ANNA MILIAN*

SIMPLE CHOOSER OPTIONS WITH MAPLE

OPCJE SIMPLE CHOOSER Z MAPLE

Abstract

This paper discusses Monte Carlo simulations of the Black-Scholes model. It is introduced with the simple example of the pricing of ʻEuropean call options on a no-dividend stock and the simulation results are compared with an analytical solution. Monte-Carlo methods are then used to price simple chooser options. Moreover, it is shown that the distribution of rate of the return from investment in simple chooser options is significantly dependent on the strike price. The presented simulation is performed using MAPLE.

Keywords: simple chooser options, Black-Scholes model, Monte Carlo method

Streszczenie

W artykule rozważa się zastosowanie metody Monte Carlo do modelu Blacka-Scholesa. Wstęp stanowi przykład wyceny europejskiej opcji kupna na akcje bez dywidendy i porównanie wyniku symulacji z rozwiązaniem analitycznym. Metodą Monte Carlo wyceniono opcję *simple chooser*. Pokazano istotną zależność rozkładu prawdopodobieństwa stopy zwrotu z tych opcji od ich ceny wykonania.

Słowa kluczowe: opcje simple chooser, model Blacka-Scholesa, metoda Monte Carlo

* Institute of Mathematics, Faculty of Physics, Mathematics and Computer Science, Cracow University of Technology; amilian@pk.edu.pl.

1. Introduction

An option is a contract between a buyer (holder) and a seller (writer) that gives the buyer the right, but not the obligation, to buy or to sell the underlying asset at an agreed price at a later date. The agreed price in the contract is called the strike price; the date is referred to as the expiration date. There are two basic kinds of options – calls and puts. A call stock option gives the holder the right to buy a specified quantity of stock at the strike price on or before the expiration date. The writer of the call option has the obligation to sell the underlying asset if the holder of the call option decides to exercise his right to buy. A put option gives the holder the right to sell a specified quantity of the underlying stock at the strike price on or before the expiration date. The writer of the put option has the obligation to buy the underlying asset at the strike price if the holder decides to exercise his right to sell. The style of an option refers to when that option is exercisable. An American option may be exercised at any time prior to the expiration date. A European option may be exercised only at the expiration date. The Monte Carlo simulation is a valuable and flexible computational tool in financial theory and practice [3, 4]. In this paper, we demonstrate how it can be applied to analyse chooser options. We price the options using Monte Carlo methods combined with the analytical Black-Scholes solution, relating to the case of the European call option, available through the MAPLE command. Using crude Monte Carlo, the distribution of the rate of return from investments in chooser options is examined. The simulations are performed using MAPLE. We use the Black-Scholes model to describe the price of the underlying asset. The following assumptions were made to derive the Black-Scholes model: there are no riskless arbitrage opportunities; there are no transaction costs; there are no dividends during the life of an option; security trading is continuous; the risk-free rate of interest and the stock price volatility are constant; the price of the underlying asset follows a geometric Brownian process

2. Model description

The Black-Scholes model is used, this is the most popular valuation model for options. The model is based on the assumptions that markets are arbitrage free and the price S of the underlying asset follows a geometric Brownian motion:

$$S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right), t \in [0,T],\tag{1}$$

where:

$W = \{W_t, t \in [0,T]\}$ – a standard Brownian motion under the risk-neutral probability P, r denotes the risk-free interest rate,

S_0 – the stock price at time 0,

T – the time to maturity of the option (expiry date),

$\sigma > 0$ – the stock price volatility.

An estimation of future volatility σ can be obtained from historical prices of stocks as the standard deviation of the stock return, by assuming that the recent realized level of volatility will continue in the future. Another estimation can be computed from current option prices (implied volatility). The estimation of σ has been widely studied in [1] and is not discussed

in this paper. Volatility is expressed in the percentage of the underlying asset price, and for stocks, it is typically between 15% and 60%.

Under the assumption of no arbitrage, the price of a generic derivative security can be expressed as the expected value of its discounted payouts. This expectation is taken with respect to the risk-neutral measure. Then today's price of a stock option that pays at some time t according to a F_t – measurable payoff function $f(t)$, is:

$$E\left(e^{-rt} f(t)\right) \tag{2}$$

Let S_T denote the price of the underlying asset at the expiry date T, and K denote the strike price. The pay-off is given by:

$$f(T) = \left(S_T - K\right)^+ = C\left(S_0, K, T\right)$$

for a call option, and by:

$$f(T) = \left(K - S_T\right)^+ = P\left(S_0, K, T\right)$$

for a put option. A closed form formula for pricing the above options is the Black-Scholes formula:

$$c = S_0 N\left(d_1\right) - K e^{-rT} N\left(d_2\right),$$

$$p = K e^{(-rT)} N(-d_2) - S_0 N(-d_1),$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

c is the price of a call option, p is the price of a put option and N is the cumulative probability function for a standard normal distribution [6].

Analytical solution

The call option price can be computed in MAPLE, as the analytical solution, based on the Black-Scholes model. It is available through MAPLE command:
> *with (finance)*:
$S0 := 50 : K := 49 : r := 0.07 : \sigma := 0.3 : \tau := 199$:

$$c := \text{evalf}\left(blackscholes\left(S0, K, r, \frac{\tau}{365}, \sigma \right)\right);$$

where τ denotes 199 days to maturity.

Monte Carlo Simulation

For the purpose of introduction, the evaluation of the price by the Monte Carlo simulation is also presented. Moreover, we compare the computed result with the analytical solution presented above.

Independent replications $S_T^{(i)}$ of the terminal stock price under the risk–neutral measure can be generated from formula (1). By the Strong Law of Large Numbers we have:

$$\frac{1}{n}\sum_{i=1}^{n} f\left(S_T^{(i)}\right) \to E\left(f(T)\right), n \to \infty, a.s.$$

An unbiased estimator of the price of European call option is given by:

$$C = \frac{1}{n}\sum_{i=1}^{n} e^{-rT} \max(S_T^{(i)} - K, 0), \tag{3}$$

where:

$$S_T^{(i)} = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right)T + \sigma x_i\right), i = 1\ldots n, \tag{4}$$

T is the option's maturity and $\{x_i\}$ are independent samples from the normal distribution with mean 0 and standard deviation $\sqrt{T}$.

As we can see below, the difference between the exact and Monte Carlo results is about 0.01:

$$d := \exp\left(-\frac{r \cdot \tau}{365}\right):$$

$$N := 10^5$$

$$X := Random\ Variable\left(Normal\left(0, \sqrt{\frac{\tau}{365}}\right)\right):$$
$$x := Sample\ (X, N):$$
$$L := 0:$$
for i **to** N **do**

$$p[i] := \max\left(S0 \cdot \exp\left(\frac{\left(r - \frac{1}{2}\cdot\sigma^2\right)\cdot\tau}{365} + \sigma \cdot x[i]\right) - K, 0\right):$$

$$L := L + p[i]$$
end do:

$$c := \frac{d \cdot L}{N};$$

Monte Carlo simulations are never exact and one always has to take the sample standard deviation into account. With 100 independent Monte Carlo calculations of c, the standard deviation of the price sample and mean are around 0.03 and 5.8514 respectively. Hence

5.8514±0.006 forms the boundary for the 95% confidence interval for the price. We present a histogram of the sample of c, based on 100 simulations:
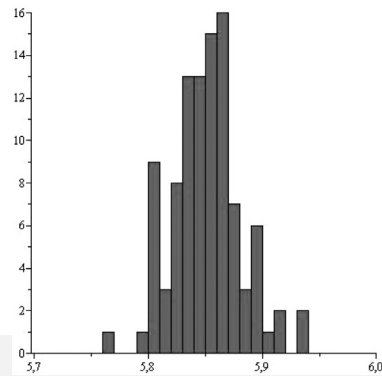

Fig. 1. Histogram of price of European
call option based on 100 simulations

3. Analysis of simple chooser options

In this paper, the main attention is focused on the analysis of simple chooser options. The Monte Carlo simulation aimed at the pricing of simple chooser options and the examination of the distribution of the rate of return from the options is described. Chooser options have been traded since July 1990 with the initial contracts traded by Bankers Trust [5]. They are purchased in the present, but are chosen to be either put or call at some specified future date. Their holder has the right to decide at some specific point in time $t(t < T)$, whether the options will finally be put or call. Hence they are sometimes named 'you-choose' or 'as-you-like' options. Chooser options are suitable when strong volatility of the underlying asset is expected but investors are not certain about the direction of the change. In the case of a rising value of the underlying asset over a period of time, the holder of the option will choose the call option because it will have a higher value than the put option. When the underlying asset falls up, the choice will be the put option. Once this choice has been made at time t, the option stays as either a call or a put to maturity. If the strike prices of the call and the put are the same, just as their expirations, such an option is referred to as a simple chooser. We will continue to call them briefly as chooser options.

3.1. Pricing chooser options

Let us denote:
$T - t$ time to maturity,
S_t stock price at t,
$C\left(S_t, K, T - t\right)$ premium of European call option,
$P\left(S_t, K, T - t\right)$ premium of European put option.

At time t, the investor will choose the call option if:

$$C(S_t, K, T-t) > P(S_t, K, T-t),$$

otherwise, he will choose the put option [2].

By the put-call parity:

$$C(S_t, K, T-t) - P(S_t, K, T-t) = S_t - K\exp\left[-r(T-t)\right]$$

the above inequality is equivalent to:

$$S_t > K\exp\left[-r(T-t)\right].$$

Hence the value of the chooser option at time t equals:

$$ch(t) = \max\left(C(S_t, K, T-t), P(S_t, K, T-t)\right) =$$

$$C(S_t, K, T-t) + \max\left(K\exp\left[-r(T-t)\right] - S_t, 0\right). \qquad (5)$$

The value of the option at time 0, when the choosing time is t, is equal to:

$$v(t) = \exp(-rt)\, E\left[C(S_t, K, T-t) + \max\left(K\exp\left[-r(T-t)\right] - S_t, 0\right)\right]. \qquad (6)$$

In [5], the relationships between the choice date and the chooser price, and between the chooser price and its strike price were examined.

Applying (6), we can price the European simple chooser option by simulation.

Example. Here we use the Monte Carlo method with $n = 100000$ simulations to price the chooser option where a maturity T is one year, the underlying asset price S_0 is 50, $r = 10\%$, $\sigma = 20\%$, $k = 50$ and $t = 0.25$ and $t = 0.25$. We simulate values of S_t and use the analytical result for $C(S_t, K, T-t)$ calculated by MAPLE command *blackscholes* $(S_t, K, r, T-t, \sigma)$. We obtain the price $v(t) = 7.01983862$. The algorithm is as following:

Set sum = 0
for I = 1 to i = 1 to n
generate S_t
set sum = sum + $C(S_t, K, T-t)$ + max($K exp(-rt)$ – St, 0)
end
set $v(t) = (\text{sum}/n)\exp(-rt)$

3.2. Rate of return

Using the Monte Carlo method, we can also analyse the profit function, which determines the profit for the holder of a chooser option on the expiry date. To obtain this goal, we have to know the values of the payoff function. We express a payoff function of the option in the following way:

$$f(T) = 1_A \max(S_T - K, 0) + 1_B \max(K - S_T, 0) \tag{7}$$

where $A = \{S_t > K \exp[-r(T-t)]\}, B = \{S_t < K \exp[-r(T-t)]\}$.

Let U and W be independent, normally distributed random variables with mean 0 and variance t and $T - t$ respectively:

$$X = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma U\right), \quad Y = \exp\left(\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \sigma W\right).$$

By (1) we have $S_t = XY$. Independent samples X and Y are generated. By definition of chooser options, if $X > K \exp[-r(T-t)]$ then $f(T) = \max(S_T - K, 0)$, lse $f(T) = \max(K - S_T, 0)$. Independent replications $f^{(i)}(T), i = 1\dots n$ give us not only the estimation of the chooser price:

$$ch(T) = \exp(-rT)\frac{1}{n}\sum_{i=1}^{n} f^{(i)}(T)$$

at $t = 0$, but also the sample of rate of return R, expressed in percentage:

$$R^{(i)}(T) = \frac{\exp(-rT) f^{(i)}(T) - ch(T)}{ch(T)} * 100\%, i = 1,\dots,n.$$

3.3. Simulation of rate of return

Figure 2 presents eight different histograms and medians of the rate of return dependent on K, based on $n = 10^4$ simulations each case, where a maturity T is one year, the underlying asset price S_0 is 50, $r = 10\%$, $\sigma = 30\%$, $t = 0.6$. The mean is equal to 0 each case.

As can be seen, a data set of the rate of return is unimodal and positively skewed, the long tail is on the right-hand side, when $K \leq S_0$. The situation reverses when $K > S_0$. A data set of the rate of return is bimodal. The right-hand tail of the distribution decreases with an increased K. Figure 3 plots the median of rate of return against the strike price.

From the investor's point of view, the most interesting case is when the median rate of return is the biggest. As can be seen in Figure 3, the worst case is for $K = 50$ and the best is for $K = 90$. Let us compare the probabilities corresponding to different value ranges of the rate of return R. The simulation results are presented in Table 1.

Let us observe that in the case of strike price $K = 50$, the probability of relative loss exceeding 50% is equal to 0.39 while in the case where $K = 90$, the probability equals only 0.15. Interestingly, for $K = 50$, the probability that relative gain exceeds 50% is equal to 0.25, while for $K = 90$ it is only 0.12. Hence, for $K = 50$, large gains and large losses have the highest probabilities. The opposite situation occurs in the case of $K = 90$, the highest probabilities have small gains and small losses. As presented, the Monte Carlo simulation proves to be very useful for the analysis of the investment risk.
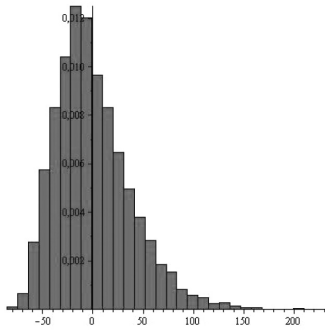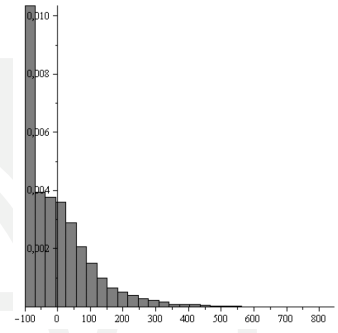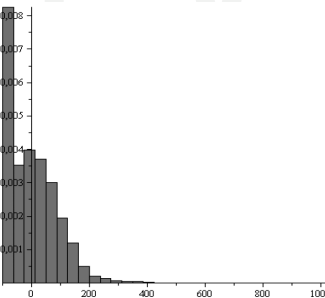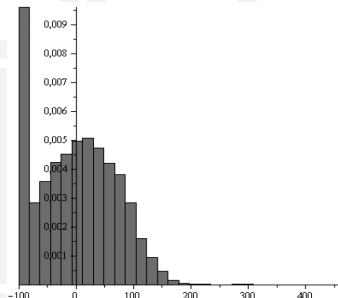
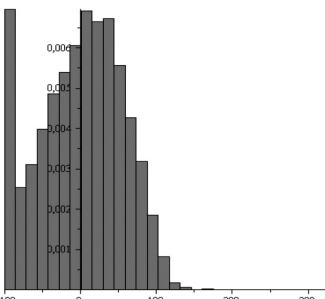$K = 10$, Median = –5.05
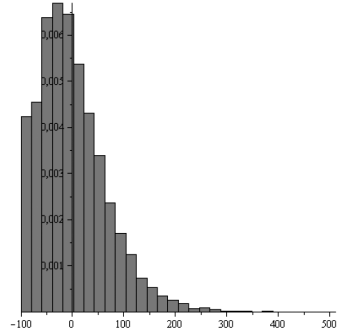
$K = 30$, Median = –9.9
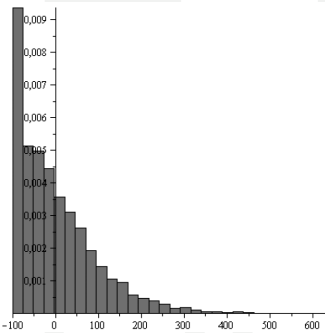
$K = 40$, Median = –21.09

$K = 50$, Median = –22.66
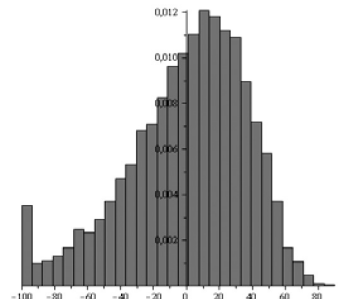
$K = 60$, Median = –8.21

$K = 70$, Median = 1.05

$K = 80$, Median = 4.86

$K = 90$, Median = 5.83
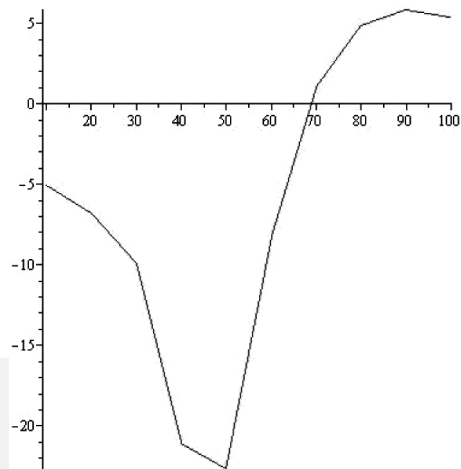
Fig. 2. Histograms and medians of rate of return

Fig. 3. Median of rate of return

Approximated distributions of rates of return for strike price $K = 50$ and $K = 90$

	$K = 50$	$K = 90$
$P(R \le -75\%)$	0.3	0.08
$P(-75\% < R \le -50\%)$	0.09	0.07
$P(-50\% < R \le -25\%)$ Wpisz tutaj równanie. -25%)	0.1	0.12
$P(-25\% < R \le 0\%)$	0.1	0.18
$P(0\% < R \le 25\%)$	0.09	0.23
$P(25\% < R \le 50\%)$	0.07	0.2
$P(50\% < R \le 75\%)$	0.06	0.1
$P(R > 75\%)$	0.19	0.02

4. Conclusions

The Monte Carlo simulation is useful in determining the distribution of the rate of return from investments in options. Knowledge of this distribution helps in determining investment risk. As demonstrated in Figures 2 and 3, and in Table 1, in the case of chooser options, the distribution and consequently, the level of risk, is significantly dependent on the strike price.

R e f e r e n c e s

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