we have said, their results were quite highly appreciated in the West, the Muscovites were not satisfied with the position to which they were actively pushed by the Petersburg academicians. They wanted to see themselves also at the forefront of the modern mathematical research. They did not want to compete with the colleagues from St. Petersburg in their favorite subjects, since this necessarily put them in the position of walking in the footsteps of St. Petersburg school.

For them it was necessary to find their own way, even further distanced from St. Petersburg ways. And they did find the way.

3. Birth of the Moscow school of function theory

In 1903 Bugaev died and his disciples, the young professors B.K. Mlodzeevskii and D.F. Egorov, became leaders in the Department of Mathematics at Moscow University. They made a lot of efforts to modernize the teaching of mathematics at Moscow University. They tried, firstly, to teach at the most modern level, and secondly, to acquaint students with the latest achievements of mathematics and the latest trends in their special courses. So Mlodzeevskii already in 1900/1901 lectured on the theory of functions of a real variable, and in the next academic year he repeated the lectures. The synopsis of these lectures compiled by a student P.A. Florenskii (1882–1937) (later the famous philosopher and theologian) has been preserved – see [7]. In this synopsis we find the exposition of the principles of set theory, and an introduction to the theory of functions of a real variable – a new discipline developed in 1890s on the basis of Cantor's set theory by the French mathematicians E. Borel (1871–1956), H. Lebesgue (1875–1941) and R. Baire (1874–1932). It is important to note that Mlodzeevskii not only introduced his students to the latest variant of the function theory, but connected it with Bugaev's ideas, with his arithmology. Cultivated in the atmosphere of Bugaev's preaching of the importance of building of the theory of discontinuous functions, the young Moscow mathematicians saw such a theory in the constructions of French mathematicians. And nothing prevented Muscovites from starting their own research on the theory of sets and functions of a real variable. If the Petersburg mathematicians were repelled by set theory because of its theological framing proposed by Cantor, the Muscovites found it rather attractive. The possibility to study efficiently the world of discontinuous functions made the new topics particularly attractive for them. Various aspects of this theory were discussed at the meetings of the students' circle organized by Florenskii at the Moscow Mathematical Society [8]. In 1908 I.I. Zhegalkin defended his thesis on transfinite numbers [9]. In 1903 Florenskii published in the literary magazine "Put" (Path) the first Russian exposition of the set theory [10]. The problems of set theory and of discontinuity were the topic of his Candidate's thesis "The idea of discontinuity as an element of the world outlook" [11], defended in 1904. The student N.N. Luzin was one year younger than his friend Florenskii and was under his influence [12, 13]. After graduating from university Florenskii was recommended by Zhukovskii for further studies at the University for the preparation of the master's thesis, but he did not use this recommendation and went to the Moscow Theological Academy – it was his conscious choice to devote himself to philosophy and theology. He delegated his function of the Secretary of the students' circle to Luzin [13]. And although Luzin, when entering the mathematics department of Moscow University, did not intend to devote himself to mathematics (his goal was to get an engineering degree and the training at the University was to be only a step in achieving this goal), in the course of training he became extremely interested in it. Under the guidance of Egorov his mathematical talent was revealed (its presence was a surprise even to himself). In 1906 he defended his graduation essay "On a method of the integration of differential equations" and passed state exams. In this way Luzin completed his studies at the university and was recommended by Egorov to continue the training for the preparation of master's thesis. By the end of 1909, he passed his master's examinations, which did not take him long as he studied these topics in his student years. Reflecting on the direction of his further studies, he took classes at the Faculty of history and philology, where he attended lectures on theoretical philosophy and on various areas of modern philosophy (in particular, L.M. Lopatin's lectures). In the autumn of 1910, when he (already in the rank of private-docent) was preparing to start his teaching at the University, an order came from the Ministry of Education to send him on a mission to Göttingen and Paris "for improvement in the mathematical sciences". Of course, he received such a gift as a result of Egorov's efforts, who exercised for this all his influence. In Göttingen he read a lot, worked (mostly in the theory of trigonometric series), and talked with the local mathematicians. In December he moved to Paris; his stay there turned out to be truly momentous. There he began to work in Hadamard's seminar, coming into personal contact with E. Picard, E. Borel, Lebesgue, Denjoy, etc. We can judge the creative atmosphere of Luzin's Parisian life in this period by his correspondence with D.F. Egorov [14]: his amazing creative enthusiasm, his contacts with the leaders of the French school of function theory – with Borel and Lebesgue, the beginning of his friendship with Denjoy. This correspondence allows one to feel the atmosphere in which the Moscow school of function theory was born. During this period, D.F. Egorov pondered a question that led him to the proof of the theorem which is known now as Egorov theorem and was published [15] in 1911 in the Comptes Rendus of the French Academy of Sciences (the correspondence [14] enables one to reconstruct the creative process of the demonstration of this theorem – see [16]). In that period Luzin was working on the problems which formed the content of his article [17], published the following year in the same journal and containing the theorem on the C-property (more extensive articles containing this result appeared in the same year in Russian [18]), known in mathematics as Luzin theorem (a similar result was published in 1905 by G. Vitali [19], which however, passed then unnoticed – see [20]).

These two articles became the foundation of the Moscow school of function theory, one of the most glorious in the first third of the twentieth century.

4. The first steps of the Moscow school of function theory

At the end of 1911 Luzin settled in Paris. His work progressed well and with Egorov's help his study tour, which ended in 1913, was extended. In the spring semester of 1914, he attended Picard's lectures on selected chapters of the function theory, the lectures of M. Bocher, a visiting professor from the United States, on the recent results in the theory of linear differential equations of the second order, Borel's lectures on the generalization of the notion of an analytic function. He participated in the sessions of Hadamard's seminar in Collège de France.

The most important, of course, was his work on problems of the theory of functions of a real variable and of set theory (he spent a lot of time reflecting on the problem of continuum) [21]. Returning to Moscow, he began in the fall his teaching at the University: a course of analytical geometry and a special course on the theory of functions of a real variable. The ground for the reception of the latter course was prepared by Egorov, who ran a spring semester seminar on the subject. As his disciple recalled later [22, c. 475]: "It is this special course and the accompanying seminar (...) that became a center from which the Moscow school of function theory grew (...)" The first generation of his disciples was raised at this seminar.

In 1915 Lusin published his thesis *The integral and the trigonometric series* [23], the defence of which was held on 27 April of the following 1916. The opponents were D.F. Egorov and L.K. Lakhtin. The historical and mathematical analysis of its content can be found in the book of A.P. Yushkevich [5, p. 572], who, in particular, wrote: "«The integral and the trigonometric series» was Luzin's invaluable contribution to the metric function theory. On the basis of the concept of measure the author studies properties of measurable functions, of the integral, of the derivative and of other central concepts of analysis". The result of the defence was a triumph. The Council decided to "(...) approve N.N. Luzin for the degree of the Doctor of pure mathematics (i.e. bypassing the Master's degree – S.D.)

because this thesis has special scientific merit (...)" [24, p. 18]. In the same year he was approved for a position of an extraordinary professor. The rise was quick and extraordinary. This was the heyday of Luzin's talent. A truly charismatic personality, he rallied around him the talented youth, who literally adored a young professor. All of them felt like the true creators of the new science and like members of a knight order, which they called Luzitania. These are the names of the first "knights": M.Ya. Suslin, D.E. Menshov, A.Ya. Khinchin, P.S. Aleksandrov, P.S. Uryson, V.N. Veniaminov, V.S. Fedorov.

The creative atmosphere of Luzitania promoted the early appearance of the first results of its members. In 1916 the notes of A.Ya. Khinchin (*Sur une extension de l'intégrale de M. Denjoy*. T. 162) appeared in Comptes Rendus of the French Academy of Sciences, in which he applied his notion of "asymptotic derivative" to the generalization of the concept of the Denjoy integral. P.S. Aleksandrov (*Sur la puissance des ensembles mesurables.* B. T. 162) demonstrated that every uncountable Borel set has the cardinality of the continuum, and D.E. Menshov (*Sur l'unicité du développement trigonométrique*. T. 163) constructed an example of a trigonometric series which has coefficients different from zero and converges almost everywhere to zero. Finally, in 1917 in the same journal a brilliant article of M.Ya. Suslin (*Sur une definition des ensembles mesurables B sans nombres transfinis*. T. 164) was published, which marked a turning point in the history of the Moscow school of function theory. A history of this note is the following. In the summer of 1916 Luzin assigned to his student the task to analyze critically the work of Lebesgue *Sur la représentation des fonctions analytiques* (1905). Trying to prove Lebesgue's assertion that the projection onto a straight line of a two-dimensional Borel set is a Borel set (Lebesgue considered this statement obvious), the student found that it was not true: using a construction introduced by Aleksandrov, he constructed an example in which such a projection is not a Borel set. W. Sierpiński, who resided in those years in Moscow and worked together with Luzin (how the fate has thrown a young Polish mathematician in Moscow – see below), described this event [25, c. 33]: "I witnessed how Suslin informed Luzin about an error of Lebesgue and handed him the manuscript of his first paper. Luzin very took seriously to the report of a young student and confirmed that he had indeed found a mistake in the work of the famous scientist".

The theory of new sets, which received the name of Suslin sets or analytical sets, became the last word in set theory and its development started immediately by Luzin himself. His first work, "where the set theory got its notable further development" [26, c. 130], was published in the same volume of Comptes Rendus as the work of Suslin. The new subject – theory of analytical or Suslin sets – became later central for the Luzin school. The milestone in its development was Luzin's book *Leçons sur les ensembles analytiques et leurs applications*, published in Paris in 1930 with a preface by Lebesgue (a Russian edition [27] appeared only in 1959).

Moscow school of function theory became one of the most striking phenomena in the European mathematical life of the first quarter of the twentieth century. Its development was rapid, despite the gravity of the situation in which Russia found itself in that period: the First World War, the Revolution and the subsequent Civil War. The attractive force of Luzin's personality (in those days it was exceptional – see the memories of L.A. Lyusternik [28]), the beauty of topics which opened before the Muscovites, the possibility for them to feel themselves at the epicenter of the nascent new mathematics, finally (and we should not forget about it!) Egorov's activity, who played the role of the unquestionable moral authority and the guardian of principles, created in Moscow the extremely favorable conditions for the scientific work, which continued even in the most unfavorable period of the years 1918–1921, when Luzin and his disciples, in search of sustenance, left Moscow. When from time to time Luzin came to the capital, all those who happened to be at that time in Moscow gathered for a seminar. Despite these harsh conditions the studies were going on very intensively. When in 1922 Luzin finally returned to the university and his seminar started to work in regular mode, the circle of his pupils was joined by L.A. Lyusternik, N.K. Bari, M.A. Lavrentiev, L.G. Shnirel'man, P.S. Novikov, L.V. Keldysh, A.N. Kolmogorov, and V.I. Glivenko. Luzin's older students became masters themselves and established their own seminars, in which they studied questions different from Luzin's topics.

The first to separate were the members of the topological circle of P.S. Aleksandrov and P.S. Uryson, including their own students A.N. Tikhonov, V.V. Nemytskii, N.B. Vedinisov, L.A. Tumarkin, and L.S. Pontryagin. A.Ya. Khinchin began to apply the methods of the measurable function theory to number theory and obtained important results in the metric number theory. Under his guidance L.G. Shnirelman and A.O. Gelfond began their research in number theory. M.A. Lavrentiev created his own school in the theory of functions of a complex variable (M.V. Keldysh, etc.). Finally A.Ya. Khinchin and A.N. Kolmogorov started their research in probability theory. The research of the school in set theory and theory of functions of a real variable. became an excellent common ground for all these studies, the results of which gained worldwide recognition, But in their research Luzin's students went in different directions, sometimes quite distant from each other (and, most importantly, from their Master). The school broke up and in the process of this disintegration a new formation began to develop, which in turn, became (together with the Leningrad school) the basis for the Soviet school of mathematics, one of the most influential ones in the second half of the twentieth century.

It is interesting to note that the arrogant attitude of mathematicians from Petersburg (in the 1920s already named Leningrad) was kept long enough. There is an anecdote, popular in Russian mathematical community. According to that anecdote, V.A. Steklov, displaying Luzin's thesis and leafing through its pages in which there were not as many formulas as there were in the works of the mathematicians from St. Petersburg, summed it up: "Is this mathematics? No, this is philosophy". In 1926, when the significance of the work of Luzin's school apparently should have been clear to mathematicians, another representative of the same school, academician Ya.V. Uspenskii in his letter to A.N. Krylov, discussing the candidates for the elections to the Academy wrote [29, p. 193-194] the following: "I feel deep disgust for this direction and firmly believe that this fashion will soon pass, especially if we take into account the criticism of Brouwer and Weyl, who raised strong objections not only against the entire colossus erected by Cantor and Lebesgue, but also against the facts which since the days of Weierstrass were considered as firmly established". The conflict lasted until the mid-30s and was put to the end by (...) I.V. Stalin. In the course of his reform of the Soviet science the Presidium of the Academy and a number of leading academic institutions (including the V.A. Steklov Mathematical institute) were transferred in 1934 to Moscow – "the headquarters of the Soviet science" had to be located close to the overlord.

The conflicting sides were forced to live and work together by the "will of the monarch". As a result, there was a fruitful synthesis of the Moscow and the St. Petersburg traditions, which laid the foundations for the Soviet mathematical school.

5. Concluding remarks on W. Sierpiński and on the parallels in the development of the Moscow and Warsaw schools of set theory and of the function theory

In our story we mentioned the name of the Polish mathematician W. Sierpiński, who witnessed the events of the Moscow mathematical life of 1915–1918 years and participated in them [25]. The events of the World War I brought him to Moscow (The entry of Russia into the war in 1914 took him on its territory. Because he was at that time a citizen of Austro-Hungary, he was interned in Vyatka. By efforts of B.K. Mlodzeevskii and D.F. Egorov he received the right of residence in Moscow, where he spent several years, closely associating with Egorov and Luzin). There he became friends with Luzin, with whom he kept creative relationship for many years [30, 31]. It was in Moscow that Sierpiński obtained, by his own admission (see a fragment of his letter to I.G. Melnikov from May 9, 1966 [30, c. 362]), his first significant results in set theory, published in 1916 in the Paris Comptes Rendus. Between 1915 and 1918 he published 36 papers, 4 of them in collaboration with Luzin. Their cooperation, despite some theoretical differences, for example, on the question of the axiom of choice, continued in subsequent years.

(On the Moscow period of Sierpinski's life and on the philosophical spirit reigning in Moscow mathematics in that time see E. Medushevski's article [32].) The "Russian component" of Sierpiński's biography cannot be reduced to the contacts with Luzin and his entourage. Born in Warsaw, after finishing the gymnasium he studied at the Warsaw University, where an outstanding representative of the St. Petersburg school G.F. Voronoi (1866–1908) was his teacher. Under his supervision Sierpiński did (1904) his first scientific study: he improved Gauss' result about the number $A(x)$ of the integer points in the circle $u^2 + v^2 \leq x$. The communications of the Polish and Soviet mathematicians in research on set theory (for example, in the theory of projective sets) and the theory of functions of a real variable are a special story, still waiting for its researcher. In conclusion I would like to draw attention to some parallels in the history of the Moscow and Warsaw schools of the theory of sets and functions.

Moscow school of function theory, as we have said, arose from Muscovites' search for topics to enable them to go out into the forefront of modern research, moreover, topics that would be independent of the interests of the Petersburg school, with which Muscovites were in the confrontational relationship. The theory of sets and functions of a real variable turned out to be such topics.

For Polish mathematicians (Sierpiński, etc.) the urgent task was to find areas of research which would allow them in the shortest possible time to create a mathematical school in Poland and, moreover, a school whose research would be at the forefront of modern mathematics.

To them, the same theory of sets and functions turned out to be such areas. The school was created in the confrontation with the old Polish mathematical center – with the mathematicians, grouped around Krakow University. The roots of the confrontation of the Moscow mathematicians and Petersburg mathematicians were ideological, and all other factors (including personal ones) were only secondary (although they would from time to time come to the forefront). In the Polish case the personal factor played a much more significant role. S. Zaremba's dominance in Krakow, his personal preferences, particularly his mathematical habits (he was "a pure classicist" and the set-theoretic direction evoked his strong antipathy) caused the departure from Krakow of many young mathematicians (including S. Banach, S. Kaczmarz, O. Nikodym).

Luzin wrote about this in his letter to Denjoy September 30, 1926 [33, c. 318-319]: "On returning from Paris to Moscow, I spent some days in Warsaw since Mr. Sierpiński invited me to meet him and to familiarize myself with his school (…) I would like to inform you about my mathematical impressions that I got in Warsaw (…).

Polish mathematicians, with whom I met, live in different cities – in Warsaw , Krakow, Lvov, Kovno, Vilno. From conversations with them I got a pretty clear view of mathematical life in Poland.

It seems to me that the mathematical life in Poland follows two completely different ways: one of them is inclined to the classical parts of mathematics, and the other to the theory of sets (functions). These ways exclude one another in Poland, being the irreconcilable enemies, and now a fierce struggle is going on between them".

The "classical side" forms a group, wrote Luzin [33, c. 319-320], around the Krakow University and the Krakow Academy and its leader is S. Zaremba. This group stands in opposition to the school of Sierpiński, the studies of which focused mainly on the theory of functions of a real variable and set theory. The representatives of this school took the leading positions in Warsaw and Lvov. These schools were in a state of war, the success of which, apparently, is predetermined: Warsaw and Lvov must win. That perspective was considered by Luzin as dangerous for the development of the Polish mathematics – this development gained unilateral character, and as a result, mathematics detached from its roots [33. c. 320]. In my opinion – wrote Luzin [33, c. 319] – such situation is dangerous because the exclusive attention to set theory and the neglect of the branches of classical mathematics seems to me to be too narrow, too one-sided".

(The situation was similar in many respects to the Russian one – there, the adherents of the traditional mathematics grouped around Leningrad mathematicians, and a new trend that was growing out of Cantor's set theory and the theory of functions of a real variable grouped around Muscovites: of Luzin and his school. And here and there the relationships were confrontational. But Russian scales made the situation not so acute: the rapid growth of research on new topics in Moscow did not threaten the development of the traditional mathematics in St. Petersburg, especially since one of the most important European schools of the time operated there – the school of Chebyshev).

Luzin told Sierpiński about his concerns, and the latter replied as follows [33, c. 320]: "Yes, this is really a serious danger, but greater than the dominance of one way is the danger of the lack of any way.

Before the advent of the Warsaw way mathematics in Poland didn't exist as there were separate scientists each of whom was interested in different things and did not have disciples.

This is why their works often had only a personal interest and were devoid of any scientific significance. Undoubtedly, this lack of personal creative initiative was caused by the lack of the public control, of the general mathematical opinions and of recognition of their works.

It was necessary, therefore, to create a broad mathematical environment, and it was created by the Warsaw school. As for our narrowness, I hope that it will decrease and disappear afterwards. The choice of the function theory as a basis for a common mathematical movement is the consequence of its simplicity".

Sierpiński proved to be right: Polish mathematics rather quickly went beyond the theory of sets and functions of a real variable and already in the 1930s established itself as one of the Europe's leading. Its potential was so powerful that even the tragedy that Polish science experienced during the Second World War (the extermination of a number of outstanding Polish mathematicians, the departure of talented young people to the West) has not stopped the process of its active development.

Luzin, discussing the situation that evolved in the Polish mathematics by 1926, of course, meant also a situation which was similar in many respects, that of mathematical Moscow at that time: the expanding of research topics by his students led to the disintegration of Luzitania. As we said before, hitherto a united community, rallied around him, their recognized master, was then divided into a number of new schools headed by his former students, who chose the direction of their research sometimes very far from his own interests. Luzin felt very painfully this decay and the loss of close ties with his disciples, trying to understand what was happening and to find the correct line of conduct. As we know, he was not so successful. The conflict that occurred with some of his disciples led to the notorious "affaire of academician N.N. Luzin" and could have ended tragically for him [34].

Many Soviet and western mathematicians stood up for Luzin; a special role in that campaign belonged to his old friend W. Sierpiński [35]. Fortunately, all ended well for him, though the wound inflicted by the circumstances of this "affaire" on the corps of the Soviet mathematical community did not heal for a long time.

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