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## ESTIMATING THE TIME OF BUILDING PROCESSES WITH PROBABILISTIC MODELS

SZACOWANIE CZASU REALIZACJI PROCESÓW  
BUDOWLANYCH Z WYKORZYSTANIEM MODELI  
PROBABILISTYCZNYCH

### Abstract

On the basis of traditional methods for the technical normalization of working time, this present paper presents the use of probabilistic models of working processes for use when estimating the execution time for selected building processes.

*Keywords: Planning and organisation in building projects, probabilistic models, risk*

### Streszczenie

Na bazie tradycyjnych metod normowania technicznego czasu pracy w artykule przedstawiono wykorzystanie probabilistycznych modeli procesów budowlanych w zagadnieniu szacowania czasu realizacji wybranych procesów budowlanych.

*Słowa kluczowe: organizacja i planowanie przedsięwzięć budowlanych, modele probabilistyczne, ryzyko*

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### 1. Introduction

Estimating the execution time for work processes has traditionally been carried out on the basis of quantitative standards which characterise processes and information (type, unit of measure, the number of units). The principal methods of quantitative standards' development, used among others to estimate the execution time for work processes, are **analytical, summary and parametrical** methods [3].

*Analytical methods* [17] are based on a detailed analysis of the organizational and technological execution of work and time examination, in order to identify the constituent elements of standard and irregular time. The investigation carried out for work process' components and model verification help to improve technical, technological as well as organizational aspects of process planning.

Time measurements are taken in conditions determined in prior analyses, and then average execution times for all components of the process are established. With regard to the level of complexity of the process, and based on averaged times, the planner can set up the following: **standard** of the time of working process (**simple process**), **merged standard** of the working process (**complex process**).

Splitting the standard of working time into components is carried out in a different way with reference to the work of workers and the work of machines (table 1) [3, 5].

Table 1

**Structure of the standard of working time of the worker and machine in the deterministic approach (example)**

<p><b>1.</b> <math>n^r</math> – standard of worker's working time</p> <p>1.1. <math>tpz</math> – preparatory &amp; finishing time</p> <p>1.2. <math>tj</math> – unit time</p> <p>1.2.1. <math>tw</math> – execution time</p> <p>1.2.1.1. <math>tg</math> – main time</p> <p>1.2.1.2. <math>tp</math> – auxiliary time</p> <p>1.2.2. <math>txt</math> – time of technological breaks</p> <p>1.2.3. <math>to</math> – attended time of workstation</p> <p>1.2.4. <math>tf</math> – time for physiological needs</p> <p>1.2.4.1. <math>tfn</math> – time for natural needs</p> <p>1.2.4.2. <math>tfo</math> – time for the rest</p>	<p><b>2.</b> <math>n_j^m</math> – standard of working time of the <math>j</math>-th machine</p> <p>2.1. <math>tpz</math> – preparatory &amp; finishing time</p> <p>2.2. <math>tj</math> – unit time</p> <p>2.2.1. <math>tg</math> – main time</p> <p>2.2.2. <math>tp</math> – auxiliary time</p> <p>2.2.3. <math>txt</math> – time of technological breaks or time of the idle ride</p> <p>2.2.4. <math>to</math> – attended time of workstation</p> <p>2.2.5. <math>tf</math> – time for physiological needs (of operators)</p>
$n^r = tpz + tw \left( \frac{100 + to + tf}{100} \right) + txt$	$n_j^m = tpz + tg + tp + txt + to + tf$

Amongst the analytical methods, an analytical-computational method and an analytical-measuring method are distinguished. **The analytical-computational method** is applied if component times of the standard are well-known. **The analytical-measuring method** is applied in such cases when there is a lack of such data, and when it is necessary to take multiple measurements and compile the results.

Measurements and observations of working time can be carried out with the help e.g.: of **the timing** (chronometry), **shutter observation** or **the photograph of a working day**.

**Summary methods** [17] allow the user to establish labour standards for working process as a whole, without splitting them into components and without the analysis of standardized and irregular time. Summary methods are divided into: **estimated-experimental**, **comparative** and **statistical** methods.

Summary methods take the following into account: worker's experience (estimated-experimental method), historical data from works performed earlier, data provided from measurements of times or other information describing the actual use of working time, which allows for standard's development (comparative method) within analyses and analogies, and finally statistical data from daily or periodic reports, enabling the right standards of the time with statistical methods to be determined.

**Parametric methods** [3] enable the planner to develop the standard of time, based on relations defined between parameters which describe the scope of the working process as well as labour intensity.

**The execution time of the simple working process** constitutes the maximum time intervals necessary to complete all work components and considers demand on the working times of every kind of resource. The execution time of the simple process  $t_i$  is calculated using formula (1) and (2).

$$t_i^r = \frac{n_i^r}{k_i^r}; \quad t_{i,j}^m = \frac{n_{i,j}^m}{k_{i,j}^m} \quad \text{dla } j \in M_i \quad (1)$$

$$t_i = \max \left\{ t_i^r, t_{i,1}^m, t_{i,2}^m, \dots, t_{i,|M_i|}^m \right\} \cdot l_i \quad (2)$$

where:

- $t_i^r$  – execution time of the  $i$ -th process based on labour demand,
- $t_{i,j}^m$  – execution time of the  $i$ -th process based on the work demand of the machine of the  $j$ -th type,
- $n_i^r, n_{i,j}^m$  – standards of working time of worker and machine of the  $j$ -th type from the  $M_i$  set,
- $k_i^r, k_{i,j}^m$  – number of workers and machines of the  $j$ -th type, realizing the  $i$ -th process,
- $i$  – working process' number and  $j$  – number of machine's types
- $l_i$  – range of the process (number of units).

**The execution time of the complex process**  $t^c$  is a sum of execution times for simple processes and the offsets (breaks) resulting from technological and organizational relations, calculated with reference to the chain of processes (paths), in which this sum achieves the maximum value.

$$t^c = \max S \rightarrow S \left\{ s_q \right\}, \quad q = 1, \dots, h \quad (3)$$

$$s_q = \sum_{p=1}^{z_q} (t_p^q + d_p^q), \quad p = 1, \dots, z^q, \quad (4)$$

where:

- $t^c$  – execution time of the complex process,
- $S$  – set of sums of execution times of processes along  $s_q$  paths,
- $q$  – indicator of the path in the network model,
- $d_p^q$  – breaks resulting from the relation between processes ( $p$ ) and ( $p+1$ ) on  $q$ -th path,
- $h$  – number of possible paths in the network model,

- $z^q$  – number of processes in the  $q$  path,  
 $t_p^q$  – execution time of  $p$ -th simple working process on the  $q$  path.

Table 2

**Model of simple working process in deterministic approach (example)**

Number of process	1	Unit of measure	m <sup>2</sup>	Number of units	3600
Name of the process:	Removal of the fertile soil (humus) to a thickness 15 cm with using of bulldozers				
Resources			Standards of		
Kinds of resources		[units]	Number of resources	demand on work of resources	execution time
1	labour	[l-hr]	4	0.0053 [l-hr/m <sup>2</sup> ]	0.00133 [hr/m <sup>2</sup> ]
2	caterpillar dozer 74 kW	[m-hr]	1	0.0025 [m-hr/m <sup>2</sup> ]	0.00250 [hr/m <sup>2</sup> ]
Duration of the process: (max → {standard time * number of units})				9.00 [hr]	

Remarks: [l-hr] – it means man-hour, [m-hr] – it means machine-hour.

Table 3

**Model of the complex working process in the deterministic approach (example)**

Number of process	1	Unit of measure	m <sup>2</sup>	Number of units	10 000																																																																	
Name of the process:	Measurement and preparatory work at the surface earthworks (trough pavement parking squares), trees felling (diameter 16–25 cm) and cutting down of bushes, with their transporting (2 km) and the removal of humus (25 cm) by bulldozer.																																																																					
ID	Name of the sub-process	u.m.	Quantity	Estimated time of execution [h]	Binary matrix of the order																																																																	
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Estimating the execution time of complex process:					<b>148,3 [hr]</b>																																																																	
<p align="center">Network model0:</p> <pre> graph LR   1[1] --&gt; 2[2]   1 --&gt; 3[3]   1 --&gt; 4[4]   1 --&gt; 5[5]   1 --&gt; 6[6]   2 -.-&gt; 3   2 -.-&gt; 4   2 -.-&gt; 5   3 -.-&gt; 4   3 -.-&gt; 5   4 -.-&gt; 5   6 -.-&gt; 7[7]   </pre>																																																																						

Remarks: [ha] – hectare, [pcs.] – pieces, [l.p.] – loading point.

A path with the maximum sum of times is established within the time analysis of the complex processes in the network model, in order to determine the course of the critical path.

## 2. Premises for the probabilistic model appliance

The main premises for the appliance of the probabilistic model in the building processes research is their random character. This is due to a lack in the explicit and credible estimation of results of experiments multiplied and repeated in similar or the same conditions. Repeatedly run experiments and the registration of these results, although being expensive and labour-demanding, allow for the statistical estimation of results' forecast using i.e. the average value and measurements of the changeability (variance, standard deviation).

The probability of obtaining a certain result in a single experiment or series of experiments is another helpful parameter. An appropriate and acceptable probability level lets us formulate a satisfactory and accurate forecast using statistical values. The quality of forecasts strongly influences the effectiveness of future actions' planning.

## 3. Characteristics of the probabilistic model

The proposed working processes' model is aimed at creating a tool for simulative estimations of execution time processes' [10]. Execution time for working process  $t_i$  is a sum of component times (table 1), hence, based on the central limit theorem, it is variable  $T_i$  that models it, which is a continuous random variable which has a distribution convergent with normal distribution.

The proposed probabilistic model of the simple working process considers both: types of working time components, included in deterministic quantitative standards of the process, and information concerning the number of working resources.

Using general properties of the expected value, that is:

$$E(a \cdot T_i + b) = a \cdot E(T_i) + b \quad (5)$$

where  $a$  and  $b$  are constant, it is possible to divide the characteristics of components' times into three groups:

- **group I** – times described by random variables normally or log-normally distributed (basic component times), defined after an empirical and/or simulation examination with the type of the distribution, the average value and the standard deviation or variance, e.g.: main time, auxiliary time (see table 1),
- **group II** – times described by the ratio  $a_i$  proportional to the sum of times established in **group I**, e.g.: attendance time for a workstation or machine, time for physiological needs (i.e. time for rest for average heavy works makes up 3% of the sum of main and auxiliary times),
- **group III** – component times described by an established period of the time (single-point distribution) depending on: e.g. the specificity of the working process, the technology of works' performance (i.e. the  $t_{pz}$  time makes up c.a. 4% of the working shift's time, which is c.a. 20 min.).

The **group I** times should be, as far as possible, examined empirically [9] and/or with simulation methods [10]. Times defined in **group II** are established based on determined standards, proportionally to the sum of times from **group I**. Times from **group III** can be examined and/or determined using the standards towards the time of the working shift.

In the proposed model, random time variables can be distributed [6, 2, 13, 14]:

- a) normally  $N(m, \sigma)$ ,
- b) normally on both sides cut  $N_{\alpha,\beta}(m, \sigma)$ ,
- c) log-normally  $LN(m, \sigma^2)$ ,
- d) regularly in established range of value  $R(t_1, t_2)$ ,
- e) discretely with a single-point  $J(t_x)$ .

Accuracy of the fit of the adopted theoretical distribution to the data obtained from measurements or simulations should be examined with a nonparametric tests [1, 12, 15, 16] such as:  $\chi^2$  test, *Kolmogorov-Smirnov* test or *Shapiro-Wilk* test.

In the probabilistic analysis of component times we appoint: the expected value  $E(T_i^I)$  of the sum of basic times from **group I**, a summary indicator  $a' = \sum_{k=1}^v a_k$  proportional towards  $E(T_i^I)$  – (**group II**) and a sum of the component times  $b' = \sum_{r=1}^w b_r$  (**group III**), where  $v$  and  $w$  mean number of component times in **groups II** and **III**.

Table 4

**Estimating the execution time of simple working process in probabilistic approach (example)**

Group	Components times [hrs]	probability level $P_{uf} = 0,9772$			multiplicity of deviation $g = 2$		
		labour			caterpillar dozer		
		aver.	dev.	distribut.	aver.	dev.	distribut.
I	$tg$	4,083	0,310	normal	1,250	0,157	normal
	$tp$	1,117	0,257	normal	0,633	0,120	normal
	$E(T_i^x) =$	6,333			2,437		
II		$a_k$	base	value	$a_k$	base	value
	$to$	0,030	6,333	0,156	0,020	2,437	0,038
	$tf$	0,015	6,333	0,078	0,015	2,437	0,028
	$a' =$	0,045			0,035		
III		$b_r$			$b_r$		
	$tpz$	0,500			0,583		
	$txt$	0,750			0,667		
	$b' =$	1,250			0,583		
		$(I + a') * E(T_i) + b' =$					
		$E(T_i^{x-1}) = 7,868$	$k_i^r = 2$	$E(T_i^{x-2}) = 3,105$	$k_{ij}^m = 1$		
		$E(T_i) = \max \{E(T_i^x)\} = 3,934$ [hrs]					

An estimation of the essential time for simple working process  $E(T_i)$  performance, due to demand on working time established for each kind of resource, should be calculated in agreement with property (5) and formulas: (6) and (7) as showed in the table 4.

$$E(T_i^x) = (1 + a^{ix}) \cdot E(T_{i,j}^{L,x}) + b^{ix}, \quad (6)$$

$$E(T_i) = \max \left\{ E(T_i^r), E(T_{i,j}^m) \right\}. \quad (7)$$

Indicator  $x$  in the equation (6) refers one by one to every kind of resource, performing the  $i$ -th process (i.e. labour  $\{r\}$ , and work of  $j$ -th type of machine  $\{m, j\}$ ).

This complex working process is portrayed by probabilistic network models. Its characteristics are appointed by the set of component characteristics of simple processes and parameters of time dependencies between processes (order and offsets matrices – see table 3).

The characteristics of a complex process [1, 4, 7, 16] incorporate: the assumed type of distribution of random variables describing the execution time of complex processes (normal or log-normal distribution), while the expected value of execution time  $E(T^e)$ , standard deviation  $\sigma(E(T^e))$ , types of relations between the processes and potential offsets, probability level  $P_{uf}$  which is acceptable for real data measurements (execution times of the process), and is at most equal to the adopted limit value  $E(T_{\max}^e)$ .

$$E(T_{\max}^c) = E(T^c) + \Delta(T^c) \quad (8)$$

$$E(T^c) = \max \left\{ \sum_{p=1}^{z^q} \left( E(T_p^q) + d_p^q \right) \right\} \quad (9)$$

$$\Delta(T^c) = g \cdot \sigma(E(T^c)) \quad (10)$$

$$\sigma(E(T^c)) = \sqrt{\sum_{p=1}^{z^q} \sigma^2(T_p^q)} \quad (11)$$

where:

$\Delta(T^c)$  – time reserved in the network model, involving the accepted  $P_{uf}$  probability level of meeting  $E(T_{\max}^c)$  value, calculated from the standard normal table  $N(0, 1)$  for the specific standard deviation of the execution time  $E(T^e)$ ,

$E(T_p^q)$  – expected value of the execution time of  $p$ -th process on the  $q$ -th path,  
 $g$  – multiplicity of the standard deviation assuming  $P_{uf}$  probability level,  
 $\sigma^2(T_p^q)$  – variance of the  $p$ -th process on the  $q$ -th path of processes.

Value  $E(T_{\max}^c)$  estimated according to relations (8)÷(11) with the probability level  $P_{uf}$  (usually 0.99865 for  $g = 3$ ) won't be overestimated. This value meets the condition:  
 $P(T^c \leq T_{\max}^c) = P_{uf}$

#### 4. Conclusions

Working processes' modelling in the planning aspect [8], in particular when estimating the execution time of working processes, which is the subject of this paper, requires a good knowledge of the processes' nature and specific parameters.

The most important issues, which provide the utilitarian character of the models introduced, are: high cost and high labour demand necessary to run multiple experiments "in-situ", in order to obtain credible data for planning [10], cost control or allocation of equipment units [8, 11] etc. Moreover, the changeability of parameters and conditions of working process performance – is, (apart from when performed in the laboratory) natural, however, hampers the collection of appropriate information.

Process realization modelling contributes to a better understanding of significant parameters and relations appearing in process performance. One should notice, that analysing the nature of process' realization and building its model, even the simplest one, is one of the first steps in overcoming the problems which can appear in process realization.

An appropriate and credible model, mapping the examined problem, can be one of many tools applicable to limiting recalled costs and workloads for examinations and observations. By having a model at ones disposal, it is possible e.g. to apply simulation methods to multiply the data obtained in examinations on so-called "small samples".

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