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Ewa Kozień (koziene@uek.krakow.pl)
Department of Strategy Management and Organization Growth, Faculty of Economics and International Relations, Cracow University of Economics

Application of approximation technique to on-line updating of the actual cost curve in the earned value method

Zastosowanie metod aproksymacji do bieżącego uaktualniania przebiegu krzywej kosztu rzeczywistego w metodzie wartości wypracowanej

Abstract

The Earned Value Method (EVM) is a method commonly used in the quantitative project management. The course of the actual curve is usually different from the course of the planned curve. In order to provide a reliable estimation of the time of the project implementation or its real cost, the method of on-line updating the actual cost (AC) original curve can be applied. The actual cost curve is usually of the *S*-curve character. The approximation method is often used in the engineering applications. In the paper, the possibility of applying the polynomial type approximation method to the on-line updating of the actual cost curve is discussed.

Keywords: Earned value method, S-curve, approximation

Streszczenie

Metoda wartości wypracowanej (EVM) jest powszechnie stosowana w ilościowym zarządzaniu projektami. Przebieg krzywej kosztów rzeczywistych jest zwykle inny niż przebieg krzywej kosztów planowanych. W celu realistycznej oceny czasu końcowego realizacji projektu bądź jego rzeczywistych kosztów może być zastosowana metoda bieżącego poprawiania przebiegu krzywej kosztów rzeczywistych. Krzywa kosztów rzeczywistych ma zwykle przebieg podobny do przebiegu krzywej *S*. Metody aproksymacji są często stosowane w zastosowaniach inżynierskich. W artykule dyskutowana jest możliwość zastosowania wielomianów trzeciego stopnia do bieżącego uaktualniania przebiegu krzywej kosztów bieżących.

Słowa kluczowe: metoda wartości wypracowanej, krzywa S, aproksymacja

1. Introduction

Currently, the method of the earned value analysis – EVM (Earned Value Method) – has been used in the quantitative project management and has been computer-assisted in the form of universal computer programs packages (e.g. EVMS for Project™) [3, 9, 17, 21, 23]. The Earned Value is a program management technique that makes it possible to indicate what will happen to work in the future, based on the actual time data [12]. Unique and huge projects require specially created computer packages. As a whole, it connects the incurred expenses and the time of the project realization. It also requires the accurate determination of the schedule as well as providing the time of activity completion. In the classic approach, the method allows the current estimation of the progress of the project, and to be precise, the deviation from the accepted schedule. The analysis of the deviation (working in favour or to the detriment – acceleration or delay of the project) allows monitoring the execution of the project in the current moment of its implementation. It is therefore a convenient tool for the control of the project execution. The proper interpretation of the earned value parameters poses a problem. One of the most interesting possibilities of applications, however, is the estimation of the real (revised) time of the project completion or real (revised) total cost of implementation, which may be predicted in every moment of its execution. It is a crucial problem; however, it is barely developed in the literature. The heretofore applied approaches are based on the knowledge of the on-line known values of the earned value (budgeted cost of work performed, EV, BCWP) and actual cost (actual cost of work performed, AC, ACWP) functions and the course of the planned value (budgeted cost of work scheduled, PV, BCWS) curve. Such an approach may be encumbered with a big error, especially when applying it in the intermediate stages of the project implementation. The reason for that lies in the fact that it does not allow for the scheme of the project lifetime cycle, legitimate for the majority of projects, which leads in a straight line to the so-called *S*-curve, incurred expenses in the function of the project implementation time. In practice, it is a problem of predicting the course of the AC function, which is the extrapolation of the course beyond the course of its definiteness. The extrapolation's limit, which has to be taken into consideration, is the real schedule – the known PV function's course.

In the paper, the author considers the possibility of applying the approximation method in the *S*-curves class to the on-line updating of the actual cost curve. In the future, it will make it possible for the on-line updating of the estimated time duration (ED) or estimate cost at completion (EAC) values in ways other than the ones known in literature [5, 6, 11, 14–16, 19, 20].

The originality of the analysis is the detailed analysis of advantages, disadvantages and crucial points of using the polynomial of the third range to estimate the actual *S*-curve. Using polynomial in approximation is a very popular and useful technique due to the simple application of the last square method in order to find the proper form of the approximate function. Therefore, the crucial part of the practical application of the proposed approximation method is using the obtained shape and values of the approximation curve beyond the range of time from the beginning of project to the actual time (AT) of its realization.

2. *S*-curve

2.1. *S*-curves in EVM

The analysis of the project implementation prompts an observation that, despite the fact that the incurred expenses vary in particular stages of the project, if presented in a cumulated form of the accrued costs incurred from the beginning of the project execution, will take the form of the known in the economics *S*-curve. The name of the curve originates from its waveform, which resembles the letter *S*, or from the first letter of the term "spending-curve".

To be more precise, three curves, which are the functions of time, may be distinguished in the EVM method: planned value curve (PV(t) or BCWS(t)), earned value one (EV(t) or BCWP(t)) and actual cost one (AC(t) or ACWP(t)). The course of all the curves refers to the *S*-curve form. The accepted in a project's stage course of the PV(t) expenses function is of a particular course defined in the interval of the design life of the project implementation ($t \in [0,\text{PD}]$, PD-planned duration). In practice, one may usually encounter a course deviation, which results in a necessity for a regular creation of a EV(t) or AC(t) curve. In reality, however, the interval of the functions' definiteness is changing during the project realization and is defined after the project completion as ($t \in [0,\text{SAC}]$, SAC-schedule at completion), or is estimated on a regular basis as ($t \in [0,\text{ED}]$, ED-estimated duration).

The notion of a fast delayed start of the project implementation is one of the practical problems. In such a case, one may imagine two limit curves of the project implementation, with the assumption that the end schedule is met when the total costs and the implementation time are met. Considering the acquired shape of the area, it is sometimes called a banana curve [3].

It should be noted that the Earned Value's *S*-curves are not the same as those used for project cash flow analysis. This does not facilitate combined interpretation of project performance in terms of work progress, receipts and expenditures [6].

2.2. Up-dating of AC curve

In a case when there are clear differences between the planned schedule and its real implementation, a process of a specific up-dating of the original schedule may occur. The process should not be performed too often. In the relevant literature, one may find a discussion on a single and multiple up-dating of the AC(t) curve's course [4, 13, 15, 16]. The approach presented under the 3 point is an alternative to the one discussed in the article.

It should be noted that the regression based methodology to interpolate characteristics of growth models are an important way to forecast project cost at completion [15]. Defining an equation for the *S*-curve model requires consideration of some issues relevant to nonlinear regression analysis [15].

2.3. Analytical formulas for *S*-curve

In the classic approach, the equation of the *S* curve takes a form of the so-called logistic curve in form (1) [16, 18, 24], where the defining parameters meet the following conditions: $a > 0$, $c > 0$ and $b > 1$. The course of the function has been presented in Fig. 1.

$$Y(t)=\frac{a}{1+b\cdot e^{-c\cdot t}} \tag{1}$$

D.F. Cioffi proposed a parametrized *S*-curve tool for managing cost of ongoing project [4, 15] introducing a different analytic function, which implements the similar tendency in form (2) [4]. Fig.1 presents the course of the curves defined by (1) and (2) for the following parameters' value: $a = 1$, $b = 100$, $c = 10$.

$$Y(t)=a\cdot\frac{1-e^{(-c\cdot t)}}{1+b\cdot e^{(-c\cdot t)}} \tag{2}$$

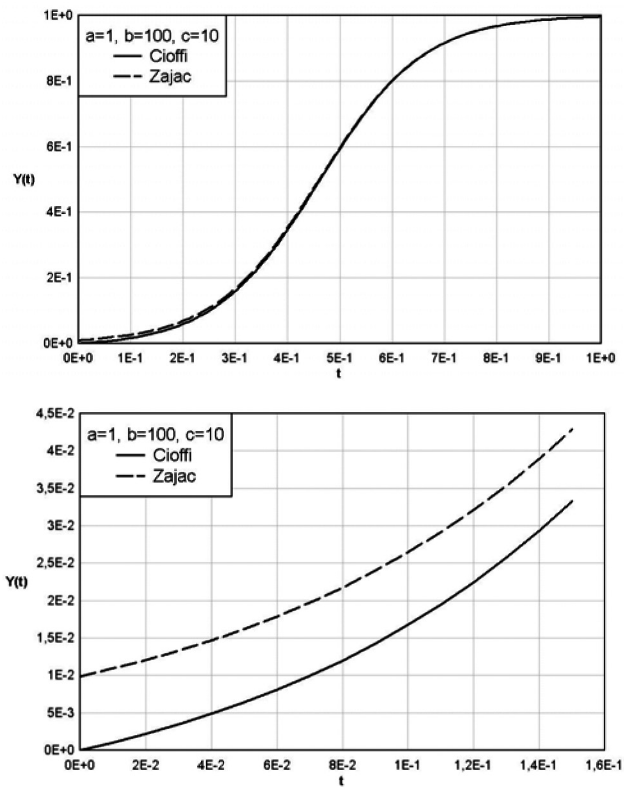

Fig. 1. Comparison of the courses of the logistic curve and D.F. Cioffi's *S*-curve

One may notice a clear similarity between the courses of the functions in a great time scale, which is visible in the left picture. The fundamental difference is shown around the time value, which equals zero. Two horizontal asymptotes are the characteristics of the logistic function course, which is visible in Fig. 1. The first one for $y = 0$, the second one for $y = a$. This means that, in order to obtain zero costs for $t = 0$, one should shift the course of the function downwards. The value of the shift, however, depends on the configuration of the a, b and c parameters, whereas the course of the second function's value, by definition, is zero for $t = 0$. The behaviour of the functions for the time value near zero is shown in the right picture. The provided observation shows the analytic function, which models the S-curve, and according to D.F. Cioffi, is more convenient for the EVM applications.

The other analytical functions of exponential type, which are used in approximation of the S-curves, are reviewed by T. Narbaev and A. De Marco [16]. They are functions proposed by G.A.F. Seber and C.J. Wild [18] – the Gomperetz one (3), proposed by Bass [2] (4) and proposed by W.W. Hines and D.C. Montgomery [10] – the Weibull one (5).

$$Y(t) = a \cdot e^{-e^{[-b(t-c)]}} \tag{3}$$

$$Y(t) = \frac{1 - e^{[-(a+b)t]}}{1 + \frac{b}{a} e^{[-(a+b)t]}} \tag{4}$$

$$Y(t) = \frac{a}{1 - e^{\left[-\left(\frac{t}{b}\right)^{c}\right]}} \tag{5}$$

In some of the applications to the approximation, the curve from the hyperbolic tangent family (6) may be used where the natural n value ($n \in N$) is accepted in relation to the postulated curve course (see Fig. 2 for some values of n). For $a = 1$, the course of the presented formula (7) has two horizontal asymptotes: $y = 0$ and $y = 1$ [1].

$$Y(t) = a \cdot \frac{1}{2} \cdot \left[1 + \tanh(nt)\right] \tag{6}$$

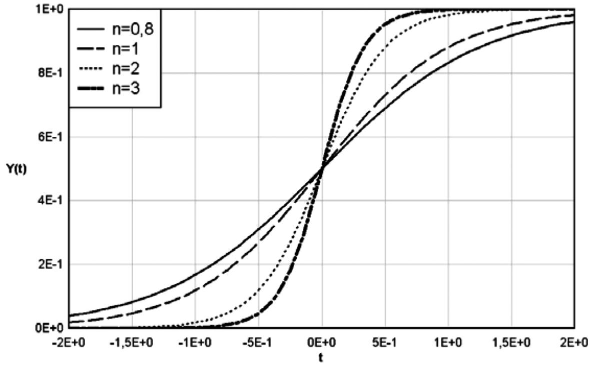

Fig. 2. Hyperbolic tangent family (n-parameter)

Let us consider the nonlinear first-order ordinary differential equation, called the logistic equation (8). Let us assume that: $a > 0$, $b > 0$. The solution of the equation with the original condition (9), where t_0 of the requested instant, whose value of the function is Y_0, takes the form (10) [13]. The obtained curve is a logistic curve with horizontal asymptotes $Y = 0$ and $Y = b/a$, which is the S-curve type (see Fig. 3). The differential equation in the considered form may be used for a description of the population growth dynamics [4]. A properly calibrated curve, as a solution of the differential equation, served M.K. Hubbert as a means to predict the decrease in oil extraction in the United States of America in the 1970's [13].

$$\frac{dY(t)}{dt} - aY(t) + bY^2(t) = 0 \tag{7}$$

$$Y(t_0) = Y_0 \tag{8}$$

$$Y(t) = \frac{\frac{a}{b}}{1 + \left(\frac{a}{b}\frac{1}{Y_0} - 1\right)e^{[-a(t-t_0)]}} \tag{9}$$

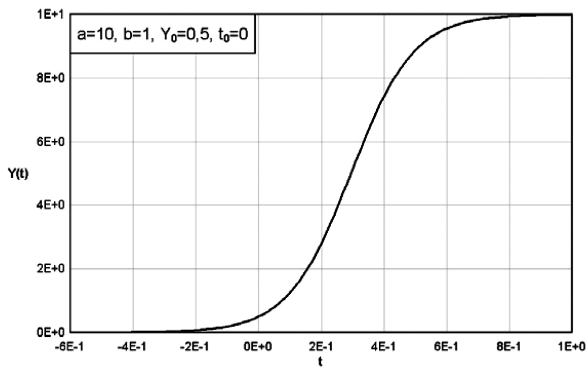

Fig. 3. The course of the curve which is the solution of the differential equation (10) for $a = 10$, $b = 1$, $Y_0 = 0.5$ and $t_0 = 0$

The S-curve can be approximated by the polynomial type families of normalized functions of the second order (11). It is visualized in Fig. 4 for $a = 0$ and $b = 1$. The values of curves are between 0 and 1. The function has its inflection point for the time variable equal to $t_{pp} = (a + b)/2$. A disadvantage of applying the second order polynomial is its definition in two sub-ranges. The advantage is its limited values of the function in the range [0, 1] for normalized formulation (11).

A. Czarniogowska [6] afer J. Evensmo and J.T. Karlsen [8] proposes application of the part of the third order polynomial function to the ex-post analysis of the actual cost function (12). They formulate the approximate formula for AC(t) and EV(t) directly taking into account values of budget at completion (BAC), estimate of completion (EAC) and the on-line determined schedule performance index (SPI). Application of formula in general formulation (13) makes it possible to easily apply the regression analysis for approximation. In formula (13), it is assumed that project starts for $t = 0$, i.e. $Y(0) = 0$. Moreover, if we assume that for the time variable equal to t_{pp} the considered function (13) has its inflection point, the additional relationship (14) may be formulated. This relationship is useful for application of the formula to approximation of the realistic AC(t) curve in the later time part of realization, when the realistic value of t_{pp} is known. A disadvantage of applying the third order polynomial is changing the S-type form of the function out of the limited range of time.

Application of the polynomial type of approximation is very useful for practical application. The least mean squares method can be used for finding the values of unknown parameters of the approximate function. In practice, there is no problem with the number of points, for which the actual cost values are known.

$$Y(t)=\begin{cases} 0 & \text{for} \quad t<a \\ 2\left(\dfrac{t-a}{b-a}\right)^2 & \text{for} \quad a\le t\le \dfrac{a+b}{2} \\ 1-2\left(\dfrac{t-b}{b-a}\right)^2 & \text{for} \quad \dfrac{a+b}{2}\le t\le b \\ 1 & \text{for} \quad t>b \end{cases} \tag{10}$$

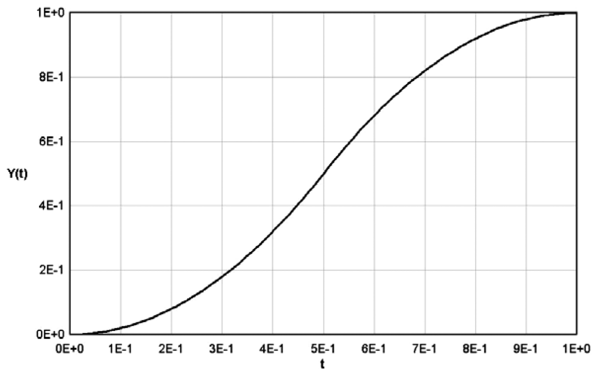

Fig. 4. Example of the polynomial second order spline function (11) for $a = 0$ and $b = 1$

$$AC(t)=\frac{EAC}{T^2}\left(3t^2-\frac{2}{T}t^3\right); \quad T=\frac{T}{SPI} \tag{11}$$

$$Y(t)=at^3+bt^2+ct \tag{12}$$

$$b=-3at_{pp} \tag{13}$$

3. Function approximation

The essence of the notion of function interpolation and approximation is creating a continuous function, which describes the course of the chosen parameter's variations in the best way possible. The values of the parameters are known in the specific point, in our case in the specific instants, namely in the discrete form. The notion of interpolation is connected with defining the function exactly going through the discrete points. The notion of approximation, however, does not contain the limit of going beyond the discrete points and set of discrete data. The considered problem of defining the real time of project implementation is a problem of approximation, in which, with data on a fragment of the project execution time, we estimate what is going to happen in the future. The notion of extrapolation is not unequivocally stated and is related to two problems: the choice of the class of the approximation function (type of the function) and the criteria of the approximation error, which, when minimized, define the notion of the best approximation for the considered problem. When the problem of nonlinear approximation is considered (e.g. the *S*-curve type approximation), the problem of selecting the chosen class of approximation curve is more important than the obtained average (discrete or integral) value of the chosen error measure – if the procedure of error minimization has been applied to the obtained values of parameters, which define the approximation curve. It also introduces the problem of interpolation and multidimensional approximation, in which the created functions depend on numerous parameters (independent variables). In the analysed case, it is a single-valued approximation, as the functions in the EVM methods are only the functions of time. It is worth pointing out that one of the approximation methods is the least squares method, and especially the linear regression in which the approximate function is a linear function with unknown coefficients subject to evaluation (it is possible to build non-linear functions by using a proper transform). The error measurement is written in the form of a quadratic function, after the variance function likeness. However, different kinds of approximation may be encountered in technology, e.g. in the Fourier series, Chebyshev, Lagrange and Legendre polynomials.

S-type function courses are essential for the EVM applications. It is crucial to choose the best values of the *a*, *b* and *c* parameters. Various possibilities of approximation, which lead to various extrapolation courses, have been presented in [1, 4, 6, 13, 15, 16, 24]. It is therefore crucial to estimate the criteria of the extrapolation, which is also connected with the criteria used in the management of a given project.

S-type function courses, discussed in the previous section, are essential for the EVM applications. It is crucial to choose the best values of the a, b and c parameters. Various possibilities of approximation, which lead to various extrapolation courses, have been presented in an approach determined in Fig. 5. Comments "better", "poor" and "average" describe the quality of approximation with relation to the PV curve. It is therefore crucial to estimate the criteria of the extrapolation, which are also connected with the criteria used in management of a given project.

It is also possible to use the calculus of the probability approach for the notion of the AC(t) curve approximation, starting from the actual time (AT) instant, in a way similar as in the case of creating the $PV_i(t)$ curve family. In such an approach, the ellipse of project completions is created as well – Fig. 6. A series of simulations with the Monte Carlo method should be used for determining the ellipse in the most convenient way.

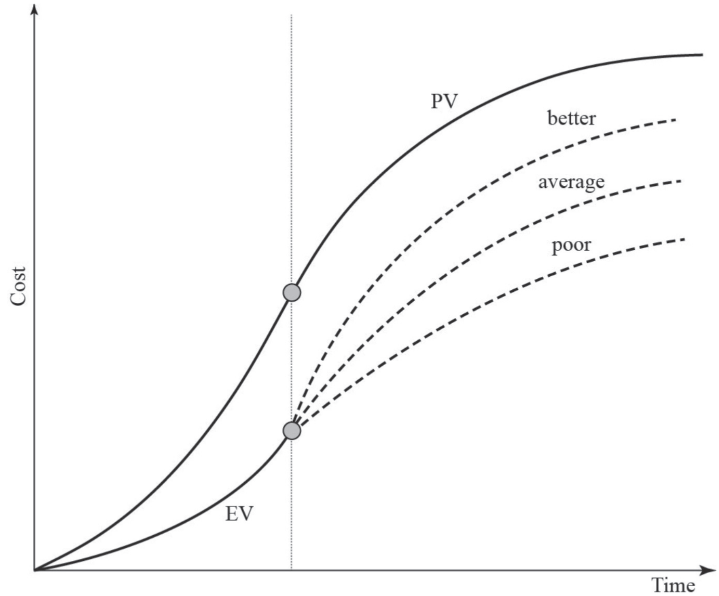

Fig. 5. Examples of various approximations of the EV(t) function course in the determined approach [3]

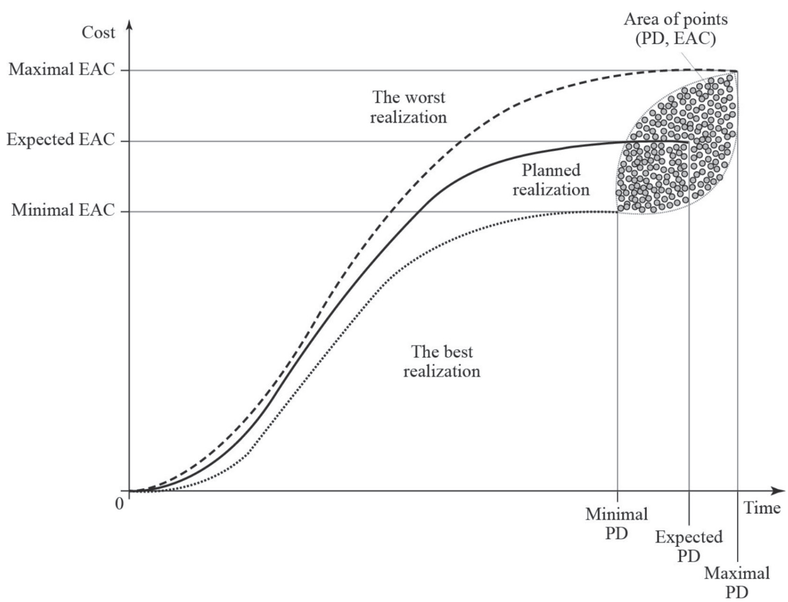

Fig. 6. The calculus of probability presentation of the AC(t) curve approximation [9]

4. An exemplary analyses

4.1. General remarks

During the project implementation in the EVM method, one has at the disposal the PV(t) function of the planned value and the AC(t) curves of the actual cost of the performed activities, as well as the EV(t) of the earned value, determined only up to the actual time (AT), $t \in [0,\text{AT}]$ instant. As has been mentioned above, usually the functions are of a character similar to the *S*-type function. It is therefore natural that in order to predict it, the function approximation method with the choice of approximate function class is used, taking into consideration all the above-mentioned limits to the approach. The values of some of the parameters may, or even should, at least when considering the range of their values, be selected with the PV(t) function course taken into account. Next, one should subjectively choose the criteria of the best approximation. For the polynomial type of approximation, the leas square method is commonly used to find the best approximation (to minimize the error of approximation).

4.2. Real investment project

In order to present the application of the estimation method, the example of a completed building and refurbishment project, described by S. Wawak [22], was taken into consideration. The AC(t) curve is defined for $t \in [0, 101]$ and is presented in the Fig. 7, where time is scaled in weeks. Looking at the curve as a whole, it has a form of the *S*-curve, but it is not the model course of the *S*-curve, especially in the final part of realization of the project.

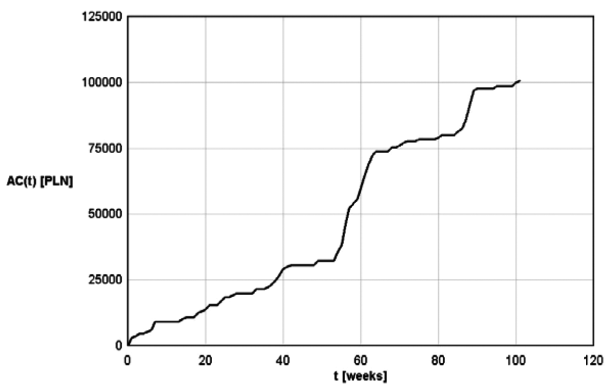

Fig. 7. The example course of the actual cost curve for the real investment project [22]

The on-line updating of the AC(t) curve is tested by taking into account the only data to chosen time value t_n. More detailed $t \in [0, t_n]$, $n \in [0,100]$. The method is applied to extrapolation of the function after its inflection point, hence $t_p = 62$, $n > 62$ and formula (14) is used for the least square method. The parameter chosen for validation of the method is the final cost of the project t_{101}. The realistic value is equal to t_{101} = 100 769. In Table 1, there are compared values of final cost obtained by the on-line type of estimation and the percentage relative error of estimation. As an example the approximation function obtained for n = 100 has form (15).

$$Y_{100}(t) = -\frac{1324249618837}{10921580194370}t^3 + \frac{123155214551841}{5460790097185}t^2 - \frac{1324249618837}{10921580194370}t \quad (14)$$

Table 1. Relative error of on-line estimation of the final cost

n	**Final cost [tys. PLN]**	**Relative error [%]**
Exact	100.769	–
70	109.547	8.7
80	108.668	7.8
90	105.941	5.1
100	101.641	0.9

4.3. *S*-curves analysed by A. Czarniogowska

A. Czarniogowska analysed the two project models, which are purely hypothetical, but were used to illustrate the discussed method in the article [6]. The actual cost functions have the form of *S*-curves and are defined in the period $t \in [0, 12]$. The on-line approximated functions obtained by the last mean squares method for t = 7 for both cases are shown in Fig. 8 together with the original functions. The assumed infection point is $t = 6$ for both cases. The analytical formula for the approximation formula for the first case has the form (16) and is shown in Fig. 8 (top). The analytical formula for the approximation formula for the second case has the form (17) and is shown in Fig. 8 (bottom).

$$Y_7(t) = -1.174301691t^3 + 21.13743044t^2 - 1.174301691t \quad (15)$$

$$Y_7(t) = -\frac{759759}{352652}t^3 + \frac{11396385}{352652}t^2 - \frac{759759}{352652}t \quad (16)$$

The approximation function for the first case is almost the same as the analysed function (see Fig. 7 – left). For the second case, due to the position of the inflection point, the obtained approximate *S*-curve starts to go down before the end of the analysed range (planed duration time – PD). It can be observed when the approximation formula has the form of polynomial of the third order. In a practical application, it means that estimated time duration (ED) is shorter that planned time duration (PD). The possibility of such experience in practical realization must be verified in comparison with other parameters of project monitoring, or the other method of estimation of ED should be applied for comparison.

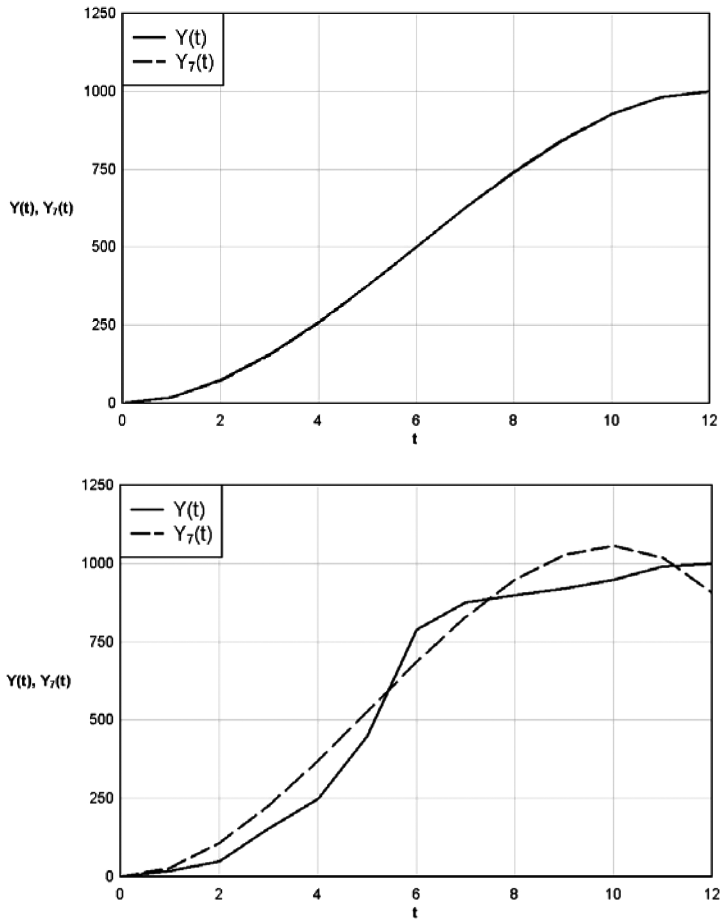

Fig. 8. The actual costs curves analysed by A.Czarniogowska [6] and their approximations

4.4. *S*-curve analysed by J. Dolezal

J. Dolezal [7] analysed the project described by the planned value function PV(t) (see Fig. 8). Its realization is monitored by the actual cost function AC(t) defined up to time equal to 6 weeks (see Fig. 8). The actual cost function is extrapolated by polynomial function of the third order by the least square method assuming the inflection point for $t = 9$ week, as it is observed for the PV(t) curve (see Fig. 9). The approximation function is defined by formula (18), and is shown in Fig. 9.

$$AC_9(t) = -\frac{2747}{102444}t^3 + \frac{24723}{34148}t^2 + \frac{846584}{179277}t \tag{17}$$

The planned final cost of the project is equal to PV(15) = 151, and estimated by the up-dated actual cost curve is equal to $AC_9(15) = 143$. The relative error of estimation of this value is equal to 5.3%.

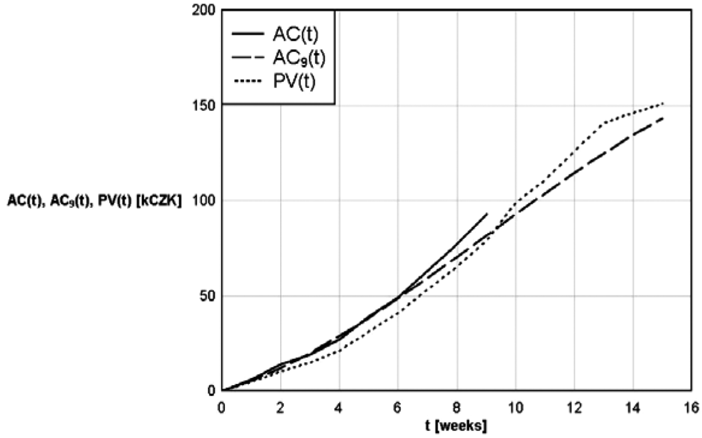

Fig. 9. The actual cost function approximation for the case analysed by J. Dolezal [7]

5. Conclusions

The proposed method of the on-line approximation of the actual cost curve shape is a flexible method with a great range of application possibilities. The essence of the presented approximation is connected with the applied family of the approximation function. It is polynomial of the third order. It makes the approximation much more convenient than the exponential type functions commonly used in literature.

The examples used in the presented article for verification of the method are taken from literature and are chosen to show the following problems of approximation:

- Irregular (non-ideal) form of the AC(t) function after S. Wawak [22],
- AC(t) of ideal form and shape with translated in time domain inflection point after A. Czarniogowska [6],
- AC(t) function defined before it reached its inflection point after J. Dolezal [7].

The analysed examples show the possibility of applying the polynomial of the third order function for the approximation of the *S*-type function. For a proper approximation, the inflection point must be defined. The obtained values of estimated time duration (ED) or estimate cost at completion (EAC) for some extraordinary cases, e.g. when the obtained approximate *S*-curve starts to go down before the end of the planed time duration (PD), must be verified in comparison with other parameters of project monitoring, or the other method of estimation of ED should be applied for comparison.

The presented method may be used as yet another assisting tool for the project manager, whose aim is to facilitate the estimation of the real project ED time or the real project EAC cost.

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