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MARCIN JONAK*

A DIGITAL METHOD FOR THE GENERATION OF ANAMORPHIC IMAGES – VISUALISED IN CONICAL REFLECTIVE SURFACES

CYFROWA METODA GENEROWANIA ANAMORFICZNYCH OBRAZÓW RESTYTUOWANYCH ZA POMOCĄ STOŻKÓW REFLEKSYJNYCH

Abstract

In this paper, the author presents a practical method for the construction of anamorphic images based on their analytical properties that are characteristic of reflective conical anamorphic images. This study is a continuation of the earlier work in which the geometrical principles from a real-life image were described. In this paper, the author presents a set of analytical formulas that are used to describe the transformation together with their digital notation in the MS Excel software. The originally developed analytical model of transformation lets the author generate anamorphic images of any designed objects on the condition that they are represented by the parametric equations. These parametric formulas make the essential condition for enhancing the creation of the final form of an anamorphic image. Some exemplary anamorphic images have been presented here together with their visualisation that has been created with the aid of a prototype, conical-shaped mirror. The solutions presented in this work provide a tool for the precise design and development of anamorphic images within the urban space of a town and in the interiors of a public use.

Keywords: transformation, anamorphic image, visualisation of anamorphic images, reflective cone

Streszczenie

W niniejszym opracowaniu przedstawiono praktyczną metodę konstruowania obrazów anamorficznych na podstawie własności analitycznych dla anamorf refleksyjno-stożkowych. Niniejsza praca jest kontynuacją zagadnień [6] związanych z określeniem geometrycznych zasad powstawania obrazów anamorficznych na bazie obrazu rzeczywistego, natomiast prezentuje ona analityczne przekształcenie oraz cyfrowy zapis takich obrazów z wykorzystaniem modułu graficznego programu MS Excel. Tak więc opracowany model analityczny pozwala generować obrazy anamorficznie dowolnych projektowanych obiektów zapisanych w formie równań parametrycznych, wspomagających opracowanie ostatecznych anamorf. Przykładowe anamorfy zaprezentowano wraz z ich obrazami restytuowanymi za pomocą prototypowego zwierciadła stożkowego. Powyższe rozwiązania dają możliwość precyzyjnego projektowania obrazów anamorficznych w zurbanizowanej przestrzeni miejskiej oraz architektonicznych wnętrzach przestrzeni publicznej.

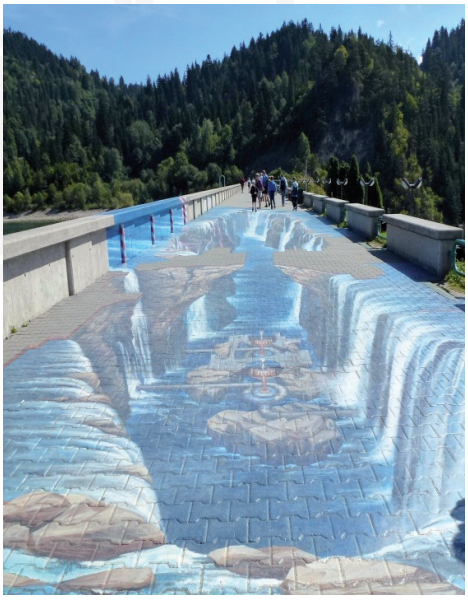
Słowa kluczowe: przekształcenie, restytucja, anamorfa, stożek refleksyjny

* Ph.D. Eng. Marcin Jonak, Division of Descriptive Geometry, Technical Drawing & Engineering Graphics, Faculty of Architecture, Cracow University of Technology.

1. Introduction

Perspective anamorphic images can be perceived as bigeneric illusions created in a process of image perception. These illusions are hidden in a double content of anamorphic images At first glance of an anamorphic image, we simply recognise the aesthetics of its original beauty that is exactly ruled by the geometric principles set up for their creation. These rules form fascinating deformations of drawn compositions. The other content of a drawing can be read by looking at it from a particular direction either directly or by looking at its reflection in a specific mirrored surface. Today, anamorphic images survive returns and revivals. They appear in a public space as planar, multi-coloured and surprising compositions that can be viewed from a specifically chosen point (Ill. 1, 2). For many years, numerous and consistentarticles have been published [1–8] in which the authors have analysed the geometry of anamorphic transformations, their creation and visualisation. The earlier studies define the complex rules governing the geometric transformation of real images into anamorphic images along with the methods used to record these transformations. They also provide the results of the analysis of the metric parameters that shape the anamorphic images constructed with the aid of a conical mirror. These publications also complement the basic publications with the simplified methods which can be used for the creation of more complex anamorphic images where complicated geometric deformations take place. These studies may prove to provide useful suggestions to focus architects' and artists' interest on this forgotten form of art.

Ill. 1. Planar anamorphic image in a closed municipal space: Main Market Square in Wieliczka; Author: Ryszard Paprocki



Ill. 2. Planar anamorphic image on a Czorsztyn dam: Author: Ryszard Paprocki

2. Ddescription of an anamorphic transformation

2.1. Elements of a transformation

The elements of a transformation contain (Ill. 3):
- an anamorphic picture plane which is a horizontal plane united with the base of a cone;
- a reflective cone of revolution with a base and height of specific dimensions; the axis of the cone is perpendicular to the anamorphic picture plane;
- a direction of projection that is parallel to the cone axis.

2.2. Principle of transformation

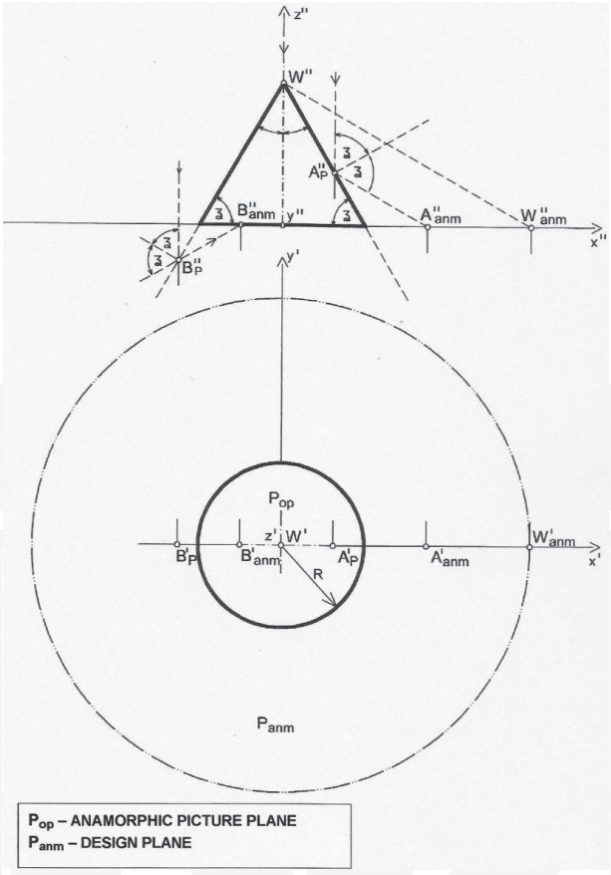
Even though both in literature [11] and in practice, the theory of anamorphic transformation is commonly discussed in terms of a central projection (theory of visual perception to be a perspective view), a simplification of the method by applying a parallel projection principle has been used in this particular piece of research. Thus, the centre of projection has been assumed to lie at infinity and to coincide with the infinite point on the axis of the reflective cone.

The assumed direction of transformation defines the **active part** P_{anm} of the cone surface in terms of its visibility from the perspective of the observer. This active part of the cone is limited with two rings of the diameters: R and (W', W'_{anm}). On the cone, we can also distinguish a **passive part** P_{op}. The field of a picture plane is similarly divided into two parts: the **passive part**, used for a design and the **active part,** upon which there will be created an anamorphic image of a designed object (Ill. 3).

Figure 3 presents a schematic diagram of the principle for the creation of an anamorphic image A_{anm} of any point A_p (taking into account the conditions described above). The basis for the creation of any anamorphic image is the well-known optical law of two angles equality: the angle of incidence equals the angle of reflection in geometric optics (angle ζ in Ill. 3). A randomly chosen point A_p is transferred along a direction described earlier onto the cone surface to receive point A_w as a mirroring point of its viewing ray. The point of intersection A_w is a point of incidence of a viewing ray that passes through A_p and lies on the active part of the cone. A normal to the cone surface is drawn at point A_w. According to the geometric optics, we can determine the angle ζ between the incident ray and a normal n and then construct the reflected ray making the same angle with n. A_{anm} will be an anamorphic image of the designed point A_p.

2.3. Analytical issues of transformation

The digital method has been elaborated with the use of some analytical equations which describe the relationships existing between a designed image and its anamorphic image. This whole process has been realised in the MS Excel software.



Ill. 3. Projecting apparatus of the discussed anamorphic transformation represented in two orthographic views

The set of analytic equations has been developed based on well-known theorem of trigonometry. Analysis of the geometric transformation that is presented in Ill. 3 lets us determine the relationships between two points: a designed point $A'_P(x_p, y_p)$ and an anamorphic point $A'_{anm}(x_{anm}, y_{anm})$ relative to it. These relationships can be notated with the following parametric formulas:

$$x_{anm} = (t_1 + r_p)*\cos(\varphi) \tag{1}$$

$$y_{anm} = (t_1 + r_p)*\sin(\varphi) \tag{2}$$

while:

$$r_p = \sqrt{x_p^2 + y_p^2}$$

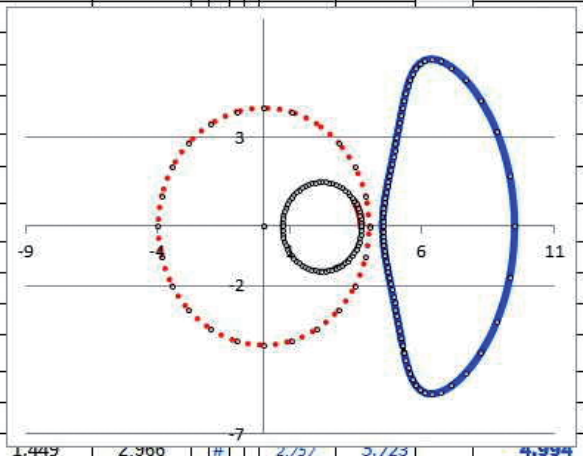
$$t_1 = \frac{\left(R - r_p\right)\mathrm{tg}\left(\zeta\right)}{2{*}\mathrm{tg}\left(2\zeta - \frac{\pi}{2}\right)}$$

$$\varphi = \arcsin\frac{x_p}{r_p}$$

where:

x_{anm} – abscissa of the anamorphic image of a designed point;
y_{anm} – ordinate of the anamorphic image of a designed point;
R – radius of the cone of revolution;
x_p – abscissa of a designed point;
y_p – ordinate of a designed point;
$\zeta = \arcsin\frac{y_p}{R}$.

	xAp	yAp	rp			t1	t1+rp		xAanm	yAanm
1	3,700	0,000	3,700	#		0,800	4,500		4,500	0,000
2	3,694	0,131	3,697	#		0,809	4,506		4,503	0,159
3	3,677									0,320
4	3,649									0,481
5	3,610									0,646
6	3,559									0,814
7	3,499									0,986
8	3,429									1,162
9	3,349									1,344
10	3,261									1,532
11	3,164									1,726
12	3,060									1,926
13	2,950									2,134
14	2,834									2,348
15	2,713									2,568
16	2,588	1,449	2,966	#		2,757	5,723		4,994	2,796
17	2,460	1,477	2,870	#		3,014	5,884		5,044	3,028
18	2,331	1,494	2,769	#		3,284	6,052		5,095	3,267



Ill. 4. A picture of MS Excel page with a semi-circle (design project) and its anamorphic image calculation

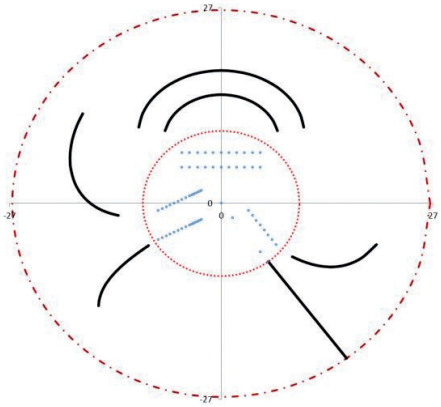
In this paper, the author presents individually elaborated method in the MS Excel [7] application. With the use of the application, it is possible to create an anamorphic image of a designed object. A default function of MS Excel, which creates what is called a 'dotted diagram with smoothing option', has been utilised in order to build a software. With the use of the described tool, it is possible to precisely define the contours of an image while visualisation may be executed by adopting various line weights, line types and colours. Ill. 4 presents a part of an Excel file in which a semi-circle of a defined radius has been designed in the passive part of a picture plane. Parametric equations of a semi-circle were inserted into the file. The anamorphic image of some specifically chosen points on the semi-circle were calculated with the application of the presented earlier equations. An anamorphic image of an object was generated with the use of the tool for 'dotted diagram with smoothing option'. Two uniquely specified points in the anamorphic image are the corresponding points A_p i A_{anm} of a transformation (compare to Ill. 3). All the examples provided below have been calculated for the specific parameters of the reflective cone of revolution dimensions (base radius $R = 3$ cm; cone height $h = 6$ cm). Let us add that the elaborated software enables its usage for calculations of any cone fulfilling the condition $h > r$, where h is the height and r is the base radius, as otherwise, the reflected ray does not pierce the active part P_{anm} of a picture plane.

In the reflection, when we look from the top onto a cone of revolution, we spot an image of a semi-circle respectively positioned on the right circular cone.

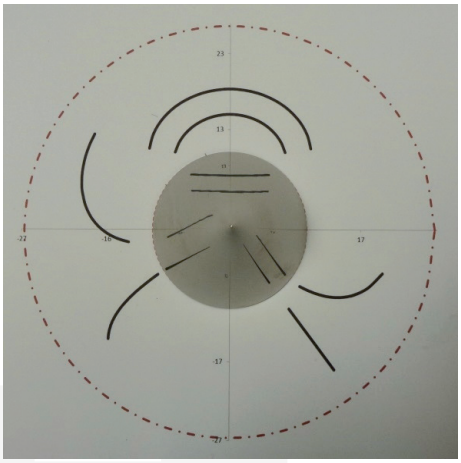
3. Examples

A designed image consists of respectively positioned parts of geometric figures that are notated with the aid of parametric equations. Any designed image is a composition of specifically designed segments of lines, circles or any planar curves which are described with aid of parametric equations. In the following examples, it has been assumed that we will obtain particular real-life images, while in some cases, the anamorphic images were enhanced with the aid of such values as colour or a graphical value.

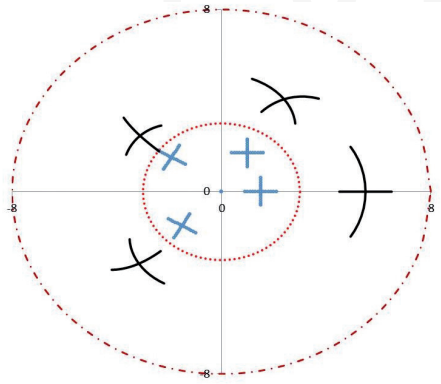
- Ill. 5 & 5a – pairs of parallel segments in various positions in the design field;
- Ill. 6 & 6a – pairs of mutually perpendicular and intersecting segments in various positions in the design field;
- Ill. 7 & 7a – squares in various positions in the design field;
- Ill. 8 & 8a – composition made of squares and triangles;
- Ill. 9 & 9a – circles and a single semi-circle in various positions in the design field;
- Ill. 10 & 10a – ellipses in various positions in the design field;
- Ill. 11 & 11a – parabolas in various positions in the design field;
- Ill. 12 & 12a – geometrical composition – a square, ellipses and arcs;
- Ill. 13 & 13a – geometrical composition – a square and some arcs inscribed into a circle;
- Ill. 14 & 14a – geometrical composition – a square and some arcs;
- Ill. 15 & 15a – geometrical composition – parabola arcs, circles and some straight segments;



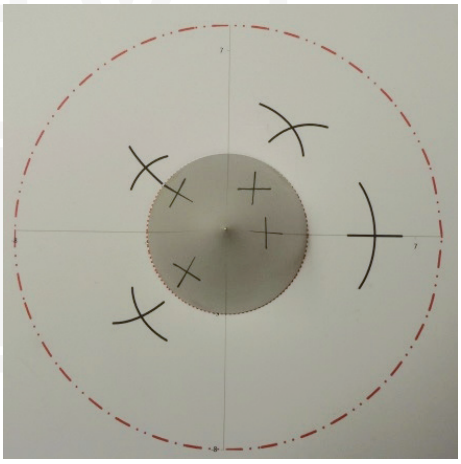
Ill. 5. Pairs of mutually parallel segments randomly positioned in a passive part of a picture plane and their anamorphic image in the active part



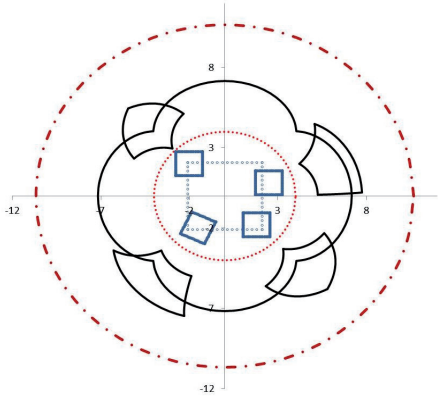
Ill. 5a. Visualisation of an anamorphic image of mutually parallel segments as a reflection obtained in the reflective cone of revolution



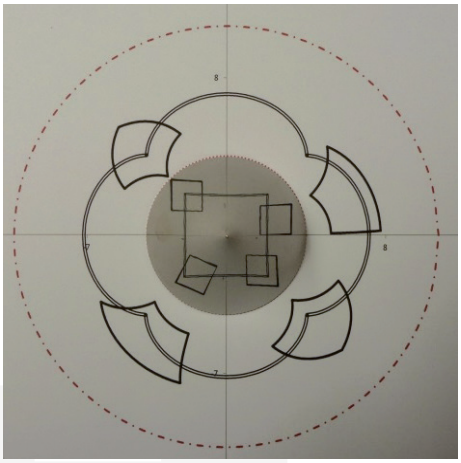
Ill. 6. Pairs of mutually perpendicular segments positioned in a passive part of a picture plane and their anamorphic image in the active part



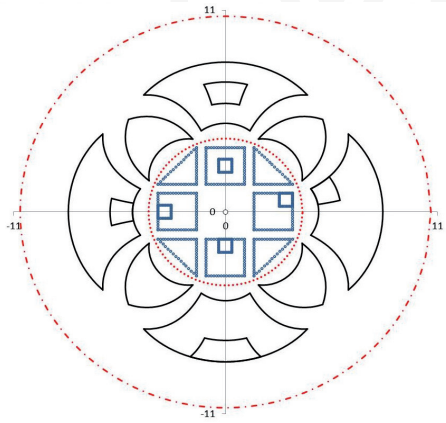
Ill. 6a. Visualisation of an anamorphic image of mutually perpendicular segments as a reflection obtained in the reflective cone of revolution



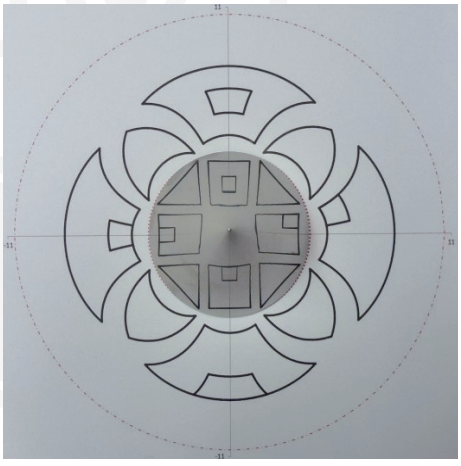
Ill. 7. Squares randomly positioned in a passive part of a picture plane and their anamorphic images in the active part



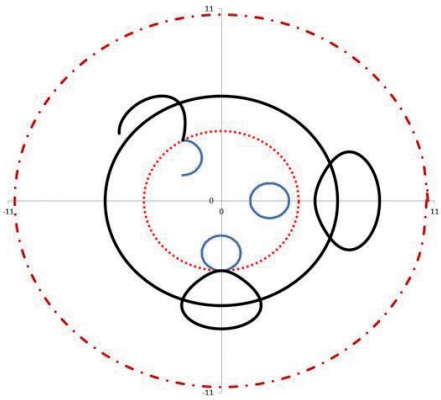
Ill. 7a. Visualisation of an anamorphic image as a reflection obtained in the reflective cone of revolution



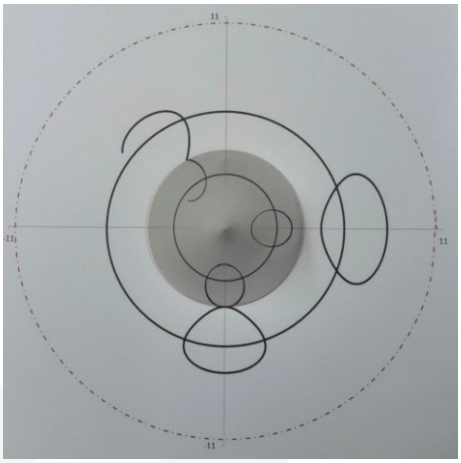
Ill. 8. Squares and triangles randomly positioned in a passive part of a picture plane and their anamorphic images in the active par



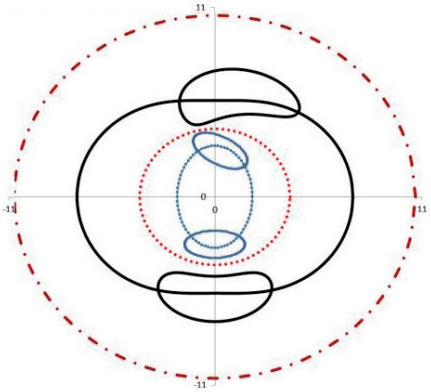
Ill. 8a. Visualisation of an anamorphic image as a reflection obtained in the reflective cone of revolution



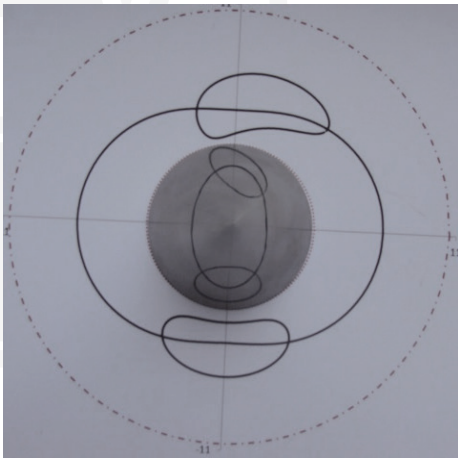
Ill. 9. Circles and semi-circles contained in a passive part of a picture plane and their anamorphic images in the active part



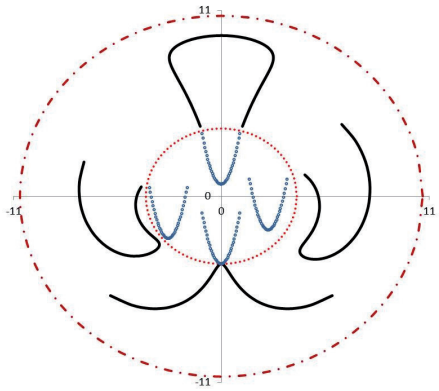
Ill. 9a. Visualisation of an anamorphic image of the circles at random positions – the image received in the reflective cone of revolution



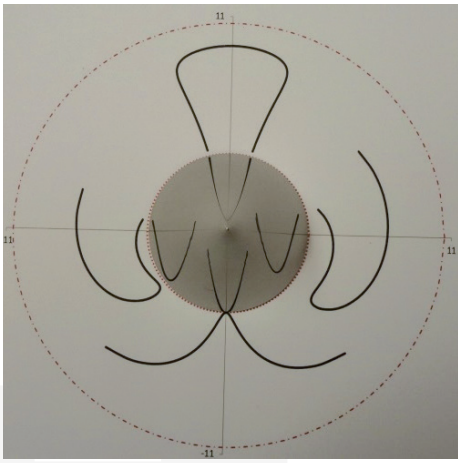
Ill. 10. Ellipses randomly positioned in a passive part of a picture plane and their anamorphic images in the active part



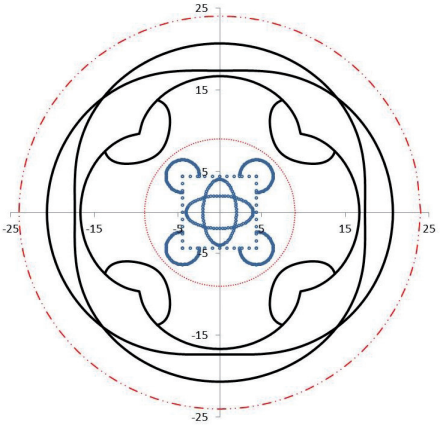
Ill. 10a. Visualisation of a designed composition containing ellipses at random positions within the contours of a reflective cone



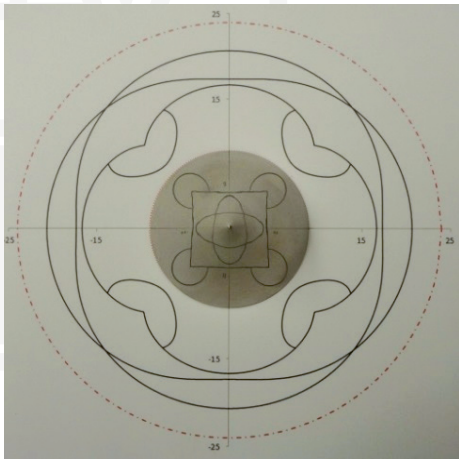
Ill. 11. Parabolas randomly positioned in a passive part of a picture plane and their anamorphic images in the active part



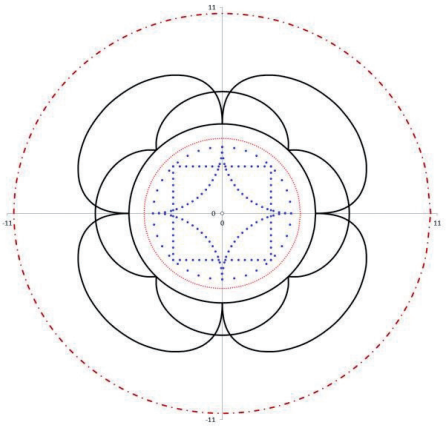
Ill. 11a. Visualisation of a designed composition containing parabolas at random positions within the contours of a reflective cone



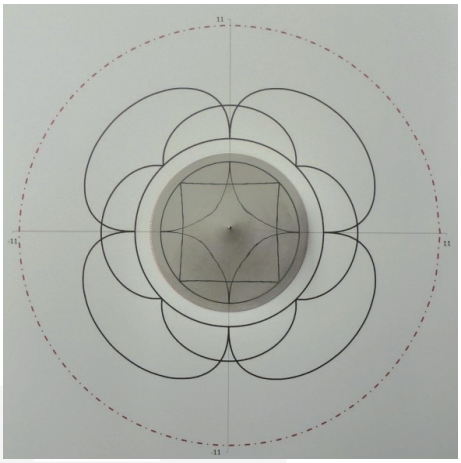
Ill. 12. Geometric composition – a design in a passive part of a picture plane and its anamorphic image in the active part



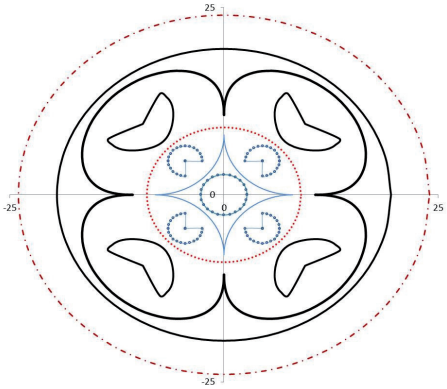
Ill. 12a. Visualisation of an anamorphic image containing a circle and an ellipse inscribed into a rectangle with the use of a reflective cone



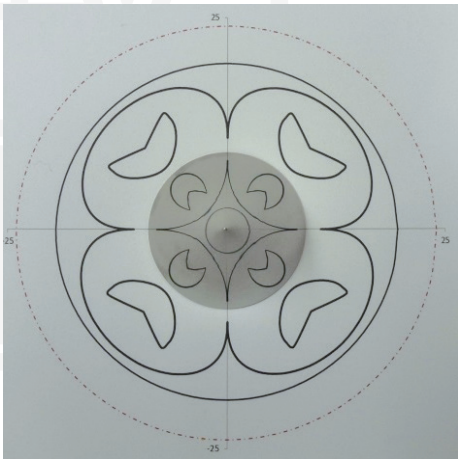
Ill. 13. Geometric composition – a design in a passive part of a picture plane and its anamorphic image in the active part



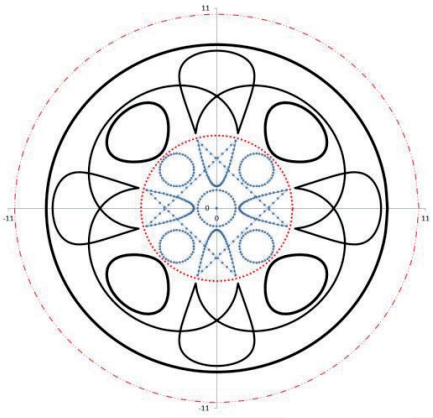
Ill. 13a. Geometric composition – its visualisation within the contours of a reflective cone



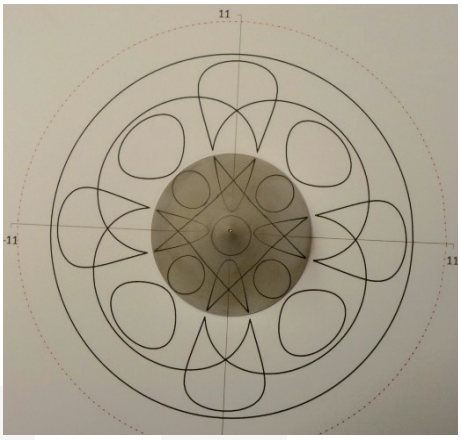
Ill. 14. Geometric composition – a design in a passive part of a picture plane and its anamorphic image in the active part



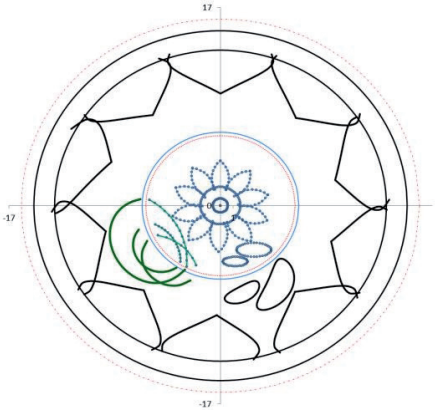
Ill. 14a. Visualisation of a designed composition within the contours of a reflective cone



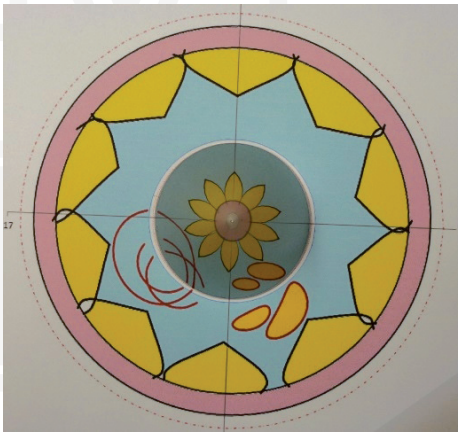
Ill. 15. Geometric composition – a design in a passive part of a picture plane and its anamorphic image in the active part



Ill. 15a. Visualisation of an anamorphic image within the contours of a reflective cone

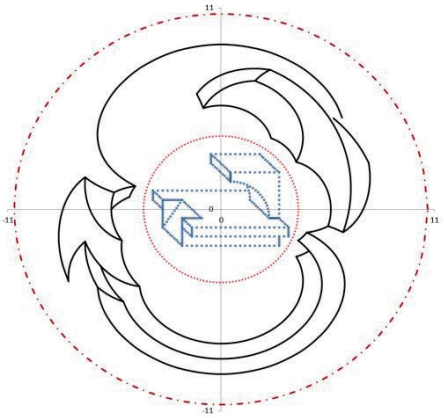


Ill. 16. Geometric composition – a design in a passive part of a picture plane and its anamorphic image in the active part

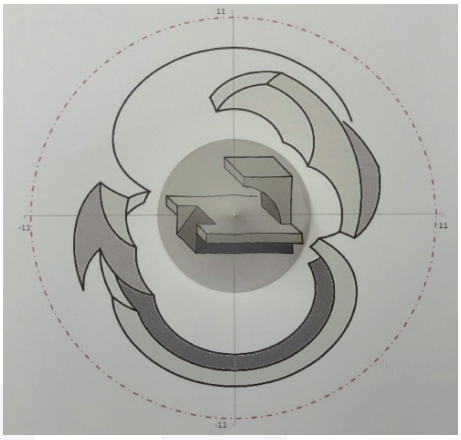


Ill. 16a. Visualisation of an anamorphic image within the contours of a reflective cone enhanced with colours

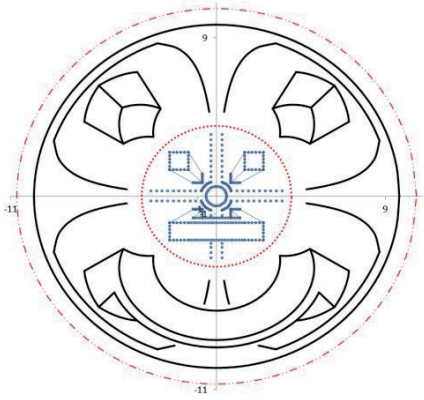
– Ill. 16 & 16a – geometrical composition with a sunflower;
– Ill. 17 & 17a – spatial object – axonometry;
– Ill. 18 & 18a – skyscrapers with a traffic circle – aerial perspective;
– Ill. 19 & 19a – spatial composition (axonometry) – a spatial frame and a cone.



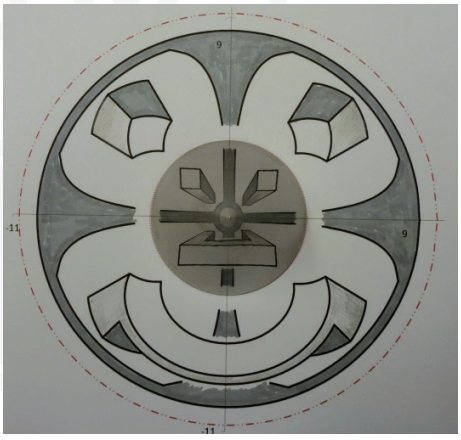
Ill. 17. Spatial object represented in axonometry – a design in a passive part of a picture plane and its anamorphic image in the active part



Ill. 17a. Visualisation of an anamorphic image within the contours of a reflective cone enhanced with a graphical value

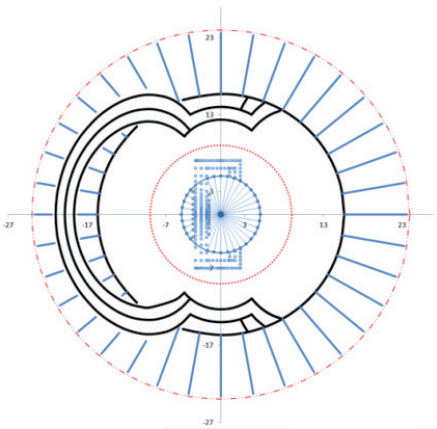


Ill. 18. Skyscrapers at the traffic circle represented in aerial perspective – a design in a passive part of a picture plane and its anamorphic image in the active part

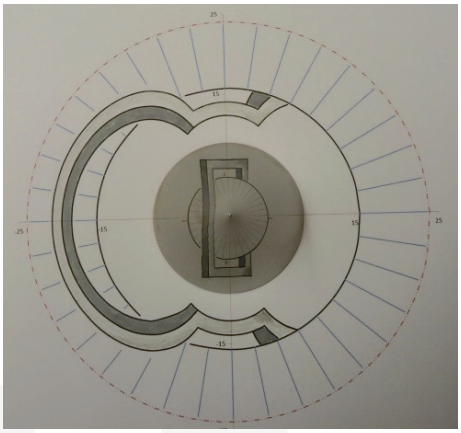


Ill. 18a. Visualisation of an anamorphic image within the contours of a reflective cone enhanced with a graphical value

The illustrations presented below include: a real-life image of each composition that has been composed in the passive part of a picture plane; its anamorphic image contained in the active part of a picture plane; in the additional place a visualisation of a composition that has been created through the use of a reflective cone.



Ill. 19. Spatial composition – cone of revolution and a frame – a design in a passive part of a picture plane and its anamorphic image in the active part



Ill. 19a. Visualisation of a spatial composition within the contours of a reflective cone enhanced with a graphical value

In practice, the generated images with aid of the developed method may serve as a starting point for further graphical elaboration by a designer. The quality of the designed image depends on the quality of the reflective mirror. The fact that we are able to correctly visualise a real-life spatial object and also that we can get rid of deformations that can commonly occur while creating intuitive anamorphic images are key advantages of visualised anamorphic images. Surprise at what is perceived is a common effect for the observer.

Ilustration sources: Marcin Jonak

Symbols

A_p – punkt projektowany
A_{anm} – anamorfa punkt projektowanego
x_{anm} – odcięta punktu anamorficznego
y_{anm} – rzędna punktu anamorficznego
R – promień stożka refleksyjnego
h – wysokość stożka refleksyjnego
x_p – odcięta punktu projektowego
y_p – rzędna punktu projektowego
ζ – kąt odbicia

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