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Optimal design of structural elements as a control theory problem

Optymalne kształtowanie elementów konstrukcyjnych jako problem optymalnego sterowania

Abstract

In the paper an application of the maximum principle to designing a cross section of a still frame under several loading schemes is presented. The optimal height of the web under minimum volume of still as a cost function is determined. In particular the implicit and explicit conditions of state variables at characteristic points of axis of symmetry under different loading schemes are presented.

Keywords: optimal control, maximum principle

Streszczenie

W artykule przedstawiono zastosowanie zasady maksimum w wymiarowaniu przekroju poprzecznego stalowej ramy poddanej wielu stanom obciążenia, polegającego na wyznaczeniu optymalnej, ze względu na minimum objętości stali, wysokości środnika. Szczegółowo przedstawiono jawne i uwikłane warunki zmiennych stanu w punktach charakterystycznych oraz w osi symetrii, a także sformułowania prowadzące do uwzględnienia kombinacji obciążeń.

Słowa kluczowe: sterowanie optymalne, zasada maksimum

1. Introduction

Optimisation of complex structural systems, which until recently remained in the domain of basic research, has now become a practical fact owing to the development of numerical methods on the one hand and computer software on the other. Optimal control theory has become an extremely valuable theory that can be used effectively as a tool for solving important problems of various scientific disciplines, including structural engineering. Some of the results obtained are presented in this paper.

Optimal control theory has been applied to the field of optimal design of structures with the use of the maximum principle. The principle is related to the onerous character (which until recently had been an unsurmountable obstacle) of the so-called multi-point boundary value problem formulated with reference to sets of ordinary differential equations. The specific nature of tasks faced by structural engineering manifests itself in the need to formulate not only the initial-boundary conditions, but also to take into account the internal point conditions.

At present, owing to effective numerical algorithms, complemented by the authors' own suggestions, particularly those referring to non-analytical objective functions and restricting the number of characteristic intervals as well as taking account of the secondary conditions of the *maximum* type listed in the Dircol-2.1 computer software, it is possible to undertake new and unconventional tasks related to optimisation – significant both from the perspective of the expansion of knowledge and the possible applications.

2. The subject of optimisation

The article presents the formulation of the problem of finding the optimal design of a steel I-frame subjected to various loads and the subsequent solution to this problem. The frame to be optimised is a load-bearing element of a hall. The static diagram of the frame (Fig. 1) results from the structure of the building and the manner of support.

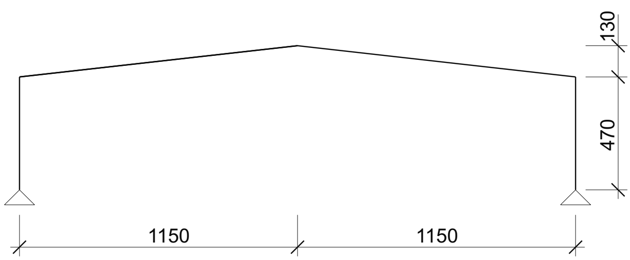


Fig. 1. The static diagram of the frame

The optimisation process will determine the course of variability of web height, whereas the dimensions of the remaining parts of the I-frame will remain constant (Fig. 2).

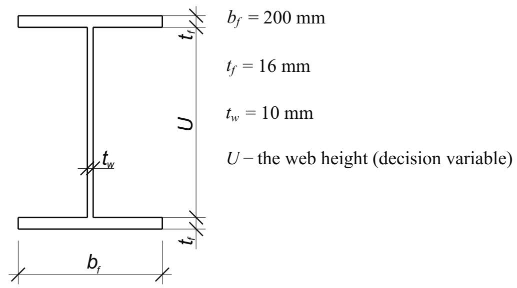


Fig. 2. Cross-section of the frame

3. Loads

In the process of designing a frame cross-section all the relevant computational situations that may occur during the service of the structure in which it is to be used should be taken into account. The computational situation should be understood as a specific combination of individual load systems – hereinafter called elementary loads. The number of elementary loads determines the number of state equations describing the structure and affects considerably the scope of the optimisation task. In the problem under consideration here the following elementary loads have been taken into account:

Load 1. frame dead load (maximum),
Load 2. roofing dead load (maximum),
Load 3. frame dead load (minimum),
Load 4. roofing dead load (minimum),
Load 5. snow load,
Load 6. wind load – wind from the left,
Load 7. wind load – wind from the right.

Symbols of the above elementary loads in the normal (perpendicular) and tangential direction to the frame axis have been listed in Table 1.

The static and kinematic values describing the structure are linearly dependant on the loads, which is why the corresponding values in the considered combinations may be aggregated. Unlike the number of elementary loads, the number of combinations does not affect significantly the scope of the optimisation task. This problem will be discussed further on in the paper. Twelve loads combinations have been considered, which are presented in Table 2.

Table 1. Elementary loads in the characteristic interspaces

Load		**Left pillar**	**Left lintel**	**Right lintel**	**Right pillar**
1		2	3	4	5
Load 1	q_{1t}	0	$\gamma_{\max} A \gamma_s \cos\alpha$	$\gamma_{\max} A \gamma_s \cos\alpha$	0
	q_{1n}	$-\gamma_{\max} A \gamma_s$	$-\gamma_{\max} A \gamma_s \sin\alpha$	$\gamma_{\max} A \gamma_s \cos\alpha$	$\gamma_{\max} A \gamma_s$
Load 2	q_{2t}	0	$q_{p.\max} \cos\alpha$	$q_{p.\max} \cos\alpha$	0
	q_{2n}	$-q_{p.\max}$	$-q_{p.\max} \sin\alpha$	$q_{p.\max} \sin\alpha$	$q_{p.\max}$

cd. Tab. 1

1		2	3	4	5
Load 3	q_{3t}	0	$\gamma_{\min} A \gamma_s \cos\alpha$	$\gamma_{\min} A \gamma_s \cos\alpha$	0
	q_{3n}	$-\gamma_{\min} A \gamma_s$	$-\gamma_{\min} A \gamma_s \sin\alpha$	$\gamma_{\min} A \gamma_s \cos\alpha$	$\gamma_{\min} A \gamma_s$
Load 4	q_{4t}	0	$q_{p.\min} \cos\alpha$	$q_{p.\min} \cos\alpha$	0
	q_{4n}	$-q_{p.\min}$	$-q_{p.\min} \sin\alpha$	$q_{p.\min} \sin\alpha$	$q_{p.\min}$
Load 5	q_{5t}	0	$q_s \cos^2\alpha$	$q_s \cos^2\alpha$	0
	q_{5n}	0	$-q_s \cos\alpha \sin\alpha$	$q_s \cos\alpha \sin\alpha$	0
Load 6	q_{6t}	$q_{w.sn}$	$-q_{w.pn}$	$-q_{w.pz}$	$-q_{w.sz}$
	q_{6n}	0	0	0	0
Load 7	q_{7t}	$-q_{w.sz}$	$-q_{w.pz}$	$-q_{w.pn}$	$q_{w.sn}$
	q_{7n}	0	0	0	0

q_{it} – load in the normal (perpendicular) direction towards the axis.
q_{in} – load in the tangential direction towards the axis.

Table 2. List of loads combinations

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
Load 1	+	+	+	+	+	+						
Load 2	+	+	+	+	+	+						
Load 3							+	+	+	+	+	+
Load 4							+	+	+	+	+	+
Load 5		+	+	+				+	+	+		
Load 6			+		+				+		+	
Load 7				+		+				+		+

4. Formulation of the optimisation task

The essence of the optimisation task presented in this article is finding the optimal design of the frame cross-section with the variable web height. The task comprises determination of the course of variability of web height so that the serviceability and the load bearing capacity limits are not exceeded. There are an infinite number of solutions fulfilling this requirement, yet of all the possible designs only the one which is characterised by the lowest objective function value may be considered optimal. The objective function in this task is the volume of steel needed for the frame, which in the optimal solution should be possibly the smallest.

The maximum principle allows formulation of the problem which makes it possible to obtain the only solution meeting the necessary conditions of the optimisation. The selection of the solution of all the obtained ones which is the best from the perspective of the adopted objective function does not guarantee that it is optimal because the set of obtained solutions may not include the optimal solution at all. The tasks of optimal cross-section design in structural engineering are characterised by so many constraints resulting from technical, design and standard requirements that the set of the solutions meeting the optimisation necessary conditions is not large and often contains only one solution with the lowest objective function value, i.e. optimal.

The frame to be optimised is symmetrical both in the aspect of its geometry and the loads to which it is subjected. However, experience shows – and the problem in question here is no exception – that the solutions obtained may be symmetrical or not. In this task the non-symmetrical solution proved to be "better" in terms of the objective function value than the symmetrical one. However, due to the fact that that the presented problem is of practical character and that the non-symmetrical solution is not very likely to be accepted by an investor, it has been decided that the formulation of the problem should include the condition for the solution to be symmetrical. In fact, the condition boils down to the adoption of the half-frame model with state variables in the axis of symmetry corresponding to the subsequent loads.

The formal structure of optimisation problems with the application of the maximum principle has been discussed in the cited publications on the subject. This article will only present the detailed formulations related to the problem under consideration, in compliance with the formalism of the minimum principle, which encompasses: equations of state, conjugate equations, objective function, Hamilton's function and functions of constraint.

5. Equations of state

As discussed above, it has been decided to adopt the half-frame model (Fig. 3). The independent variable x is measured from the bottom of the left pillar up to joint 2 and next horizontally to the axis of symmetry. The equations of state are formulated in two characteristic interspaces (Table 3), taking into account the state variables at the support, joint 2 and at the roof ridge (Tables 4 and 5).

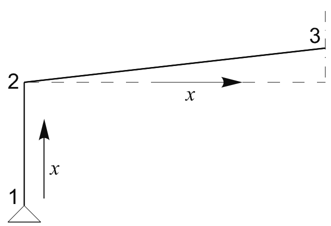


Fig. 3. The half-frame model

In the case of symmetrical loads, the conditions for static and kinematic values at the ridge (point 3) are formulated similarly to a vertically sliding fixed joint. If the elementary load is not symmetrical, as is the case of loads 6 and 7, a corresponding load of the "mirror reflection" type is introduced. The left half of the frame bearing load 6 is a mirror reflection of the right half of the frame bearing load 7 and *vice versa*. The corresponding conditions for state variables at a point in the frame's axis of symmetry result from the above.

Table 3. Equations of state in the characteristic interspaces

State variables		**Equations of state**	
		Pillar	**Lintel**
Load i, $(i = 1-7)$	v_i	$v_i' = \varphi_i$	$v_i' = \varphi_i / \cos\alpha$
	φ_i	$\varphi_i' = M_i / EI$	$\varphi_i' = M_i / (EI \cos\alpha)$
	M_i	$M_i' = Q_i$	$M_i' = Q_i / \cos\alpha$
	Q_i	$Q_i' = -q_{it}$	$Q_i' = -q_{it} / \cos\alpha$
	N_i	$N_i' = -q_{in}$	$N_i' = -q_{in} / \cos\alpha$
	w_i	$w_i' = N_i / EA$	$w_i' = N_i / (EA \cos\alpha)$
V		$V' = A$	$V' = A / \cos\alpha$

v – normal displacement
φ – deflection angle
M – bending moment
Q – transverse force
N – longitudinal force
w – tangential displacement
q – load
α – the lintel inclination angle

The frame to be optimised is described by the total of 43 equations of state – 6 for each of the 7 loads plus one equation describing the volume, introduced because of the adopted objective function.

6. Boundary conditions and internal point conditions of the state variables

If the frame retains the same static diagram when subjected to several load conditions, the state variables conditions in each of these loads are the same. Therefore it is only necessary to present the state variables conditions for one symmetrical load and for two "mirror reflection" non-symmetrical loads, and they are presented below. In both cases the state variables conditions may be divided into two groups: explicit conditions and implicit conditions.

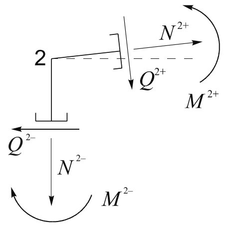
The number of the necessary state variables conditions is equal to the number of equations multiplied by the number of characteristic intervals. Thus, for one load condition described with six equations in two characteristic intervals, the number of conditions that should be formulated amounts to 12.

The explicit conditions for the symmetrical loads have been listed in Table 4, whereas the implicit conditions result from Figures 4, 5, 6 and 7.

Table 4. Explicit conditions for state variables at characteristic points for symmetrical loads

	1	**2**		**3**
		2⁻	**2⁺**	
v_i	0			
φ_i		C		0
M_i	0	C		
Q_i				
N_i				
w_i	0			
Symbols: C – condition of continuity, 0 – predefined value				

The conditions of equilibrium at joint 2:

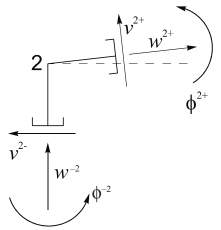


$$\sum X = 0: \qquad -Q^{2-} + Q^{2+}\sin\alpha + N^{2+}\cos\alpha = 0$$

$$\sum Y = 0: \qquad -N^{2-} - Q^{2+}\cos\alpha + N^{2+}\sin\alpha = 0$$

Fig. 4. Sectional forces at joint 2

The conditions of displacements compatibility at joint 2:

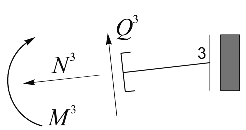


$$\delta_X^{2-} = \delta_X^{2+}: \qquad -v^{2-} = -v^{2+}\sin\alpha + w^{2+}\cos\alpha$$

$$\delta_Y^{2-} = \delta_Y^{2+}: \qquad w^{2-} = v^{2+}\cos\alpha + w^{2+}\sin\alpha$$

Fig. 5. Displacements at joint 2

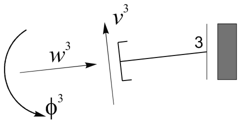
The condition of forces equilibrium at joint 3:



$$\sum Y = 0: \qquad Q^3\cos\alpha - N^3\sin\alpha = 0$$

Fig. 6. Sectional forces at joint 3

The condition of displacements compatibility at joint 3:



$$\delta_X^3 = 0: \qquad -v^3 \sin\alpha + w^3 \cos\alpha = 0$$

Fig. 7. Displacements at joint 3

The following conditions have been formulated for each symmetrical load:

- 6 explicit conditions (Table 4),
- 2 implicit conditions of forces equilibrium at point 2,
- 2 implicit conditions of displacements compatibility at point 2,
- 1 implicit condition of forces equilibrium at point 3,
- 1 implicit condition of displacements compatibility at point 3.

The total of 12 conditions have been formulated for each symmetrical load.

As regards the non-symmetrical loads (loads 6 and 7), the total number of 24 conditions must be formulated. The explicit conditions have been listed in Table 5. The implicit conditions at joint 2 are in this case the same as the ones related to the symmetrical loads, whereas the conditions at point 3 require discussion in greater detail.

Table 5. Explicit conditions for state variables at characteristic points for non-symmetrical loads 6 and 7

	1	**2**		**3**
		2⁻	**2⁺**	
v_6	0			
φ_6		C		
M_6	0	C		
Q_6				
N_6				
w_6	0			
v_7	0			
φ_7		C		
M_7	0	C		
Q_7				
N_7				
w_7	0			

The roof ridge joint equilibrium condition is to have the total forces and total moments acting on both sides of the joint neutralise each other to zero.
The following occurs in direction X:

$$S_{6X}^{3-} + S_{6X}^{3+} = 0$$

If we adopt the half-frame model, the component in direction X on the left side of the joint is subject to the following relations:

$$S_{6X}^{3-} = Q_{6X}^{3-} + N_{6X}^{3-} = Q_6^{3-}\sin\alpha + N_6^{3-}\cos\alpha = Q_6^3\sin\alpha + N_6^3\cos\alpha\,.$$

The following relations occur between sectional forces operating under load 6 and also under load 7, which is the mirror reflection of load 6 (Fig. 8):

$$S_{6X}^{3+} = Q_{6X}^{3+} + N_{6X}^{3+} = Q_6^{3+}\sin\alpha - N_6^{3+}\cos\alpha = -Q_7^3\sin\alpha - N_7^3\cos\alpha$$

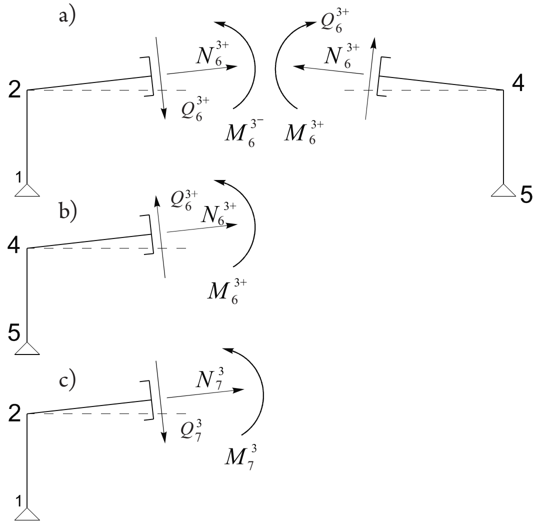


Fig. 8. Sectional forces at the roof ridge: a) in the real diagram for load 6, b) in the mirror reflection of the right part of fig. a), c) in the left part of the frame subjected to load 7, which is a mirror reflection of load 6

Hence, the relation between the transverse and longitudinal forces operating under load 6 and 7 may be written in the following form:

$$S_{6X}^{3-} + S_{6X}^{3+} = 0 \rightarrow Q_6^3\sin\alpha + N_6^3\cos\alpha - Q_7^3\sin\alpha - N_7^3\cos\alpha = 0\,.$$

The following relations occur in the vertical direction:

$$S_{6Y}^{3-} + S_{6Y}^{3+} = 0$$
$$S_{6Y}^{3-} = Q_{6Y}^{3-} + N_{6Y}^{3-} = -Q_6^{3-}\cos\alpha + N_6^{3-}\sin\alpha = -Q_6^3\cos\alpha + N_6^3\sin\alpha$$
$$S_{6Y}^{3+} = Q_{6Y}^{3+} + N_{6Y}^{3+} = Q_6^{3+}\cos\alpha + N_6^{3+}\sin\alpha = -Q_7^3\cos\alpha + N_7^3\sin\alpha$$

Hence, the next joint equilibrium condition under load 6 and 7 takes the following form:

$$S_{6Y}^{3-} + S_{6Y}^{3+} = 0 \rightarrow -Q_6^3\cos\alpha + N_6^3\sin\alpha - Q_7^3\cos\alpha + N_7^3\sin\alpha = 0$$

From Fig. 8 also stems the condition that the moments must be equal to each other:

$$M_6^3 = M_7^3$$

The next three conditions are formulated for the kinematic values (Fig. 9).

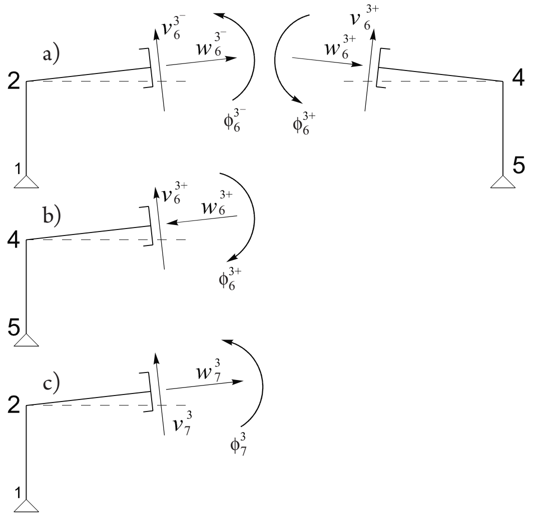


Fig. 9. Kinematic values at the roof ridge: a) in the real diagram for load 6, b) in the mirror reflection of the right part of fig. a), c) in the left part of the frame subjected to load 7, which is a mirror reflection of load 6

The conditions of displacements compatibility hold in this case (Fig. 9).
The following occurs in horizontal direction:

$$u_{6X}^{3-} = u_{6X}^{3+}$$
$$u_{6X}^{3-} = v_{6X}^{3-} + w_{6X}^{3-} = -v_6^{3-} \sin\alpha + w_6^{3-} \cos\alpha = -v_6^3 \sin\alpha + w_6^3 \cos\alpha$$
$$u_{6X}^{3+} = v_{6X}^{3+} + w_{6X}^{3+} = v_6^{3+} \sin\alpha + w_6^{3+} \cos\alpha = v_7^3 \sin\alpha - w_7^3 \cos\alpha,$$

which gives rise to the following condition:

$$u_{6X}^{3-} = u_{6X}^{3+} \rightarrow -v_6^3 \sin\alpha + w_6^3 \cos\alpha = v_7^3 \sin\alpha - w_7^3 \cos\alpha$$

The following occurs in the vertical direction:

$$u_{6Y}^{3-} = u_{6Y}^{3+}$$
$$u_{6Y}^{3-} = v_{6Y}^{3-} + w_{6Y}^{3-} = v_6^{3-} \cos\alpha + w_6^{3-} \sin\alpha = v_6^3 \cos\alpha + w_6^3 \sin\alpha$$
$$u_{6Y}^{3+} = v_{6Y}^{3+} + w_{6Y}^{3+} = v_6^{3+} \cos\alpha - w_6^{3+} \sin\alpha = v_7^3 \cos\alpha + w_7^3 \sin\alpha,$$

which gives rise to the following condition:

$$u_{6Y}^{3-} = u_{6Y}^{3+} \rightarrow v_6^3 \cos\alpha + w_6^3 \sin\alpha = v_7^3 \cos\alpha + w_7^3 \sin\alpha$$

As regards the angle of rotation, the following relations occur:

$$\varphi_6^{3-} = \varphi_6^{3+}, \qquad \varphi_6^{3-} = \varphi_6^{3}, \qquad \varphi_6^{3+} = -\varphi_7^{3},$$

hence the condition:

$$\varphi_6^{3-} = \varphi_6^{3+} \rightarrow \varphi_6^{3} = -\varphi_7^{3}$$

As regards loads 6 and 7, the following conditions have been formulated:

- 10 explicit conditions (Table 5),
- 4 implicit conditions of forces equilibrium at point 2,
- 4 implicit conditions of displacements compatibility at point 2,
- 3 implicit conditions of forces equilibrium at point 3,
- 3 implicit conditions of displacements compatibility at point 3.

A total of 24 conditions have been formulated as regards loads 6 and 7.

The suggested formulation of conditions in the axis of symmetry referring to the "mirror reflection" loads, with the prior adoption of the half-frame static diagrams, significantly reduces the scope of the optimal design tasks and extends the range of control theory applications in structural computations.

7. Constraints

At the stage of state variables and decision variables constraints formulation, certain dependencies specified in the technical provisions and related to the load bearing capacity and serviceability limits must be taken into account. Provided that the static diagram has been predefined and the cross-section geometric characteristics remain constant, the values which depend linearly on the load may be aggregated. The said values include static and kinematic state variables. It is therefore possible to introduce – at the stage of formulating the constraints – the combinations of stresses which were identified in the preliminary analysis, e.g. the normal stress in a given combination is the sum of the stresses exerted by the elementary loads which are components of this combination. In the problem under consideration here, the introduced constraints refer solely to normal stresses and deflections. It is a certain simplification of the constraints related to the load bearing capacity and serviceability limits as they are defined in the requirements of the standards, yet it does not obscure the problem of constraints discussed in this paper and enables a clear presentation of how the functions of the *maximum* or *minimum* type could be applied in structural optimisation for the purpose of reducing the number of constraints. It must be emphasised simultaneously that taking into account the constraints functions compliant with the requirements of the standards now in effect does not pose any problem in the optimisation process.

Normal stress in a given combination of loads may be determined on the basis of the following relations:

$$\sigma_{Ki} = \frac{\left| \sum_{j(Ki)} M_{j(Ki)} \cdot z \right|}{I} + \frac{\left| \sum_{j(Ki)} N_{j(Ki)} \right|}{A},$$

where $\sum_{j(Ki)} M_{j(Ki)}$, $\sum_{j(Ki)} N_{j(Ki)}$ are sums in the sets of axial moments and forces exerted by the elementary loads in combination K_i. For example, in combination K_3, $j(K_3) = \{1, 2, 5, 6\}$. In this way an expression has been formulated for the maximum edge stress for each of the twelve combinations. However, all of them should be lower than the highest permissible stress level. The above approach leads, without any special operations, to the formulation of 12 constraint functions, which complicates solving the optimisation problem. Application of the *maximum* function of the form: $\sigma_{max} = \max\left(\left\{\sigma_{K1}, \dots \sigma_{K12}\right\}\right)$ makes it possible to reduce the number of the constraints functions to one function g_1 in the form: $\sigma_{perm} - \sigma_{max} \geq 0$. This operation, however simple it may seem, is an original achievement in the field of engineering structures optimisation with the use of optimal control theory.

Similarly, in order to accommodate the serviceability limit, another constraint function has been formulated referring to the maximum normal displacement:

$$y_{Ki} = \left|\sum_{j(Ki)} v_{j(Ki)}\right|$$

$$y_{max} = \max\left(\left\{y_{K1}, \dots y_{K12}\right\}\right)$$

$$g_2 : \qquad y_{perm} - y_{max} \geq 0$$

8. Formulation of the optimisation necessary conditions

The maximum principle formalism has been applied with regard to the frame to be optimised, and in particular:

1. The structure to be optimised has been described with the use of equations of state of the following type:

$$y_i' = f_i[\tilde{y}(x), U(x), x] \quad i = 1 \dots 43 \qquad \text{(Table 3)}$$

2. State variables constraints have been formulated:

$$g_j(\tilde{y}(x), U(x)) \geq 0 \quad j = 1, 2$$

3. Hamilton's function has been formulated:

$$H = \sum_{i=1}^{43} \lambda_i \cdot f_i[\tilde{y}(x), U(x), x] + \sum_{j=1}^{2} \mu_j \cdot g_j[\tilde{y}(x), U(x)]$$

4. Hamilton's function has been used as the basis for writing the set of differential equations with conjugate variables:

$$\lambda_i' = -\frac{\partial H}{\partial y_i} \quad i = 1 \ldots 43$$

If the formal requirements specified in points 1–4 have been fulfilled, the maximum principle enables formulation of the optimisation necessary condition, namely: of all the possible solutions, which are the decision variables functions and their corresponding state variables and conjugate variables functions, the optimal solution is the maximum of Hamilton's function. The necessary condition of the optimisation, expressed in words above, may be written in the following form:

$$H[\tilde{\lambda}_{opt}(x), \tilde{\mu}_{opt}(x), \tilde{y}_{opt}(x), U_{opt}(x)] = \max_U H[\tilde{\lambda}(x), \tilde{\mu}(x), \tilde{y}(x), U(x)],$$

from which stems the following equation: $\frac{\partial H}{\partial U} = 0.$ allowing determination of decision variable U, provided that no constraints are active. Otherwise, the decision variable is determined from the active constraint.

9. Numerical solution

Application of the maximum principle has allowed formulation of a differential-algebraic set of equations in categories used in control theory, complete with boundary conditions and internal point conditions. The set constitutes the so-called multi-point boundary value problem, which has been solved with the use of the Dircol-2.1. computer software. Of all the values occurring in the formalism of the maximum principle, determination of the decision variable is particularly important from the point of view of the constructor, which is in this case the height of the I-frame web as well as the 43 state variables y_i, in which the frame remains within the limits of the load bearing capacity and serviceability. It has appeared that of all the possible options only the constraint resulting from the serviceability limit is active at a point (Fig. 10). Hence, it follows that, apart from the intervals in which the web height

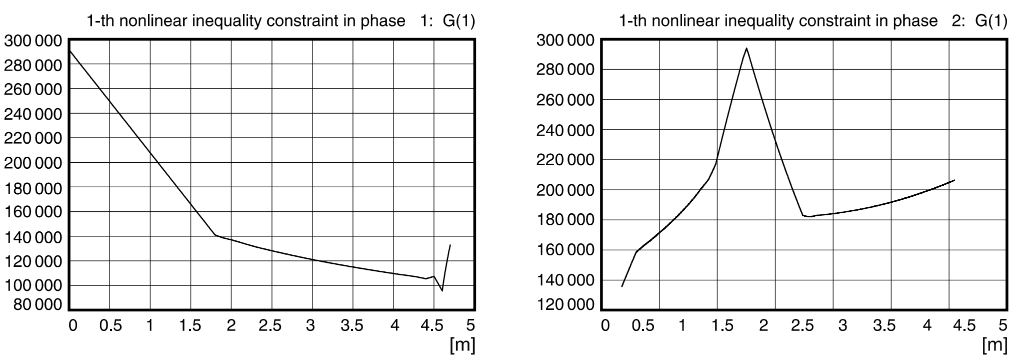


Fig. 10. The course of constraint $g1$ in characteristic interspaces [kPa]

reaches extreme values, the solution has been derived from equation $\frac{\partial H}{\partial U} = 0$. The diagrams presenting constraints g_1 and g_2 (Fig. 10 and 11) are to be found below, and so are: the optimal course of the decision variable (Fig. 12), the Hamilton function (Fig. 13) and the course of state variables in the optimal solution for load 1 (Fig. 14).

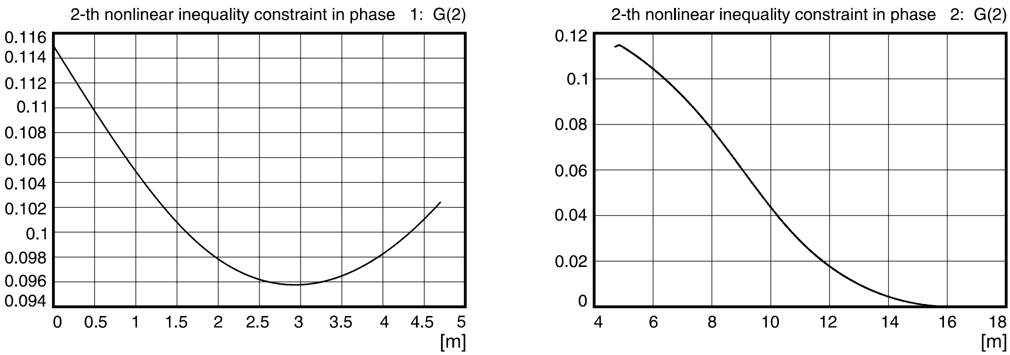


Fig. 11. The course of constraint $g2$ in characteristic interspaces [m]

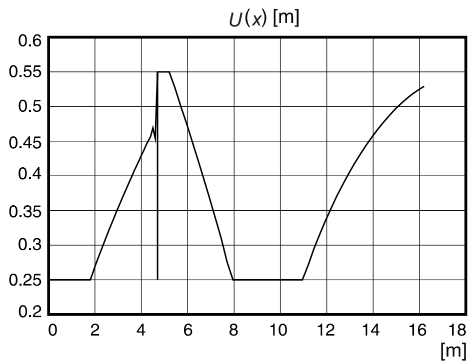


Fig. 12. Optimal course of the decision variable

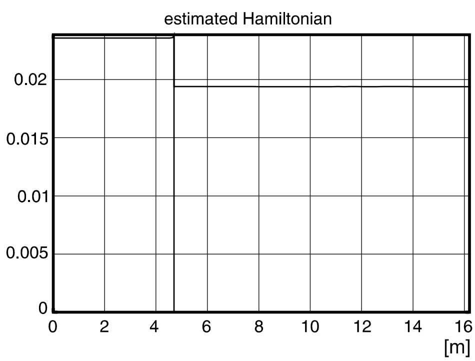


Fig. 13. The Hamilton function

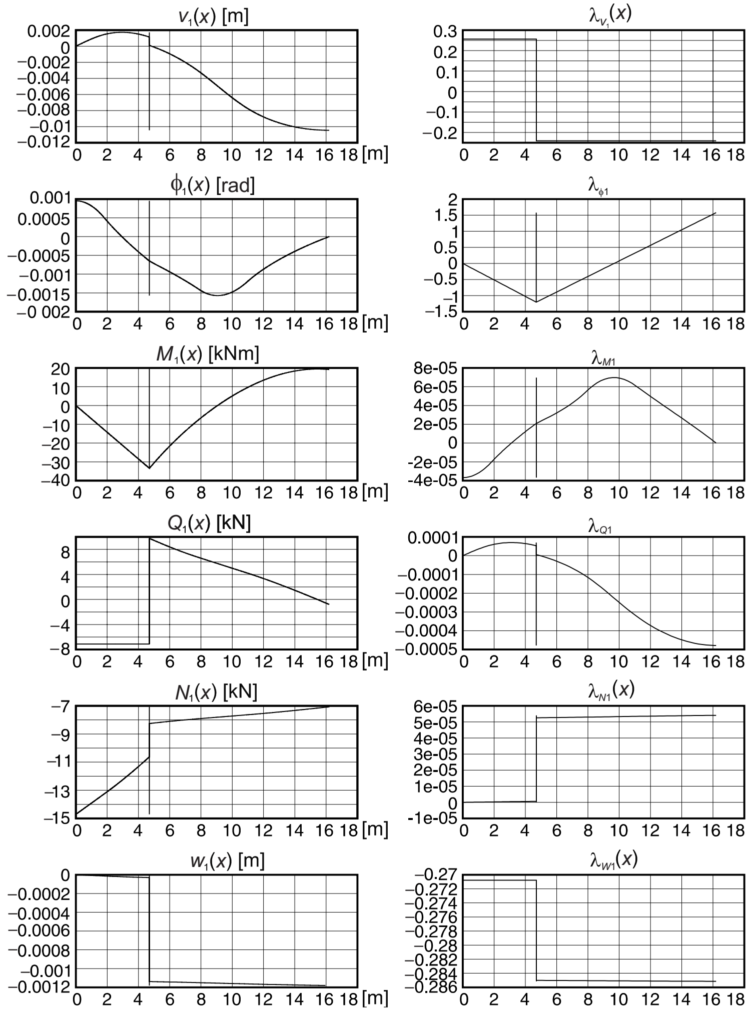


Fig. 14. State variables and conjugate variables in the situation of subjecting the frame to the first elementary load

Additionally, all the remaining values occurring in the formulated multi-point boundary value problem have also been determined.

10. Conclusions

Within the framework of the research on application of theory of optimal control based on the maximum principle in the practical design of engineering structures, a number of problems have been solved, making it possible to tackle several important design problems in the process of structure optimisation.

One of such problems is the modelling – in terms of control theory – of structures which are symmetrical in the aspects of their geometry and systems of loads yielding acceptable in practice, symmetrical distributions of decision variables. A particular emphasis has been placed in the paper on the problem of formulating the conditions for state variables at characteristic points, including at the structure's axis of symmetry. Modelling the structure with the use of the half-frame model reduces the number of characteristic interspaces by half, which considerably facilitates, or even in certain cases enables, solving the optimisation problem, which would be difficult or impossible to solve if the real model were to be used.

Another important problem solved with the use of optimal control theory in structure optimisation is the introduction of the combination of loads into the mathematic model, which has been presented in this paper. The proposed approach, consisting in the application of the *maximum/minimum* functions, enables taking account in the mathematic model of any number of load combinations, without expanding the scope of the optimisation task.

The paper presents only those computational results which may be of interest for a constructor. Due to the character of the journal, numerous details related to very specialist problems from the field of control theory have been disregarded. For the same reason not all the results obtained in the solution of the multi-point boundary value problem formulated in the task have been presented.

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