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MODELLING THE DELIVERY AND LAYING OF CONCRETE MIX

MODELOWANIE DOSTAW I WBUDOWANIA MIESZANKI BETONOWEJ

Abstract

The aim of this paper is to indicate solutions which fulfil the conditions for laying concrete mix before it starts setting. Using models $M/M/N/-/N$, for organization I for the so-called delivery directly after unloading, with automatic transfer of information using the RFID system, and $M/M/1/FIFO/N/F$, for organisation II with transportation units working in closed cycle. Of example, organisation I, with RFID system is more advisable as it is characterised by a shorter waiting time on construction sites; and longer at the concrete-mixing plant. At the same time, it increases the efficiency of working units compared with organisation II which has self-regulating transportation units running in closed cycle.

Keywords: queueing theory, construction, production organisation

Streszczenie

Celem artykułu jest badanie zmienności, gwarantowanych czasów urabiania mieszanki betonowej wraz ze wskazaniem rozwiązań spełniających warunek ukończenia wbudowywania każdej porcji, jeszcze przed chwilą początku wiązania cementu. Zastosowano modele teorii kolejek: $M/M/N/-/N$ – organizacja I, dla przypadku „realizacji dostaw bezpośrednio po rozładunkach” przy automatycznym przesyłaniu informacji z budowy do betonowni systemem *RFID* oraz $M/M/1/FIFO/N/F$ – organizacja II, z funkcjonowaniem jednostek transportowych w „cyklu zamkniętym” i samoregulacją ich pracy. W przykładzie betonowania przeszła estakady, przy zastosowaniu organizacji I, wskazano mniejsze czasy oczekiwania samochodów z mieszanką na budowie, a większe oczekiwania pojazdów niezaladowanych w betonowni, jak też większą wydajność zespołów wbudowujących mieszankę, w porównaniu z organizacją II.

Słowa kluczowe: teoria kolejek, budownictwo, organizacja produkcji

1. Introduction

Large distances between concrete plants and construction sites, as well as distortions in traffic of concrete mixers, cause variability in the transportation time of concrete mix. Furthermore, the time of unloading/pumping and laying the mix is not constant. Therefore, the required waiting time, both on the part of the concrete laying team operating the pump for transport of the mix, and on the part of vehicles queuing for the unloading procedure, [4, 11, 12, 16–18]. In the case of concrete laying, in the case of each of the connected mix batches, the condition must be met that concrete laying and its monolithic connection with the surrounding mix (laid earlier or later) should occur still before the beginning of cement setting [10, 17].

Standard analyses operate with site units (namely brigades or their selected parts, smaller teams, or even operators with the equipment) which, in the case of resignation from inter-operational storage, directly cooperate with the transport units, [1, 2, 14, 15]. These are, for example, excavation crews removing soil with heavy construction equipment, or gangs working with cranes erecting prefabricated elements which are continuously transported to the site [5, 6]. Usually, optimisation is oriented towards minimising the cost of the works. In the case of works involving concrete, there is a need to lay and connect the mix portion still before cement starts setting. Therefore, an important element of the analysis involves determining the time taken from mixing the cement with water at the concrete plant until such times as it is laid and along with other batches of mix on site [17, 19].

2. Typical systems in the queuing theory

The work of construction crews and transportation can be described using queuing theory models. In such models, the queuing system represents the functioning of an object rendering a service, e.g. the operation of a computer processor performing consecutive calculations, or on a construction site, the work of a concrete laying crew with a pump for the transfer of a concrete mix from vehicle-mounted concrete mixers.

Rich literature on the subject, e.g. [3, 7–9, 13], includes the analyses of many queuing systems, ‘open’ systems with an infinite source of customers, and with a source limited to N units circulating within a closed system. The processes of arrivals and servicing can be described with both typical theoretical distributions and with random distributions. There are systems with one or many parallel channels, with unlimited queues, or with ‘impatient customers’ who do not join queues that are too long or resign from service in the event of too long a waiting time.

In the case of cooperation between site teams (e.g. a crew with equipment) with transport units functioning in a closed cycle, which immediately leave and carry out the next delivery after unloading at the site, or in the case of loading material (e.g. soil) and immediately removing it and returning for another load to be removed, the $M/M/m/FIFO/N/F$ queuing system is applied. According to the notation by Kendall and Lee [8], such a model means

a queuing system with Markov process related to the circulation time (corresponding to the time where the customer remains outside the system, that is time interval from the customer's leaving the system until his return to the system), and also with Markov process related to the servicing time, with the station featuring m service channels, queue according to the FIFO model (where customers are serviced in the order of arrival), and with N units functioning in a closed cycle F . The model, therefore, assumes that the processes of circulation and servicing meet the conditions of being stationary, memoryless and independent [7–9].

Furthermore, it was adopted that the intensity of the stream of customers amounts to $\lambda = \frac{1}{\bar{t}}$, where \bar{t} refers to the average time that customers remaining outside the system, and μ refers to the intensity of the servicing stream, and where $\rho = \frac{\lambda}{\mu}$, $\rho < 1$.

According to [8], the probability of the service apparatus being idle amounts to:

The probability p_i and p_k of having i and k customers in the system amounts to [35]:

$$p_0 = \left(\sum_{i=0}^m \frac{N!}{i!(N-i)!} \rho^i + \sum_{k=m+1}^N \frac{N!}{m!(N-k)!m^{k-m}} \rho^k \right)^{-1} \quad (1)$$

$$p_i = \frac{N!}{i!(N-i)!} \rho^i p_0, \quad i=1, \dots, m, \quad (2)$$

$$p_k = \frac{N!}{m!m^{k-m}(N-k)!} \rho^k p_0, \quad k=m+1, \dots, N, \quad (3)$$

In cases where $I = 0$, the service apparatus is idle, and there is no customer in the system, while in the case of $1 \leq i \leq m$, i customers are being serviced while the queue status still equals 0. It is only in the case of $m+1 \leq k \leq N$ that m customers are serviced, while $k-m$ wait in the queue.

The average number of customers q waiting in the queue is calculated according to the following formula [3, 7–9]:

$$q = \sum_{j=0}^{N-m} j p_{m+j} = \frac{N!}{m!} p_0 \sum_{j=0}^{N-m} \frac{j}{m^j (N-m-j)} \rho^{m+j}. \quad (4)$$

3. The queuing model for orders directly after unloading

When analysing the supply and unloading of concrete mix at a building site, apart from the organisation where N concrete mixers circulate in a closed cycle, other solutions can be applied with 'ordering deliveries directly after unloading'. In order to describe the organisation with 'ordering deliveries...', one can apply the $M/M/N/-/N$ queuing system.

Let us assume that we have N transport units (concrete mixer trucks), while the time of loading the mix, transportation to the site, and unloading/pumping are independent random

variables with exponential distributions and with average values of $\frac{1}{\lambda}$ and $\frac{1}{\mu}$, respectively, whereas $\rho = \frac{\lambda}{\mu}$, $\rho < 1$.

Assume $p_j(t)$ is the probability of a situation where at time t , there is a 'surplus' of i concrete mixers ahead of the pump. Directly after unloading each delivery, another delivery is ordered, with the immediate loading of another concrete mixer at the production plant and its transport to the site. Therefore, the sum of transport units at the site and those ordered equals the maximum number N .

When analysing the graph of transitions presenting the evolution of the number of concrete mixers at the site, three characteristic cases can be observed (Fig. 1), [8, 9]:

- ▶ no surplus, 0 transport units to be unloaded, $i = 0$;
- ▶ the number of units ahead of the pump amounts to i , $i = 1, \dots, N - 1$;
- ▶ the number of units amounts to N , $i = N$.

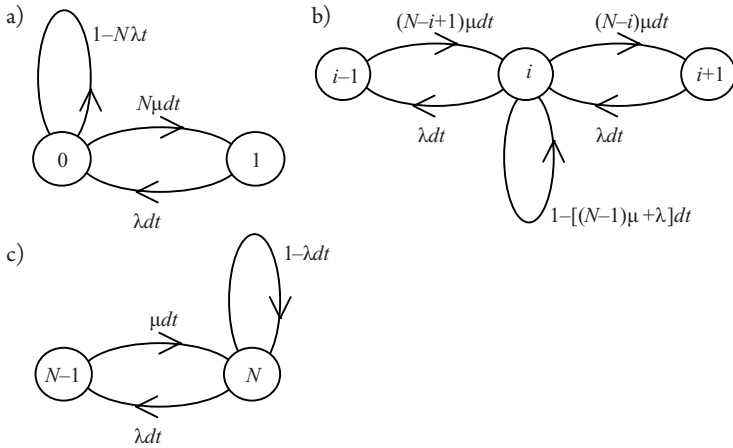


Fig. 1. Characteristic states of inventories: a – for $i = 0$; b – for $i = 1, \dots, N - 1$; c – for $i = N$, [8]

If the number of ongoing orders substitutes the surplus volume, the order of status numbering is reversed and we receive the $M/M/N/-/N$ model, namely for $m = N$ [8].

In such case, in the steady state, probability of service apparatus idle state amounts to the following:

$$p_0 = \lim_{t \rightarrow \infty} p_0(t) = \frac{\rho^N}{\sum_{j=0}^N \rho^j} \cdot \frac{N!}{j!} \tag{5}$$

The average number of concrete mixers unloaded within a time unit amounts to:

$$Q = \lambda(1 - p_0), \tag{6}$$

while the average number of transport units at the site amounts to:

$$\bar{n} = \sum_{j=1}^N j p_j, \quad (7)$$

whereas the average number of orders (loading commencements) in a time unit can be determined on the basis of the following relation:

$$\mu(N - \bar{n}) = \lambda(1 - p_o). \quad (8)$$

4. Guaranteed delivery and pumping time and direct costs

In the analysed systems, $M/M/m/FIFO/N/F$ and $M/M/N/-/N$, the operation of concrete mixer trucks and pumps is interpreted with Markov processes referring to arrivals and servicing. Furthermore, the exit process complies with exponential distribution:

$$f(t) = \lambda e^{-\lambda t}, \quad t \geq 0, \quad (9)$$

with the cumulative distribution function:

$$F(t) = 1 - e^{-\lambda t}, \quad t \geq 0. \quad (10)$$

After transformation and taking logs, the following is obtained:

$$\lambda t = \ln \frac{1}{1 - F(t)}. \quad (11)$$

Value t – the longest duration of delivery/pumping τ_{dp} can be calculated after substitution of the cumulative distribution function $F(t)$ with the required probability level to be guaranteed to observe t , and of $\frac{1}{\lambda}$ with the average number Q of concrete mixers unloaded by the pump at the site.

The most favourable number of transport units (N) for the concrete laying team with the pump can be determined by minimising the total direct costs resulting from the operation of the servicing system, and additionally, from the increase of the unit cost of the concrete mix with the increased time of the start of setting τ_{pw} (required to meet the terms of high guarantee).

The sum of the direct costs of system operation and the increased cost of the mix per load amounts to:

$$K_b = \frac{K_{bp} + \bar{n}K_b + k_m(t)}{Q}, \quad (12)$$

where, according to [16–18]:

- K_{bp} – cost per machine hour of mix pump operation, $K_{bp} = €57 / \text{h}$;
- K_{bm} – cost per machine hour of concrete mix rental, $K_{bm} = €36 / \text{h}$;
- \bar{n} – average number of customers in the system, according to (7);
- $k_m(t)$ – additional unit cost due to application of the mix with increased time of commenced setting, $t < \tau_{pw}$, $k_m(0 < t \leq 3) = 0$, $k_m(3 < t \leq 12) = (\text{€}2,22 \text{ per commenced } 0.5 \text{ hour})$,
- Q – average number of unloading procedures per hour of team operation, according to (3.4).

5. Exemplary results of calculations according to M/M/N/-/N and M/M/1/FIFO/N/F models

The example uses data acquired from the empirical measurements of concrete laying for the span of the trestle at the crossing of Wielicka and Powstańców Śląskich i Wielkopolskich Streets in Krakow, where 1057 m^3 concrete mix was used – this was delivered through the city streets in concrete mixer trucks with a nominal capacity of 8 m^3 from a plant situated 7.5 km away from the site. The following average time values were determined [16]:

- ▶ loading at the concrete plant and transport of mix to the site (31 minutes, 53 seconds), $\frac{1}{\lambda} = 0.531389 \text{ h}$;
- ▶ circulation' (according to section 2, including the passage of the vehicle for the mix, its loading and return to the site – 50 minutes, 20 seconds) $\frac{1}{\lambda^*} = 0.838885 \text{ h}$;
- ▶ pumping of the mix from each delivery (16 minutes, 44 seconds), $\frac{1}{\mu} = 0.278833 \text{ h}$.

Two solutions are analysed below:

- ▶ organization I, described by model $M/M/N/-/N$, with 'deliveries directly after unloading', where loading of the next transport unit at the plant begins immediately after completion of the unloading of the concrete mixer at the site, after automatic sending of such information, for example, using *RFID* system;
- ▶ organization II, described by model $M/M/1/FIFO/N/F$, using N transport units circulating in a closed circuit (in a 'self-regulating circuit').

5.1. Results of calculations in the case of deliveries directly after completion of unloading

For the above data: $\lambda = 3.586372$; $\mu = 1.881861$; $\rho = 1.905758$; and for $N = 1, \dots, 10$, when applying the $M/M/N/-/N$ model, according to formulas (5-7), probability was calculated of idleness of the service apparatus p_0 , average numbers of transport units at the site \bar{n} and average numbers of concrete mixers unloaded in time unit Q have been presented in Fig. 2 According to (11, 12), average values were calculated for delivery/pumping times $\bar{\tau}_{dp}$, as

well as the greatest (limit) durations $\tau_{dp,90\%}$, $\tau_{dp,95\%}$, $\tau_{dp,99\%}$ and direct costs $K_{b,90\%}$, $K_{b,95\%}$, $K_{b,99\%}$, which were not exceeded at the levels of probability of 90%, 95%, 99%, respectively.

When analysing the calculation results, the lowest direct costs (considering the cost related to increase in the time of cement setting commencement) of using three concrete mixers are recorded for the solution. In this case, the cost is minimal and amounts to $K_{b,95\%} = \text{€}38.08$ per delivery/unloading. This is with a 95% guarantee of not exceeding the duration of delivery/pumping amounting to $\tau_{dp,95\%} = 2.06$ hours (from when the cement is mixed with water at the concrete plant until the end of pumping the mix at the site). The probability of pump idleness in such a case amounts to $p_o = 0.1963$, while the average length of the queue is $q = 0.66$.

In the case of using a greater number of transport units, the probability of pump idleness intensely decreases, for example, at $N = 4$ transport units, the probability of pump idleness amounts to $p_o = 0.085$. The application of four concrete mixer trucks, however, is related to the prolongation of concrete mixer trucks waiting for unloading by, on average, 0.17 h, and to the unfavourable share of long servicing, e.g. greater than $\tau_{dp,95\%} = 2.59$ h, which then occur with the probability of 5%.

Therefore, in the existing conditions, the most favourable solution is the solution using $N = 3$ concrete mixers.

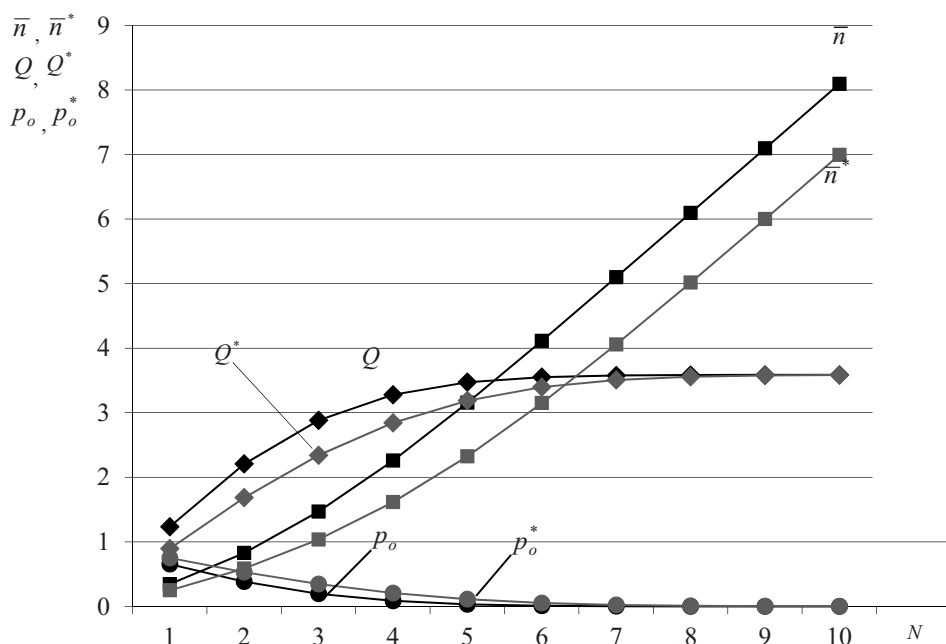


Fig. 2. Variability: p_o , p_o^* – of probabilities of pump idleness; \bar{n} , \bar{n}^* – average numbers of transport units at the site; Q , Q^* – average numbers of concrete mixers unloaded during a time unit; respectively according to models $M/M/N/-/N$ and $M/M/1/FIFO/N/F$, depending on the number N of transport units (own study)

5.2. Results relating to concrete mixer vehicles operating in closed circuits

When creating large monoliths, solutions are often applied in which N concrete mixers operate in a closed circuit – immediately after unloading, each unit drives to the concrete plant where the mix is loaded and it then returns to the site. This is an operational scheme that can be described with typical a queuing system: $M/M/1/FIFO/N/F$. In this case, for the data as above $\lambda^* = 1.192058$, $\mu^* = 3.586372$, $\rho^* = 0.332385$; the characteristics of the model were calculated according to formulas (1–4) have been presented in Fig. 4.

Calculated according to $M/M/N/-/N$ and $M/M/1/FIFO/N/F$, particular model characteristics reveal very clear differences, mainly at small N values. In the case of the most favourable solution with the application of the $M/M/N/-/N$ model (with delivery ordering directly after unloading) at $N = 3$ concrete mixers, efficiency Q has much greater values, with smaller values of pump idleness probability p_0 and lower direct costs of system operation K_b , than the values of Q^* , p_0^* , and K_b^* obtained in the model $M/M/1/FIFO/N/F$. Therefore, it is determined that in the case of creating large monoliths using concrete mixers, it is more favourable to apply the solution with deliveries directly after unloading, namely in the case of functioning according to the $M/M/N/-/N$ model, rather than applying the organisation with N transport units operating (without control) in the closed cycle, according to the $M/M/1/FIFO/N/F$ model.

6. Conclusions

The application of the queuing theory allows for analyses that consider the random variability of the duration of particular processes, in particular, modelling and analysing the cooperation of different groups of workers with transport units, in order to determine the following:

- ▶ guaranteed durations of deliveries/pumping with the assumed probabilities of their observance, as is necessary when planning the execution of concrete works where different batches of the mix must be cast on top of ('or alongside') each other before the cement starts setting.

On the basis of studies of the concrete-laying crew with a pump cooperating with concrete mixers (supplying the mix to the site via municipal roads from the concrete plant situated 7.5 km away), the following conclusions can be reached:

- ▶ it is more favourable to implement the organisation with 'deliveries directly after unloading' (where loading of the next transport unit at the plant begins immediately after completion of unloading of the concrete mixer at the site, after the automatic sending of such information, e.g. using *RFID* system), rather than the organisation using N transport units circulating in the closed circuit (in the 'self-regulating circuit');
- ▶ in such cases, the efficiency of cooperating site and transport units is greater, while the probability of pump idleness is smaller, and the direct costs of building production are lower as a result of the shorter waiting time for the unloading of vehicles with the mix at the site and the longer waiting time of unloaded concrete mixers at the plant.

References

- [1] Abdelhamid T.S., El-Gafy M., Salem O., *Lean Construction: Fundamentals And Principles*, American Professional Constructor Journal 2008.
- [2] Akhavian R., Behzadan A.H., *Evaluation of queuing systems for knowledge-based simulation of construction processes*, Automation in Construction, 7/2014, 37–49.
- [3] Asmussen S., *Applied Probability and Queues*, Springer-Verlag, New York-Berlin Heidelberg 2010.
- [4] Belniak S. et al., *Zastosowanie metod matematycznych w wybranych problemach zarządzania w budownictwie*, Wydawnictwo Politechniki Krakowskiej, Kraków 2015.
- [5] Carmichael D.G., *Shovel-truck queues: a reconciliation of theory and practice*, Construction Management and Economics, 4(2) 1986, 161–177.
- [6] Farid F., Koning T.L., *Simulation Verifies Queuing Program for Selecting Loader-Truck Fleets*, Journal of Construction Engineering and Management, 120(2) 1994, 386–404.
- [7] Filipowicz B., Dzikowska K., *Systemy i sieci kolejkowe w przykładach i zadaniach*, Wydawnictwo ABART, Kraków 2008.
- [8] Filipowicz B., *Modele stochastyczne w badaniach operacyjnych*, Wydawnictwa Naukowo-Techniczne, Warszawa 1996.
- [9] Gross D., Shortle J.F., Thompson J.M., Harris C.M., *Fundamentals of Queuing Theory*, John Wiley & Sons Inc., Hoboken, New Jersey 2008.
- [10] Jamróży Z., *Beton i jego technologie*, Wydawnictwo Naukowe PWN, Warszawa 2015.
- [11] Jaworski K., *Metodologia projektowania realizacji budowy*, Państwowe Wydawnictwo Naukowe, Warszawa 2009.
- [12] Lu M., Shen X., Chen W., *Automated Collection of Mixer Truck Operations Data in Highly Dense Urban Areas*, Journal of Construction Engineering and Management, 135(1) 2009, 17–23.
- [13] Sadeghi N., Robinson F.A., Gerami S.N., *Queue performance measures in construction simulation models containing subjective uncertainty*, Automation in Construction, 60/2015, 1–11.
- [14] Więckowski A., *Sozoeconomiczny model realizacji procesów cyklu obiektu budowlanego*, Monografia 246, Politechnika Krakowska, Kraków 1999.
- [15] Więckowski A., *Principles of the NNM method applied in the analysis of process realisation*, Automation in Construction, Elsevier Science BV, 11(4) 2002, 409–420.
- [16] Więckowski A., *System bezmagazynowej pracy jednostek budowy i transportu*, Politechnika Krakowska, Monografia 375, Kraków 2010.
- [17] Więckowski A., *Transport mieszanek betonowej*, Wydawnictwo Politechniki Krakowskiej, Kraków 2013.
- [18] Więckowski A., *The analysis of the concrete mix delivery organisation to the construction site*, Creative Construction Conference, Budapest, Hungary 2013, 472–481.
- [19] Więckowski A., Skibniewski M., *Errors of Calculations in M/M/1/FIFO/N/F Model with Limited Duration of Shift*, The 25th International Symposium on Automation and Robotics in Construction, Vilnius 2008, 708–716.

