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Jan Wrona (jwrona@pk.edu.pl)
Faculty of Environmental Engineering, Cracow University of Technology
Marek Prymon
Thessla Green sp. z o.o.

A prototype of a small Stirling refrigeration unit

Prototyp małego urządzenia chłodniczego realiuzującego obieg Stirlinga

Abstract

This paper presents original mathematical models which can be used for the size optimization of particular elements in the design process of cooling appliances using the Stirling cycle. The models were used to design a prototype of the Stirling cooling device. The project employs a unique piston–cylinder kinematic pair which enables dry fiction work. Original platelet and ball ceramic regenerators were designed. The presented model assumes adiabatic transformations of the medium in the cylinders as this approach yields more realistic results in comparison to a simple isothermal Schmidt analysis. One cycle of the device (one rotation of the shaft) is divided into elementary angles Φ, where the state of the gas is considered as constant. As a result, states of the gas in individual components of the working space are determined in any given, discrete time steps of the Stirling cycle.

Keywords: Stirling cycle, Stirling engine, Stirling cooler, Stirling cycle numerical modelling, optimisation of Stirling engine, cogeneration, heat transfer

Streszczenie

Artykuł przedstawia oryginalny, opracowany przez autorów model matematyczny, który może być użyty do projektowania i optymalizacji elementów urządzeń pracujących w obiegu Stirlinga. Model został użyty do zaprojektowania prototypu chłodziarki Stirlinga. Prototyp zawiera unikalne rozwiązanie węzła kinematycznego tłok–cylinder umożliwiające pracę w warunkach tarcia technicznie suchego oraz prototypy wymiennych, opracowanych przez autorów regeneratorów ceramicznych: kulkowego oraz płytkowego. Model zakłada adiabatyczne przemiany czynnika w cylindrach, które to podejście daje bardziej realistyczne wyniki w porównaniu do izotermicznej analizy Schmidta. Cykl pracy urządzenia (jeden obrót wału) podzielono na elementarne kąty Φ, w których stan gazu rozpatrywany jest jako ustalony. W efekcie otrzymano stany gazu w poszczególnych objętościach składowych przestrzeni roboczej w danych, dyskretnych chwilach cyklu Stirlinga.

Słowa kluczowe: Obieg Stirlinga, Silnik Stirlinga, Chłodziarka Stirlinga, Modelowanie numeryczne obiegu Stirlinga, Optymalizacja obiegu Stirlinga, Wymiana ciepła

Nomenclature

c_p	specific heat at constant pressure, J/(kgK)
c_v	specific heat at constant volume, J/(kgK)
M	total mass of gas in the machine, kg
m_C	mass of gas in warm cylinder, kg
m_E	mass of gas in cold cylinder, kg
m_{HC}	mass of gas in warm exchanger, kg
m_{HE}	mass of gas in cold exchanger, kg
m_R	mass of gas in regenerator exchanger, kg
p	pressure, Pa
Q	heat flux, W
R	gas constant, kJ/(kgK)
T_C	temperature in warm cylinder, °C
T_{HC}	temperature in heat exchanger, °C
T_R	temperature in regenerator, °C
T_{HE}	temperature in cold exchanger, °C
T_E	temperature in cold cylinder, °C
W	work of thermal cycle, J
$V_C = V_C(\Phi)$	volume of hot cylinder, m^3
$V_E = V_E(\Phi)$	volume of cold cylinder, m^3
V_{HC} = const	volume of hot exchanger, m^3
V_{HE} = const	volume of cold exchanger, m^3
V_R = const	volume of regenerator, m^3
Φ	actual instantaneous shaft angle position
$\varkappa$	coefficient of compressibility

1. Stirling cycle

Devices employing the Stirling cycle were initially built as heat engines. The systems engineered and based on the Stirling cycle may be considered as an alternative to the commonly employed internal combustion engines. The main applications of such devices are: industrial external combustion engines, cooling devices, cogeneration [5, 6, 8, 10, 12, 13, 16, 17, 20]. The development of the Stirling engine was not as dynamic as the evolution of the steam engine or the internal combustion engine. The main obstacle in the design were the shortcomings of the materials necessary to build the working unit and the very complex thermodynamic description, which was difficult to define at that time. The theoretical efficiency of the Stirling cycle is equal to the Carnot cycle. A Stirling machine is a device employing a thermodynamic cycle which is described as a group of thermodynamic processes consisting of two isotherms and two isohores.

In the real Stirling device, the enclosed gaseous working medium is continuously translocated within the working space going through cyclical pressure changes. The gas moves in the working space and is subject to thermodynamic transformation; however, it never vacates any of the working spaces (the cylinders and the heat exchangers). The working gaseous medium remains in all working spaces of the device during the cycle. The dead volume of the working spaces on the device must be minimised.

Stirling cycle devices are divided into four groups: alpha, beta, gamma and an additional configuration represented by the so-called thermoacoustic device with the travelling wave. The last of these is used mainly in cryogenics because of the absence of moving elements in the direct vicinity of the heat exchangers, but in comparison to traditional Stirling devices, their operational efficiency is lower [3, 4, 7, 9, 14, 15, 18].

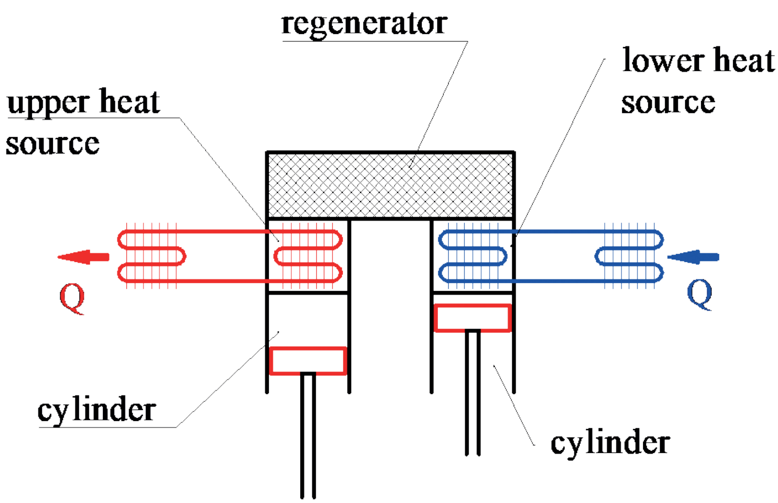


Fig. 1. Diagram of Stirling device in the alpha-configuration

The alpha-configured devices are the best devices (Fig. 1). It is the result of the least number of irreversible processes and the smallest dead volume of all configurations. The disadvantage of this configuration is the necessity to seal both pistons and to adapt the transfer drive with a phase shift.

Currently, the Schmidt analysis is the primary and simpliest tool for the initial size assessment of the Stirling engines, assuming that the cylinders represent isothermal spaces [2, 4, 9, 10, 15, 18, 21].

2. Mathematical models

The purpose of the authors' work was to develop a simplified mathematical model which would allow fast, rough dimensioning of Stirling devices and could be used in the optimisation procedure based on heuristic methods. These methods require multiple model calculations for different values of decision variables in a one-step iterative. The developed model assumes that changes to the thermodynamic working fluid take place in the individual sections of Stirling machines and are treated as separate control volumes. Figure 2 shows the space discretisation of the workspace in the thermodynamic model.

This model assumes an adiabatic transformation of the medium in the cylinders, which yields more realistic results than the isothermal Schmidt analysis.

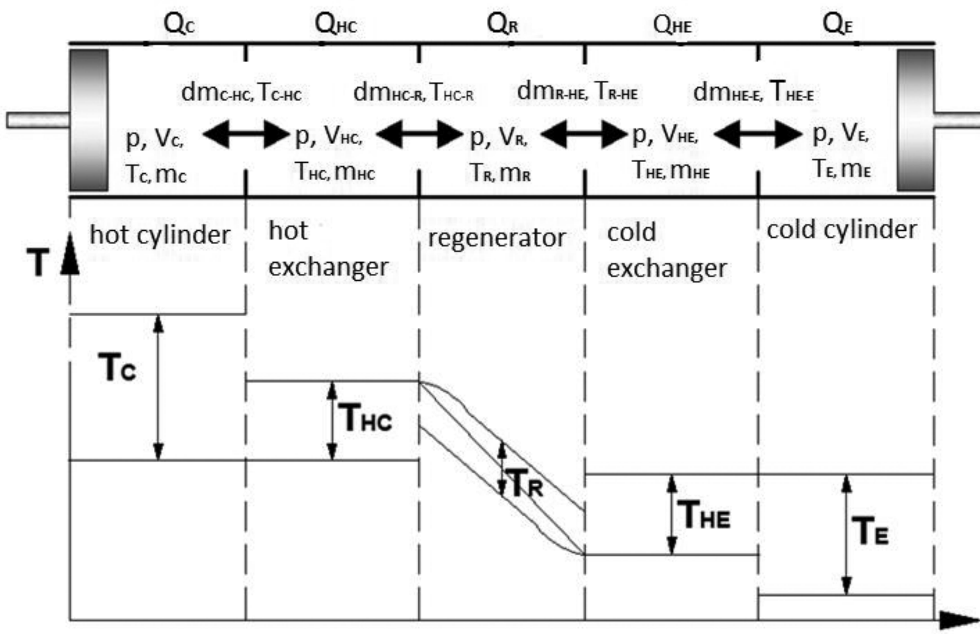


Fig. 2. Diagram of the space discretisation in the thermodynamic model

The correctness of the results obtained from this model were verified by comparing them with the results obtained from the full 3D CFD model. To start the calculation, the adiabatic model needs the initial conditions. Therefore, the first step is the perform of a simple isothermal analysis described by equations (1–5). The simplified adiabatic model is described by equations (6–14).

The cycle operation (one rotation of the shaft) was divided into elementary angles Φ, where the state of the gas is considered as constant. Gas parameters in each section of the device in discrete time steps Stirling cycle (assuming that $\Phi = 2Pi / \text{frequency}$) were obtained. The model assumes irreversibility of the processes in the heat exchangers, including resistance of the flow of gas and heat losses through the regenerator and the housing.

2.1. A simplified numerical adiabatic model

The values of particular parameters were calculated form the formulas as follows:

ISOTHERMAL MODEL

Mass balance

$$M=m_C+m_{HC}+m_R+m_{HE}+m_E \tag{1}$$

The equation of state for each gas volume

$$M=pV/RT, \tag{2}$$

The pressure in the working space as a function of the momentary shaft position

$$p(\Phi)=\left(MR\left(\frac{V_C(\Phi)}{T_C}+\frac{V_{HXC}}{T_{HXC}}+\frac{V_R}{T_R}+\frac{V_{HXE}}{T_{HXE}}+\frac{V_E(\Phi)}{T_E}\right)^{-1}\right) \tag{3}$$

Average gas temperature of the regenerator

$$T_R=\frac{(T_{HC}-T_{HE})}{\ln\left(\frac{T_{HC}}{T_{HE}}\right)} \tag{4}$$

Work cycle calculations were obtained by integration of the formulas

$$W_C=\oint \mathrm{pd}V_C(\varnothing),\ W_E=\oint \mathrm{pd}V_E(\varnothing) \tag{5}$$

$$W=W_C+W_E$$

ADIABATIC MODEL

The first law of thermodynamics for any volume space can be presented by equations 6 and 7

$$đQ+c_p\mathrm{Tdm}=đW+c_v\mathrm{d}(\mathrm{mT}) \tag{6}$$

for the adiabatic cylinder

$$c_p\mathrm{Tdm}=đW+c_v\mathrm{d}(\mathrm{mT}) \tag{7}$$

The law of mass conservation has the form

$$dm_C+dm_{HC}+dm_R+dm_{HE}+dm_E=0 \tag{8}$$

and the equation of state

$$Vdp+pdV=R(Tdm+mdT) \tag{9}$$

$$\frac{dp}{p}+\frac{dV}{V}=\frac{dm}{m}+\frac{dT}{T}$$

For a given moment of time T_{gas} = const

$$\frac{dp}{p}+\frac{dV}{V}=\frac{dm}{m}\rightarrow dm=m\left(\frac{dp}{p}+\frac{dV}{V}\right)\rightarrow dm=\frac{1}{RT}\left(dpV+dVp\right) \tag{10}$$

For individual sections of the device

$$dm_C=\frac{1}{RT_C}\left(pdV_C+\frac{dpV_C}{\varkappa}\right) \tag{11}$$

$$dm_E=\frac{1}{RT_E}\left(pdV_E+\frac{dpV_E}{\varkappa}\right)$$

$$dm_{HC}=\frac{1}{RT_{HC}}dpV_{HC}$$

$$dm_R=\frac{1}{RT_R}dpV_R$$

$$dm_{HE}=\frac{1}{RT_{HE}}dpV_{HE}$$

where: $\varkappa=\frac{c_p}{c_v}$

By substituting (11) and (8) after transformation, we obtain the differential equation for pressure as a function of shaft position

$$dp=\varkappa p\left(\frac{dV_C}{T_C}+\frac{dV_E}{T_E}\right)\left(\frac{V_C}{T_C}+\frac{V_E}{T_E}+\varkappa\left(\frac{V_R}{T_R}+\frac{V_{HC}}{T_{HC}}+\frac{V_{HE}}{T_{HE}}\right)\right)^{-1} \tag{12}$$

For both cylinders we obtain from equation (9)

$$dT_C=T_C\left(\frac{dp}{p}+\frac{dV_C}{V_C}-\frac{dm_C}{m_C}\right) \tag{13}$$

$$dT_E=T_E\left(\frac{dp}{p}+\frac{dV_E}{V_E}-\frac{dm_E}{m_E}\right)$$

Heat flux for: hot and cold exchanger and the regenerator

$$\text{đ}Q_{HC}=\frac{dpV_{HC}c_v}{R}-c_p\left(T_{C-HC}dm_{C-HC}-T_{HC-R}dm_{HC-R}\right) \tag{14}$$

$$\text{đ}Q_{HE}=\frac{dpV_{HE}c_v}{R}-c_p\left(T_{E-HE}dm_{E-HE}-T_{HE-R}dm_{HE-R}\right)$$

$$\text{đ}Q_{R}=\frac{dpV_{R}c_v}{R}-c_p\left(T_{HC-R}dm_{HC-R}-T_{R-HE}dm_{R-HE}\right)$$

With proper time steps of discretisation, the solution obtained from the system of equations allows the determination of the parameters of the device in any given conditions and the size of its elements (exchangers, cylinder diameter, piston stroke, phase shift) [10, 11, 22].

Figure 3 shows the dependence *p-V* as the result of the adiabatic and isothermal analysis (Shmidt) of the prototype Stirling cooler being designed. The red line shows the dependence *p-v* assuming isothermal process in the cylinders. The green line shows the *p-V* dependence assuming an adiabatic gas process in the cylinders where the energy cycle is being maintained by the heat exchangers.

It is noticeable that the energy input required when isothermal change takes place is lesser compared with energy required with adiabatic process in the cylinder.

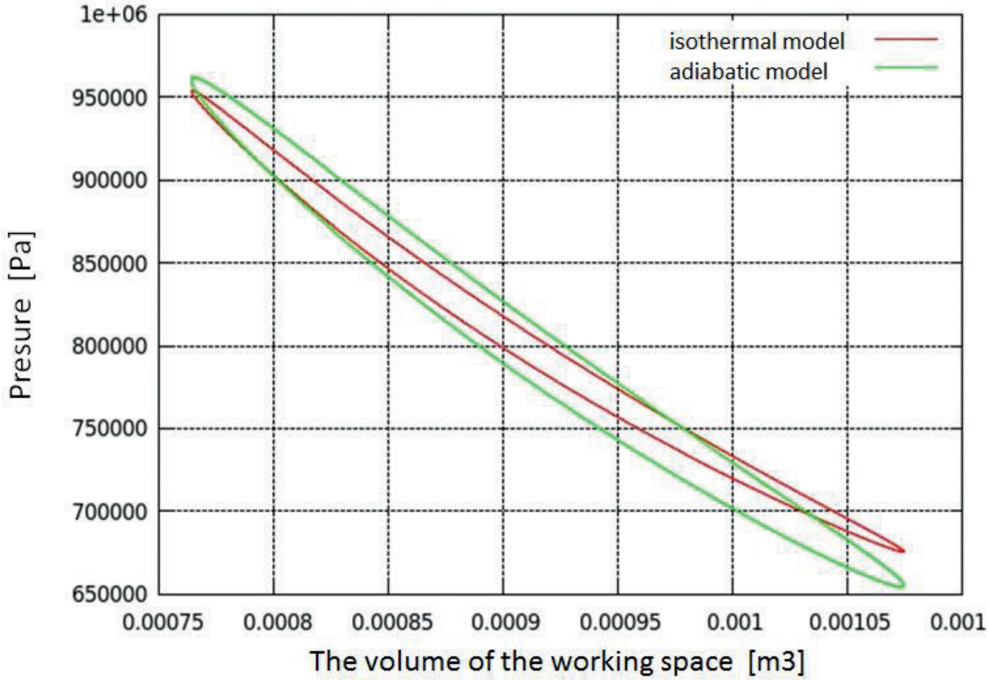


Fig. 3. Diagram *p-v* of the Stirling coolers showing the isothermal and adiabatic conversions of gas cylinders

The above formulated mathematical description does not allow analysis of the impact of the shape factor in the cylinder and the impact of the actual instantaneous piston speed on the work of the engine. It also does not allow effective measurement of the heat exchangers' performance due to their thermic-flow characteristics being in one direction. The working medium in the real Stirling engine has oscillatory characteristics.

The advantage of this kind of simplified adiabatic modelling is the fast calculation rate compared to calculations based on full Novier-Stokes equations.

2.2. CFD model

In order to validate the operation of a simplified adiabatic model, the results of calculations were verified by the full 3D CFD model which was developed by the authors. Mapping of the workspace with the heat exchangers and regenerator was performed. The simulation has taken into account the movement of the pistons in the cylinders (moving mesh). The CFD model has enabled a thorough analysis of the temperature field, speed and pressure of the working medium in the working space of the device. Furthermore, the model allowed the determination of the impact of the instantaneous piston speed on gas flow and heat transfer.

CFD Model assumptions:

- Laminar flow - on the basis of calculations made with the simplified model, instantaneous values of the Reynolds number were determined.
- A semi-structural hybrid network was used.
- The symmetry of the device was used - modelling half of the working space.
- Porous deposit model of regenerator was programmed.
- Cylinder spaces were modelled adopting a 'moving mesh' piston movement simulation.
- Piston movement was simulated with a Ross-Yoke mechanism model using numeric derivatives.

3. Results

3.1. Thermal power and temperature

The adiabatic model was shown to be in agreement with the full 3D CFD model. The global heat flow, gas velocities in different sections, and global and local pressure were taken into consideration. In the results, a phase drift can be observed in momentary functions of heat flux and some differences in temperature values can be seen.

Table 1 shows the comparison of the obtained thermal power of both the adiabatic and CFD models.

Table 1. Comparison of thermal power equipment obtained from the CFD and adiabatic models

	Model CFD	Adiabatic model
cooling power - upper source [W]	-368	-313
cooling power - lower source [W]	220	215
mechanical power – net [W]	218	214

Figure 4 shows the instantaneous temperatures values in the individual sections of the Stirling cooler during the full cycle of operation [10, 11, 22].

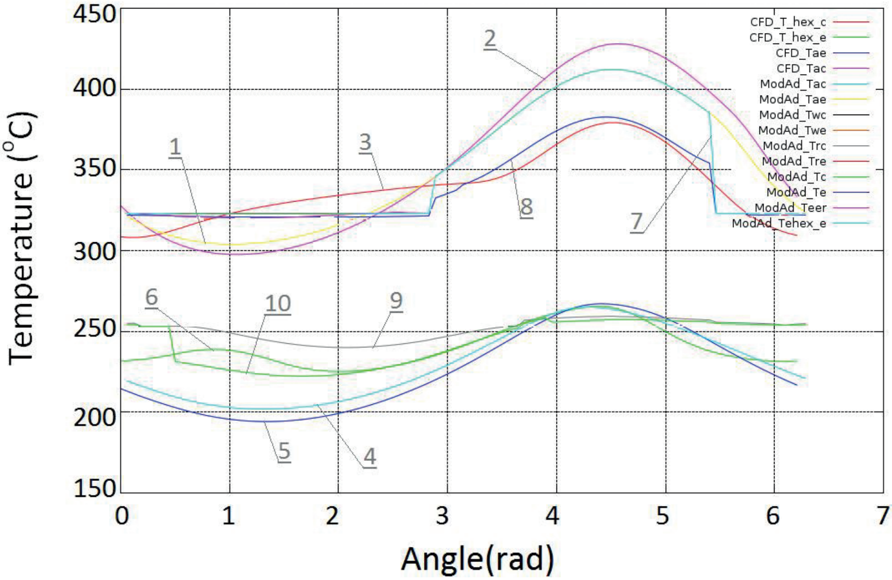


Fig. 4. Comparison of the instantaneous temperature of each section of the working chamber during the cycle operation where: 1 – hot cylinder (adiabatic model), 2 – hot cylinder CFD, 3 – hot exchanger CFD, 4 – cold cylinder (adiabatic model), 5 – cold cylinder CFD, 6 – cold exchanger CFD, 7 – gas boundary (hot exchanger/hot cylinder), 8 – boundary gas (hot exchanger/regenerator), 9 – boundary gas (cold exchanger/cold cylinder), 10 – boundary gas (cold exchanger/regenerator).

3.2. Model of the device

Based on the results obtained from both models, the prototype of the Stirling cycle cooling device was designed (Figs. 5, 6, 7). During the designing process, particular emphasis was put on the future possibility of modifying the device by exchanging the subassemblies. It may be possible to modify the diameter of some subsections, the length of the heat exchanger or the whole regenerator which itself contains an interchangeable head. Particularly noteworthy is the kinematic pair piston-cylinder solution, which enables dry friction work. The gas cycle in Stirling engines is closed – no gas exchange occurs; therefore, the device must be kept hermetic.

The devices where the piston-cylinder kinematic pair demand an oil lubrication, contact between the gas and the lubricant causes the translocation of the latter into the working space. This is the reason why cylinder oil lubrication is either troublesome or completely impossible in the hermetic Stirling devices. The most critical problem is the possibility of the lubricating oil entering the regenerator ducts.

On the basis of many years of research and experience related to the construction of Stirling machines, it can be stated that the best materials for the regenerator cartridges are [2, 3, 7, 8, 14, 23]:

- ceramic or metal balls,
- ceramic material in the form of thin tiles or foam,
- metal wire strands,
- metal wool,
- spongy metal,
- corrugated metal wires placed inside straight metal pipes,
- wire mesh,
- metal rectilinear tubes.

Fig. 5. View of the model of Stirling cooler elements with the Ross-Yoke kinetic mechanism and heat exchangers

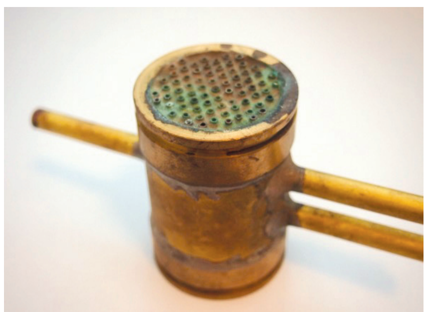


Fig. 6. One of the heat exchangers

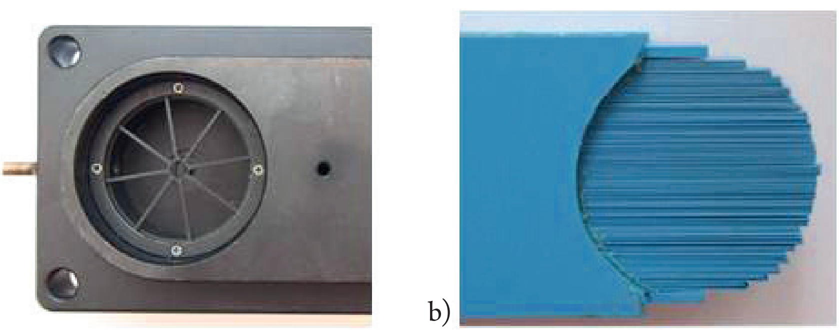
a)  b)

Fig. 7. Prototypes of the ball (A) and platelet (B) regenerators of the refrigerator

To maintain high energy output of the regenerator, the ducts must be of a very small hydraulic radius and the oil-free pistons need to work in dry friction working conditions. Figure 7 presents a prototype of the ball (A) and plate (B) regenerator (designed by the authors) which was made of ceramic materials.

This kind of design forces the adaptation of entirely different materials for the cylinder bearing surface or piston ring to those which are used for traditionally built compressors [1, 11, 19, 20, 21]. It is suggested that the cylinder sleeves must be built with an aluminium based alloy and the piston rings with a polytetrafluoroethylene composite. Tests [1, 19, 20] have shown that the tribological properties of such association promise preferable sliding cooperation in comparison to rings made from tarflen-graphite where the cylinder bearing surface is made of a chrome-plated steel alloy.

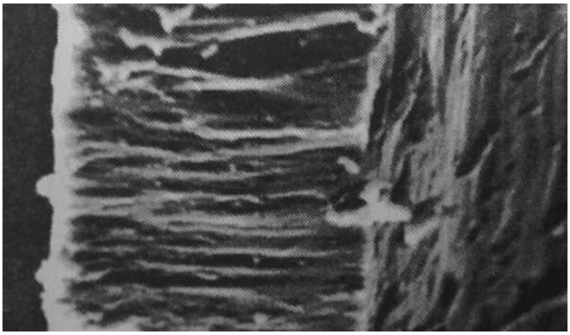


Fig. 8. Cross section of the oxide layer (magnification x 300) [19]

The oxide layer of the cylinder bearing surface has a tubular fibrous structure. The miniscule tubes are set perpendicular to the cylinder wall (Fig. 8), their pores are filled with a solid state like lubricant and additionally decrease friction, thus diminishing the abrasive wear of the oxide layer and the piston rings.

A squirrel cage motor with permanent magnets will be used for the drive of the prototype. The motor and kinetic mechanism of the device are oversized in order to account for the possibility of expansion and testing with a wide range of loads, rotational speeds and pressures.

4. Conclusions

The paper presents numerical models developed by the authors that can be used to design devices performing a Stirling cycle. Based on the results of the numerical models, the authors designed the experimental Stirling cooler.

The first model is a simplified numerical model with time discretisation based on the ideal adiabatic analysis. The results from this model have been compared with the results of the other model – the 3D CFD model in which the authors mapped the entire working space including the heat exchangers and the regenerator. In the CFD model, the dynamic mesh used allows for the simulation of piston movement in the cylinders.

The authors have achieved the compliance of adiabatic model with the full 3D CFD model based on the full system of Navier-Stokes equations regarding global heat flux and some temperature values differences, and global and local pressure. In the results, a phase drift can be observed in momentary functions of heat flux and some differences in temperature values can be seen.

Both model calculations compatibility confirm that a developed numerical simplified adiabatic model with time discretisation, due to the very short calculation time, may find future applications in the design and optimisation of *Stirling* devices.

The research on the Stirling cooler will be used to verify the results obtained from numerical models and applied innovative design solutions. The experimental confirmation of the numerical results will become the basis for creating tools in the form of computer programs for the design of Stirling devices.

The prototype of the Stirling cycle cooling device has a modular design with possible modification by exchanging the sub-assemblies (exchangers, cylinders, regenerator). The regenerator itself contains an interchangeable head for easy replacement by either the plate or the ball regenerator. To maintain high energy output of the regenerator, its ducts must be of a very small hydraulic radius.

The unique kinematic pair piston-cylinder employed enables dry friction work. Oil lubrication of the piston-cylinder kinematic pair could easily plug the regenerator ducts.

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