TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
ENVIRONMENTAL ENGINEERING | ŚRODOWISKO

2-Ś/2015

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NUMERICAL EVALUATION OF 2D ELECTRICAL RESISTIVITY TOMOGRAPHY FOR SUBSOIL INVESTIGATIONS

METODY NUMERYCZNE 2D TOMOGRAFII ELEKTROOPOROWEJ STOSOWANEJ W BADANIACH PODŁOŻA GRUNTOWEGO

Abstract

The use of numerical methods for the design and analysis of hydraulic engineering structures requires an accurate determination of the model of subsoil structure. The article presents one of the geophysical methods – electrical resistivity tomography (ERT), which allows for precise, spatially instant recognition of the substrate and the phenomena occurring within it. The methodology, the applied algorithm for numerical calculation, and data processing procedure are discussed. This article also introduces a method of inversion which relies on finding the actual model and presents examples of its application.

Keywords: Electrical Resistivity Tomography ERT, 2D inversion, investigations of subsoil

Streszczenie

Zastosowanie metod numerycznych do projektowania oraz analizy konstrukcji hydrotechnicznych wymaga określenia modelu budowy podłoża. W artykule zaprezentowano metodę tomografii elektrooporowej ERT stosowaną dzięki ciągłemu, przestrzennemu rozpoznaniu podłoża i zjawisk w nim zachodzących, m.in.: w geotechnice, hydrotechnice, a także w zagadnieniach obejmujących projektowanie konstrukcji inżynierskich. Przedstawiono metodologię, stosowane algorytmy do obliczeń numerycznych oraz procedury przetwarzania danych. Przybliżono metodę inwersji polegającą na znalezieniu modelu rzeczywistego ośrodka oraz zaprezentowano przykłady jej stosowania.

Słowa kluczowe: metoda tomografii elektrooporowej ERT, 2D inwersja, badania podłoża

DOI: 10.4467/2353737XCT.15.230.4616

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1. Introduction

Geophysical methods enable the identification of a surface zone vulnerable to changes caused by human activity and also by nature. They use the rules and the laws of physics, and the basis for their use is to differentiate the physical properties of the medium. These methods can be used comprehensively in solving diverse thematic issues: the recognition of geological and engineering subsurface, environmental monitoring, and detection of anthropogenic objects. From the whole range of engineering geophysics methods, taking into account the accuracy and the ability to use a variety of field conditions, the electrical resistivity tomography method (ERT) is the most versatile as well as the method of ground penetrating radar (GPR) and multichannel analysis of surface waves (MASW). These methods are primarily used to solve geotechnical issues. The article presents the method of electrical resistivity tomography and shows some examples of research with its use.

2. Research methodology

The basis for the application of geophysical methods are the differences in the physical properties of the medium (e.g. electrical resistance, magnetic permeability, density, dielectric constant, etc.), and the dependence of the properties on the medium structure.

The method of electrical resistivity tomography (ERT) belongs to a group of geophysical methods which offers non-invasive investigation of subsoil. Assumption methods were developed in the early twentieth century by the Schlumberger brothers. The ERT method is based on the study of changes in the electric field generated by a system of electrodes which are DC powered. The apparent resistivity of rocks, representing the result of the entire heterogeneous, complex anisotropic layers, is determined in accordance with Ohm's law (1) by measuring the intensity and the voltage between the measuring (potential) electrodes [1].

$$\rho_a = k\frac{\Delta V}{I} \tag{1}$$

where:

ρ_a – apparent resistivity [Ωm],
ΔV – measured voltage [mV],
I – current emitted into the subsoil [mA],
k – geometrical factor depending on the individual distance between the electrodes.

Various types of electrode combinations can be used, such as Wenner, Schlumberger, or Dipole-Dipole arrays, differing among others, in: (i) degrees of profile coverage, (ii) penetration depth and (iii) sensitivity to vertical and horizontal changes in resistance. Each type of electrode configuration has its advantages and limitations.

Geophysical measurements using resistivity methods can be performed by traditional Vertical Electrical Sounding (VES) and Electrical Profiling (EP) techniques. Vertical electrical sounding enables tracing of changes in electrical resistivity with the increasing depth of penetration on middle-point of the measuring system, as a result of increasing the spacing of current electrodes (Fig. 1). Electrical profiling consists of a predetermined series of measurements taken along a line measuring system with a specific profile sampling step, with a constant distance between the electrodes. The information received from the electrical sounding and profiling are one-dimensional, which means that the electrical resistivity changes are determined either vertically (in the case of sounding probes) or horizontally (for resistivity profiling) [2, 3]. The penetration-range of these methods is determined as approximately one third the spacing of current electrodes.

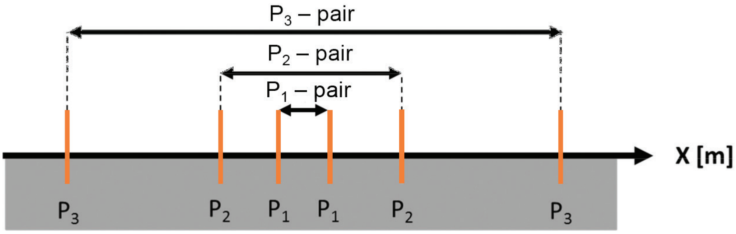

Fig. 1. Vertical Electrical Sounding [after: 4, revised]

The ERT method is a combination of both of these measurement techniques. Measurements are performed along the profile, following a sequence, which is run by a switching unit and controlled by a computer (usually integrated within a resistivity measurement apparatus) – based on the automatic selection of electrodes within the defined system, i.e. electrode array (e.g. Schlumberger array). The electrodes are selected from all those connected to the cable until all the programmed combinations are completed (Fig. 2). The number of recordable measurements can be as many as several thousand. The maximum distance between the measuring system depends on the length of the profile and translates into the depth range of the research, which is $^1/_3 – ^1/_6$ the distance between the extreme electrodes [5].

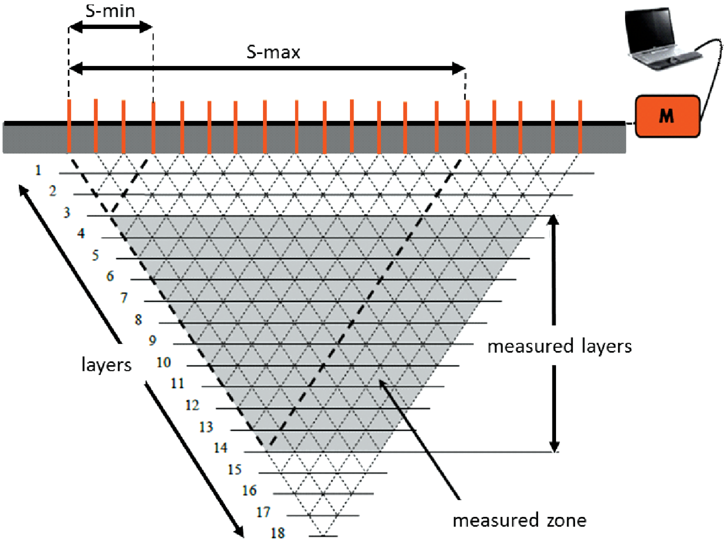

Fig. 2. Representation of the measure zone [after: 4, revised]

The measurement results are presented in the form of cross-sections showing the distribution of apparent resistivity on 2D imaging in an x-z plane: x – along the profile and z – depth. The distribution of resistivity describes by the medium bound by the surface above and by the depth of penetration bottom. The trapezoidal cross-sectional shape is the result of decreasing number of measurements with increasing distance between current electrodes.

3. Data processing and inversion introduction

The procedure for data processing and interpretation are performed using the inversion method. The main objective of the inversion is to find the actual model whose parameters are suitable for the measurement data. The first stage solves a *direct problem* that allows determination of the theoretical parameters for the assumed model of the medium. In the case of electrical sounding, a 1D model is established, which is a flat-parallel layers system, where the change of the resistivity is established in only one vertical direction (Fig. 6a). The ERT method assumes a 2D model of the medium, which allows for variability of parameters in both directions vertical and horizontal (Fig. 6b). When creating the model, external conditions are introduced, known as a *priori* information. The two-dimensional model uses discretization of the subsurface into a number of blocks using a rectangular mesh with 2 or 4 nodes per node spacing – distance between adjacent electrodes [5]. The 2D model takes into account the variability of the parameters in the vertical plane cross-section, while along the perpendicular direction the parameters are fixed, within each of the blocks. The two-dimensional model allows the imaging of more complex structures and phenomena of limited spatial extent, which are impossible with the described traditional methods using flat-parallel models.

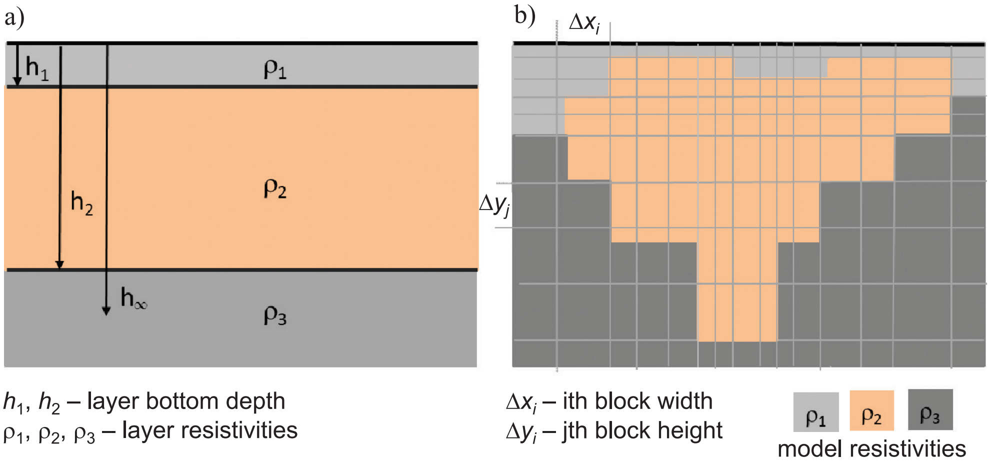

Fig. 3. a) The model of layers based on 1D measurement used by VES method; b) The 2D block model of the subsurface used by ERT method [after: 3, revised]

The solution of a *direct problem* 2D, for such assumed medium geometry relies on solving the equation of the distribution of electrical conductivity as a function of the length profile – x and the depth – z, which is described be the Poisson equation [6–8]:

$$\frac{\partial}{\partial x}\left(\sigma\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)-k_l^2\sigma V=-\frac{I}{2}\delta(x)\delta(z) \tag{2}$$

where:

σ – electrical conductivity [S/m],
V – electrical potential [V],
I – current [A],
k_l – wave number,
δ – Dirac delta.

The differential equation (2) is solved using numerical methods: the finite difference method or the finite element method. The finite difference method is faster and easier, but it offers results in low accuracy solutions. There are problems associated with matching mesh to the surface, as well as difficulties with the boundary conditions. Finite difference method does not give as good results for a large denivelation area [5]. In such cases, if the data set contains topography, the default choice is the finite element method. Additionally, topographical reduction is introduced, which takes into account the morphology of the surface.

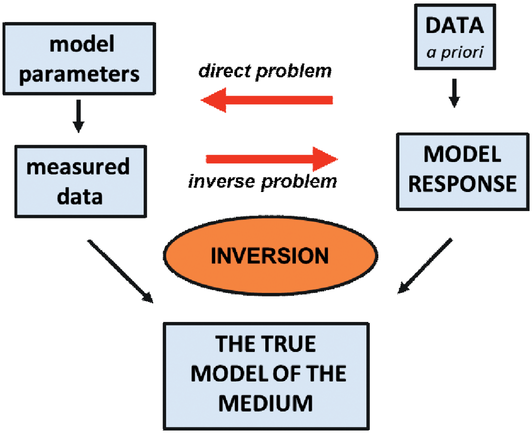

Fig. 4. Schematic diagram of a *direct* and *inverse problem*

The determined elements of the model are then used to calculate the *inversion*. The objective of the inverse problem is the iterative minimization of error fit of parameters using a theoretical model with measurement parameters (optimization of processing parameters to minimize the error). A schematic diagram of the *direct* and *inverse problems* is shown in Fig. 4. To eliminate the ambiguity of the solution of the inverse problem, it is necessary to impose boundary conditions that control the changes of the model – because there are many possible models of resistivity medium having the same solution. One of them is the smoothness condition – using the method of *smoothness inversion* in the 2D inversion, and creating a model of gentle, gradual changes in parameter values. The smoothness inversion method is based on the modified Gauss-Newton and Marquardt-Levenberg algorithms used to solve the inverse problem. The Gauss-Newton method (linear least-squares method) is to minimize the sum of squared differences between the theoretical values, resulting from the model and the observed values. It is the simplest solution of the 1D inversion giving satisfactory results for structures with little sophistication, in the absence of disturbing bodies. Adding the *Lagrange multiplier* λ_L, (also called the *damping factor* or *factor Marquardt*), which attenuates the magnitude (size) changes in the model parameters related to noise and interference measurements, the Gauss-Newton algorithm converts to the Levenberg-Marquardt algorithm. Whereas, if introducing the smoothing matrix *C* (so-called flatness filter) to the Levenberg-Marquardt algorithm, it leads to a smoothing of the assumed model and impose a smoothness condition directly on the model parameters. The resulting algorithm used for the smoothness inversion method has the form [5, 9, 10]:

$$\Delta q = \left(J_i^T J_i + \lambda_L C\right)^{-1} J_i^T g_i - \lambda_L C r \tag{3}$$

where:

Δq – model perturbation vector (vector correction),
J – Jacobian matrix of partial derivatives,
J^T – transposed Jacobian matrix,
$J^T J$ – matrix mathematical model,
i – index indicates the i-th iteration,
λ_L – Lagrange multiplier – damping factor,
C – smoothing matrix – flatness filter,
$g = (g_1, g_2, g_3, \ldots\ldots, g_N)^T$ – discrepancy vector representative of the differences between the measured and calculated apparent resistivity values,
r – vector containing the logarithm of the model resistivity values.

One advantage of this method is that the damping factor and flatness filters can be adjusted to suit different types of data. The objective of minimizing the difference between the parameters of the theoretical model and the measured data uses the *conventional Gauss-Newton method*, when the Jacobian matrix of partial derivatives *J* is recalculated after every iteration. It is much slower than the *quasi-Newton method*, but in areas with large resistivity contrasts of greater than 10:1, it gives better results. If there are too many data sets and less memory capability – a low power computer – it is desirable to shorten the inversion process e.g., by using a quasi-Newton method. In this option the Jacobian *J* is calculated only for the first iteration and then approximated. This technique can be more than 10 times faster than the conventional least-square method. A third option is to use the *combined method*, where the matrix *J* is recalculated for the first two or three iterations, after which the quasi-Newton method is used [5, 9]. In many cases, this provides the best compromise. After each iterating process, the damping factor is reduced and optimized so as to reduce the number of iterations. It requires converging by finding the optimum damping factor that gives the least RMS error [11]. To sharpen the boundary between structures, select abrupt changes in values to apply the *robust inversion*. The robust inversion algorithm is described by the formula [8, 12]:

$$\Delta q = \left(J_i^T R_d J_i + \lambda_{Li} W^T R_q W\right)^{-1} J_i^T R_d g_i - \lambda_{Li} W^T R_q W q_{i-1} \tag{4}$$

where:

Δq – model perturbation vector (vector correction)

W – weighting matrix of the inverse of the measurement errors on the diagonal

R_d – diagonal matrix with odds ratios coordinate vectors g and d on the diagonal

R_q – diagonal matrix with odds ratios coordinate vectors g and q on the diagonal

$R_d = \text{diag}\ \{g_1/d_1, g_2/d_2, \ldots\ldots\ldots\ldots.. g_N/d_N\}$, $R_q = \text{diag}\ \{g_1/q_1, g_2/q_2, \ldots\ldots\ldots\ldots.. g_N/q_N\}$

The robust constraint is less sensitive to very noisy data points, but might give a higher apparent resistivity RMS error [5]. Assuming the *robust inversion* options, the inversion process tends to create models of areas consisting of a fixed resistivity value. The block structures formed then show the geological structure with clearly sharp boundaries. The 2D inversion process ends when successive iterations no longer cause significant changes to the final model, or the RMS errors achieved is satisfactory. The inversion method is used in two- or three-dimensional resistivity inversion, and it is used in the program interpretation Res2dinv or Res3dinv.

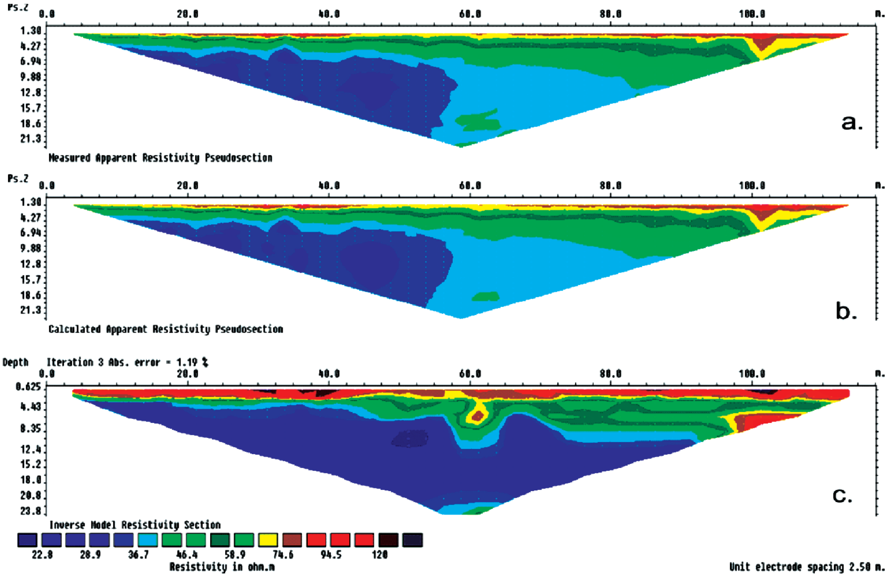

Fig. 5. Screenshot of the program Res2Dinv showing ERT inversion method: a – cross-section of apparent resistivity measured, b – cross-section of apparent resistivity calculated on the basis of the model, c – actual model of resistivity obtained by the inversion

The resulting two-dimensional distribution of the electrical resistivity of the subsoils is as close as possible to the actual distribution of resistivity along the test profile. Further interpretation is based on the description obtained from the inverse geoelectrical model and referencing it to the subsurface conditions – the geological structure. Fig. 5 shows the steps of creating an actual model of resistivity, starting from measured apparent resistivity pseudosection, through the calculated apparent resistivity pseudosection to the inverse model resistivity section. We can compare the observed apparent resistivity pseudosection with an inversion model.

4. The possibilities of applications

The ERT method is one of the most popular geophysical methods. It is used in geological mapping (i) to identify geological structure, (ii) to determine depth and thickness of the layers. In hydrogeological studies, ERT measurements are recommended for the detection of aquifers and spread-range of mineralized water in the subsurface. Within geotechnical and hydrotechnical engineering studies, ERT serves to determine the state of earth structures, especially locations of zones of loosening and relaxation, determination of weakness of the subsoil and the locations of sinkholes and voids. In environmental studies the ERT method is very effective in the location of pollution sources and mapping the aureole around potential sources of contamination resulting from landfill, waste water treatment, fuel tanks, and soil contamination by heavy metals. The ERT method can be led by monitoring pollution migration changes. It also plays a significant role in the detection of underground anthropogenic objects and planning of archaeological research by the location and identification of underground architectural remains [13].

5. Examples of application of the ERT method

The ERT method was used as a method enabling identification of the geological structure of the subsoil. It has been also used to detect underground anthropogenic objects and geotechnical structures. The ARES equipment from GF Instruments manufacturer was applied. The procedure and interpretation of the data was performed using Geotomo RES2DINV Software [5].

5.1. Application of ERT in geological structure research and anthropogenic objects identification

The ERT method recognized a landslide zone structure on the railway line Krakow–Warsaw, in Sadowie village, which runs in an artificial dig [14]. Landslide movements of the adjacent slopes have been recorded there for many years (1934–2011). After heavy rainfall in 2010–2011, the reactivation of landslide movement and soil displacement occurred, causing the track to be pushed out almost 1 m at a height. The landslide niche has moved beyond the edge of the slope, destroying the road running nearby and came to the borders of the residential area. Three parallel profiles with a length of 150–180 m were taken along the landslide. A Schlumberger array was used for the electrode spacing of 5 m. In the realized 2D inversion process, the robust model inversion constraint was used for the results of measurements carried out on the slopes. As a result of research and available data, it was found that the Quaternary sediments consist of loess (silty clay, silt) layers of thickness – of 3–4 m. Below them, there are Miocene clays with a thickness of about 30 m. Marls form the oldest bedrock in the area. The dominant feature of both Quaternary and Miocene clays in their part of the top section is their high humidity. Increased infiltration of rainwater (surface) through a layer of silt and silty loam to the series causes high saturation with water. Major changes of saturation between subsurface zones and the top of clays cause significant response of the physical properties of the grounds, which marks a significant decrease in resistivity, clearly visible in the cross-sections. That is why the localized boundary between Quaternary formations and Miocene clays should be treated not as a potential slip surface for periodically occurring mass movements, but only as the zone of soil displacements, even a few metres thick. Near the railway embankment, a zone of strong saturation, which has a low resistivity, was also interpreted. Replaced in this part of the soils marks already high values of resistivity. Occurrences of carbonate rocks are visible in the subsurface zone with much higher values, sometimes up to resistivity of 400 Ωm. They can significantly affect the direction of water run and infiltration. Permanent ground vibrations, caused by passing trains, are also conducive to the formation of the landslide. The results also showed construction elements such as piles in the subsurface, which are designed to protect and strengthen the slope.

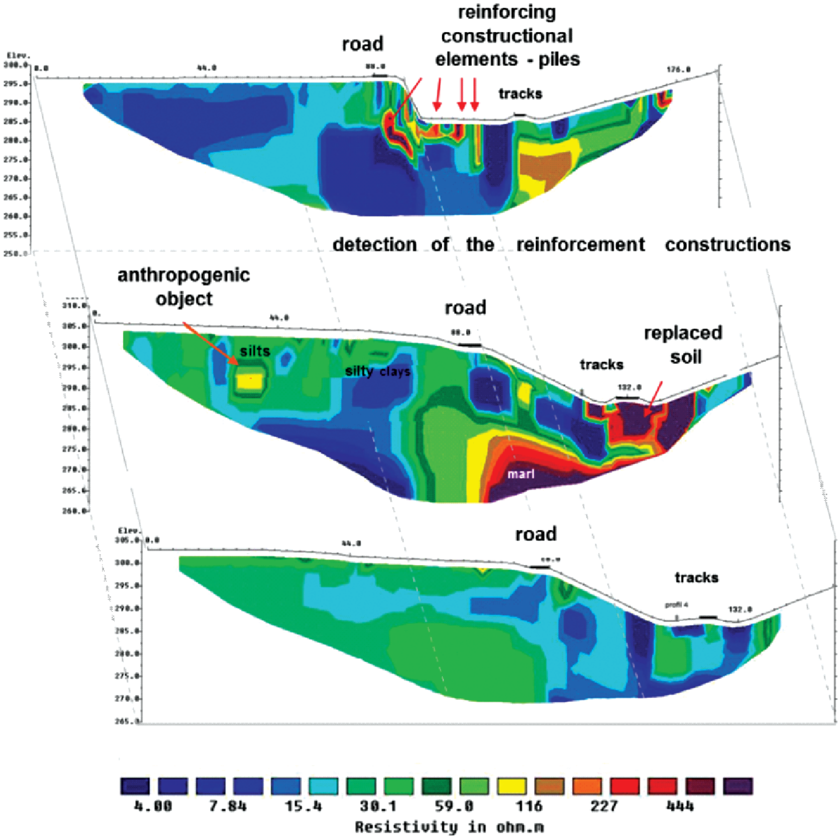

Fig. 6. ERT cross-section in Sadowie landslide area

5.2. The ERT in non-invasive river dike judgement

In the frame of protection against floods the river embankments' quality and stability were monitored, using the electrical resistivity tomography method. The reconnaissance studies were carried out in the area of Nowa Huta in order to evaluate the state of a fragment of the flood embankment of the Vistula River [15]. The aim of the study was to indicate: (i) loosened and weakened areas in the body of the river dike, and (ii) location of the place where penetration of water occurs. The ERT profile was 117.5 m in length and was performed along the crown of an embankment through the geotechnical hole, which allowed the correlation of geophysical results with the hole-data. Measurements were performed with the dipole-dipole array, with electrode spacing of 2.5 m. The default and *robust inversion* options were used. The ERT cross-section obtained for *robust inversion* was more clearly and sharper than for default inversion. The flood embankment was about 4 metres high and was made of a mixture of uncontrolled soils. Silt, silty sands, sand and loamy sands as well as clays, sandy clays and compact clays, basement fine sands and medium sand were found. The water table level is about 1.4 m below the surface.

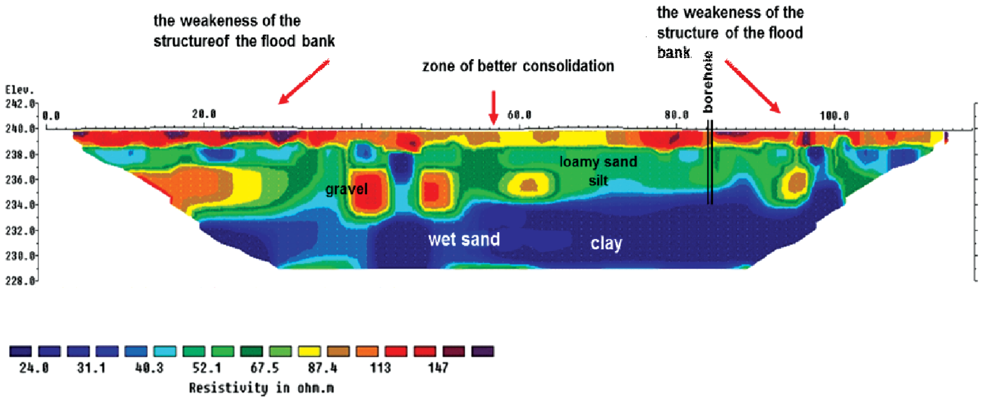

Fig. 7. Resistivity section along the crown of the flood embankment

Analysing the results of the cross-section (Fig. 7) we can distinguish three main anomalous zones: first at depth of about 1.5 m – zone with high resistivity 100–150 Ωm, which is strongly weathered and composed of loosened silty sands. Soils with better consolidation and reduced resistivity values (about 80–90 Ωm) occur in this zone between 45–75 m at the crown of flood embankment. In the second zone at the depth of 1.5 – 5m, there are soils with variable resistivity values, between 50–150 Ωm. One can isolate areas of more or less consolidation indicating the inhomogeneous structure of river dike. The low resistivity zone of 5 m depth, represents a third zone – a zone of flood embankment base – where the soils represents typical river terrace deposits: clay and silt, and watered sand with other blended gravel. At high water level, the zone of elevated resistivity values can cause water leakage and infiltration.

6. Conclusions

Geophysical surveys play an increasing role in the geotechnical recognition of subsoils. A meaningful advantage is their non-destructive and non-invasive character and obtaining continuous information from the medium. Electrical resistivity investigations have a wide range of applications, and they are used to recognize geological structure and to identify occurring engineering geological phenomena. They also serve to determine the aquifers and range of their extent. In environmental studies they are used for ground water protection, especially to determine the sources of contamination and their migration. They are also used to detect underground anthropogenic objects. From the point of view of the geotechnics, they may be useful for monitoring of the processes taking place in the subsoils: landslide risk assessment, mapping of slope deformations, slope stability monitoring, and river dike quality and stability investigation. They are applied to define and indicate vulnerable zones susceptible to weakness, damage and suffosion. As shown by the provided examples, they are also used to predict or determine places in order to implement geotechnical control.

The technical or methodological development in recent years – measuring equipment and the use of more advanced measurement techniques – has resulted in the use of more sophisticated algorithms and improved data processing procedures (e.g. *smoothness inversion*, *robust inversion*) using numerical methods. These make it possible to create more accurate models taking into account the medium's shape and dimensions and its spatial distribution. As shown in the article we are able to present research results in the form of 2D images by applying the electrical resistivity tomography method. The method is also much more precise than the traditional soundings and profiling techniques. An additional advantage is the speed of the obtained results. The use of the procedure interpretation enables the results to be obtained practically already on site. The wide range of applications, flexibility and efficiency bring additional benefits. The low costs of investigations relative to other methods undoubtedly increase its advantages.

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