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THE PATHS OF MATHEMATICS IN LITHUANIA: BISHOP BARANAUSKAS (1835–1902) AND HIS RESEARCH IN NUMBER THEORY

ŚCIEŻKI MATEMATYKI NA LITWIE: BISKUP BARANOWSKI (BARANAUSKAS) (1836–1902) I JEGO BADANIA Z TEORII LICZB

Abstract

Bishop Antanas Baranauskas is a prominent personality in the history of the Lithuanian culture. He is well known not only as a profound theologian, a talented musician creating hymns, a literary classicist and an initiator of Lithuanian dialectology, but also as a distinguished figure in the science of mathematics. The author of this article turns his attention to the mathematical legacy of this prominent Lithuanian character and aspires to reveal the circumstances that encouraged bishop Antanas Baranauskas to undertake research in mathematics, to describe the influence of his achievements in the science of mathematics, to show the incentives that encouraged him to pursue mathematical research in Lithuania as well as to emphasize his search for a connection between mathematics and theology.

Keywords: *number theory*, *geometry*, *infinity and theology*

Streszczenie

Biskup Antoni Baranowski (Antanas Baranauskas) należy do prominentnych osobistości w historii kultury litewskiej. Znany jest jako istotny teolog, utalentowany muzyk tworzący hymny, literacki klasyk i inicjator dialektologii Litwy; zajmował się również matematyką. W artykule przedstawiono okoliczności, w związku z którymi biskup A. Baranowski zajął się badaniami matematycznymi. Zarysowane zostało znaczenie jego wyników w rozwoju badań matematycznych na Litwie. Podkreślono również związki pomiędzy matematyką i teologią.

Słowa kluczowe: *teoria liczb*, *geometria*, *nieskończoność i teologia*

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1. Introduction

Bishop Antanas Baranauskas (Antoni Baranowski, 1835–1902) is a prominent personality in the history of the Lithuanian culture. He is well known not only as a profound theologian, a talented musician creating hymns, a literary classicist and an initiator of Lithuanian dialectology, but also as a distinguished personality in the science of mathematics.

Fig. 1. Bishop Antanas Baranauskas (Antoni Baranowski, 1835–1902), Kaunas, end of XIX. Source: Vilnius University Library

There were a number of authors who had written about the bishop's merits to mathematics in Lithuania. One of the most important of the authors was a writer and an encyclopedist prelate Aleksandras Jakštas-Dambrauskas (1860–1938). He knew the bishop personally and in 1906 wrote a journal article *Bishop Antanas Baranauskas as a Mathematician* [6, 7]. This article, which also appeared as a separate publication, was based exclusively on the correspondence between the author and the bishop. It presented the beginnings of Baranauskas' interests in mathematics through his articles that discuss various aspects of number theory and geometry as well as the concept of infinity in the context of the philosophy of mathematics.

Another author and a priest, Juozas Tumas-Vaižgantas (1869–1933), published a book about Baranauskas, as a writer when printing in the Lithuanian language was prohibited. The book was based on his Lithuanian literature lectures and included a chapter called *Sins of Mathematics* [16], where he described the bishop's biggest achievements in the field of mathematics.

Viktoras Biržiška (1886–1964), a professor of mathematics in the interwar Lithuania, acknowledged Baranauskas's achievements in mathematics by writing an article about him for the Lithuanian encyclopedia [5].

In 1970s Baranauskas' creative works, including those on mathematics, attracted interest of many researchers again. Regina Mikšytė (1923–2000), a researcher in literature, dedicated a big part of her life to studying the creative legacy of this prominent Lithuanian figure. She published a monograph where she also briefly discussed the bishop's activities in mathematics. In 1993, the year of revival, the book was improved, augmented and published again [12, 13].

The first people after the Second World War to return to the discussion of Baranauskas' works in mathematics for the Lithuanian reader were Petras Rumšas (1921–1987), a specialist in the didactics of mathematics, and Aleksandras Baltrūnas (1949–2005), an expert in the history of mathematics, who published a number of articles in the Lithuanian press [14, 4].

In 1985 the celebration of the 150th anniversary of Baranauskas' birthday raised a new wave of inquiries into the bishop's life and work. The writer Rapolas Šaltenis (1908–2007) published the book *Our Baranauskas* in which the chapter "The Inferno and the Swallow in Mathematics" recounted the reasons for the bishop's interest in mathematics [15]. The academician Jonas Kubilius (1921–2011), after having researched archival material and various texts on mathematics, prepared a scientific study which he first published in the book *Literature and Language*. The same study was later published as a collection of separate articles and finally, after some modificatios in 2001 it was issued as a special edition *Antanas Baranauskas and Mathematics* [8–10]. The distinction of this study is a professional evaluation of the bishop's mathematical legacy by a mathematician who worked on number theory. Eugenijus Manstavičius, a mathematician at Vilnius University, also wrote a number of articles about Antanas Baranauskas and his achievements in mathematics [11].

It is evident that in the 20th century Antanas Baranauskas and his works in mathematics were given quite a lot of attention. The articles published by various mathematicians evaluated his mathematical achievements and defined his contribution to the science of mathematics.

The author of this article turns his attention to the mathematical legacy of this prominent Lithuanian figure and aspires to reveal the circumstances that encouraged bishop Antanas Baranauskas to undertake research in mathematics, to describe the influence of his achievements in the science of mathematics, to show the incentives that encouraged him to pursue mathematical research in Lithuania as well as to emphasize his search for a connection between mathematics and theology.

2. Mathematics and Theology

The future bishop showed an inclination to mathematics already in his childhood. One of his first biographers J. Daubaras wrote, that "Antanukas (Little Antanas) was good at school, especially at *sums*" [7, p. 7-8] (i.e. mathematics). The basic arithmetic operations like counting to a big number, addition, subtraction and multiplication he learned at Anykščiai parochial school; other parts of mathematics he learned independently. His fascination with mathematics from early years can also be illustrated by his determination to solve a mathematical problem that he heard in the parochial school: "you have 100 rubles and you need to buy 100 animals. How many bulls, cows and calves can you buy if a bull costs 10 rubles , a cow 5 rubles and a calf half a ruble?" [16, p. 66]. It took him 2 weeks to solve the problem but he did it. Later "(…) he learned algebra at the Academy (Saint Petersburg Roman Catholic Theological Academy – J.B.) and from Thomas Aquinas"[1, p. 3]. He maintained his interest in mathematics through his studies of theology at the Catholic universities in Munich, Rome and Louvain. After forty years , in 1884, when Baranauskas became a suffragan bishop of Samogitia and settled in Kaunas, he turned to a deeper study of mathematics. He immersed himself in the world of mathematical calculations after having broadened his knowledge in algebra by reading textbooks by Russian authors such as Konstantin Burenin (К.П. Буренин, 1836–1882), Alexander Malinin (А.Ф. Малинин, 1835–1888) and August Davidov (А.Ю. Давидов, 1823–1885) and having got acquainted with the theory of geometry from an unnamed school-textbook. The secretary of the bishop, Rev. Juozas Laukaitis (1873–1952), remembered that once during the time of *recollectio* (i.e., spiritual retreat), when the bishop was meditating on the inferno, a question came into his mind: "how many people would the inferno accommodate?" Knowing that "the thickness of the Earth crust is 50 kilometers "he worked out the capacity of the inferno and after two years of calculation made a remark: "if from the beginning of the world all people had gone to the inferno, inside the Earth, they would have occupied only a small corner of it" [15, p. 146]. Such remarks in no way should be associated with the teachings of the church because, according to the writer Rapolas Šaltenis, he simply, "as if being under the spell, chased an uncatchable swallow" [15, p. 147]. His inclination to mathematics could also have been caused by its universality, "because the mathematical fields are free from institutional politics" [1]. On the other hand, he was attracted to mathematics because "indulging in mathematics feels like swimming into the middle of the sea and diving to the very depth; understanding that you are getting further and further from the shore as well as deeper and deeper towards the bottom. However, when you look back at the work you have done, you see that you are standing close to the shore in the water no more than up to your ankles" [15, p. 147]. This happened to Baranauskas when he got captivated by the operation of raising to a power, which, as he admitted himself, "absorbed all his efforts". Being highly inspired he made "many discoveries" in this field, as he remarked himself: "discoveries that were new for me, but known in mathematics for centuries" [2]. Getting deep into the mathematical intricacies he proved for himself the correctness of mathematical statements that were already known to mathematicians. For example, he re-created the proof of Newton's Binomial Formula.

3. Leisure Time Dedicated to the Number Theory

He continued to deepen his knowledge about the powers of numbers and started creating tables of numbers to the power of two, three and higher. He did not make merely mechanical calculations but used them to describe his mathematical insights. He was able to notice that the difference of the squares of two adjacent natural numbers equals the sum of these numbers [10, p. 14][1]. Antanas Baranauskas shared his mathematical insights with his pen-friend, the German linguist Hugo Weber (1832–1904). The linguist helped the bishop to establish

[1] A. Baranauskas rediscovered himselfa method already known in mathematics at the time.

a relationship with a teacher of mathematics at Eisenach gymnasium Carl Hossfeld, who provided Baranauskas with the new textbook in number theory written by Gustav Wertheim (*Elemente der Zahlentheorie*, issued in 1887). This encouraged Antanas Baranauskas to explore further the secrets of prime numbers as a branch of his favorite number theory. At that time, as he admitted in his letter to Weber "an obsessive question stuck in my mind" – does numeration have an end? And what end?" [13, p. 223-224]. So he got involved into calculating prime numbers, first in the hundreds of thousands and then in the first million. After having finished these tiresome direct calculations he was planning to calculate how many of them are there in ten million(10^7) but remarked that" the calculating process turned out to be so complicated that in half a year only one tenth of the plan was accomplished" [6, p. 5]. He understood that if he continued with his calculations he would need, for example, a few years to search in the range of ten to the power of eight (10^8), tens of years if to talk about ten to the power of nine (10^9) and several hundred years for ten to the power of ten (10^{10}). After discussing the improvement of calculations with Hossfeld, he discovered that higher mathematics does not have an easier way to do this. So he had to rely on his own resources and noticed some regularities, i.e. the symmetry that exists between the spaces of certain numbers. In the already mentioned textbook by Wertheim he found a formula determining the number of prime numbers that do not go beyond a given bound. After the discussions with Hossfeld he grasped the idea of the formula and presented his conclusion. He shared the results with Carl Hossfeld ,who made a mistake, while rewriting the text , and in addition he published the article *The Remark about a Formula of the Number Theory* (*Bemerkung über eine Zahlentheoretische Formel*) in the journal "Zeitschrift für Mathematik" (1890, No. 25, p. 382–384) [6, p. 6]. Afterwards Baranauskas continued his calculations and discovered that his formula is simpler than the one in Wertheim's textbook, which was created by the German mathematician Ernst D.F. Meissel (1826–1895). The bishop reviewed the process of his reasoning and prepared the article *About formulae used to calculate the number of prime numbers that do not go beyond the given bound* (*O wzorach służących do obliczenia liczby liczb pierwszych nie przekraczających danej granicy*). With the help of the Polish linguist Jan Baudouin de Courtenay (1845–1929), the article was published in "Rozprawy Wydziału Matematyczno-przyrodniczego" (1895, Vol. 28, p. 192-210) [6, p. 6] issued by the Academy of Sciences in Krakow. It was important that in the article Antanas Baranauskas included the table listing the values of the function that determines the number of prime numbers [4, p. 30-32]. Though this work by Baranauskas does not contain a rigorous proof, it got his name mentioned in the scientific work *History of the Theory of* Number (New York, 1952) by the American mathematician Leonard E. Dickson (1874–1954) [10, p. 34].

4. A Glance at Geometry

About 1891 the bishop turned from the theory of numbers to solving geometrical problems. He directed his attention to one of the three oldest mathematical problem – squaring the circle. In other words, the bishop was preoccupied with the challenge "of constructing a square with the same area as a given circle". Though back in 1882 the professor Carl L.F. von Lindemann (1852–1939) from Munich had proved the number π is transcendental, the bishop, possessing knowledge only in elementary geometry, got an interesting expression, presented as follows: $\pi = 3 + 0{,}1\sqrt{2}$. Shortly afterwards he shared his thougts about this new discovery with Rev. Dambrauskas. However, the priest explained to the bishop that the result is approximate and that an identical formula was already found in the 14th century by the great Italian Dante Alighieri. This news stopped the bishop‘s ambitions to write a thesis in Latin on the above-mentioned subject and dedicate it to the Pope Leo XIII [6, p. 7].

At the end of the 19th century, when the Lithuanian language was still banned, American Lithuanians did a great job by issuing Lithuanian textbooks that were in short supply; however, there was a problem with the Lithuanian terminology – some words did not exist in the language. With the mediation of the Rev. Dambrauskas, Most Rev. Antanas Baranauskas, who had already distinguished himself in Lithuanian language research, started to create terminology of geometry. These are a few examples of the terminology proposed by him after a discussion with the linguist Weber: the terms *smailus kampas* (acute angle), *status kampas* (right angle), *daugiakampis* (polygon), *trikampis* (triangle), *lankas* (arc), *erdvė* (space) were created on the basis of Lithuanian words and the terms such as *punktas* (point), *linija* (line), *kvadratas* (square), *kubas* (cube) were created on the basis of international words. Some of the words proposed by him did not take root in the Lithuanian language and were subsequently changed, e.g. *ratas* to *skritulys* (circle), *ratlankis* to *apskritimas* (circumference), *skersinis* to *skersmuo* (diameter), *stipinas* to *spindulys* (radius), *skerskampė* to *įstrižainė* (diagonal), *gija* to *styga* (chord), *trapezas* to *trapecija* (trapezoid), etc. [12, p. 258-259].

5. About the Limits of Mind and the Infinity

After geometry the bishop started to examine complicated sequences of numbers called transcendental progressions: $a_1, a_2, a_3, \ldots, a_n$, $(a_1 = a^a, a_2 = a_1^{a_1}, \ldots, a_n = a_{n-1}^{a_{n-1}})$. Analyzing the problem he noticed that, when a =2, the first three numbers of the progression can be easily calculated, when a = 3, two numbers can be easily calculated, when a = 5, only one number can be easily calculated, and when a equals more than 5, such a possibility ceases to exist altogether. Such observations lead the bishop to recognizing the limits of the human mind. He reflected all this in his study *About Transcendental Progression and about the Limits and Power of the Human Mind* (*O progresji transcendentalnej oraz o skali i siłach umysłu ludzkiego*), which was issued as a separate publication in 1897 in Warsaw [6, p. 9].

The end of 1897 brought big changes into Antanas Baranauskas’ life – on the 23rd of October he was appointed to the position of the bishop of Seinai (Sejny). Therefore on Christmas, on the 25th of December, he moved to this peripheral Lithuanian-Polish town that had become the episcopal see. After having settled in a new place the bishop continued his predilection for mathematics and looked at the “Queen of Science” as a philosopher and theologian.

In the above-mentioned article about transcendental progression the bishop wrote: “There are truths that surpass the human mind; and therefore, there are minds that are more powerful than the mind of a human being. There is an infinite set of truths. And hence, there is the mind that has the infinite power of comprehension. The infinity of the object for comprehension indicates that there must be the infinite subject that comprehends" [6, p. 9] This confirmed the theological trend of the article; the return to the proof of the existence of God.

The considerations about the transcendental progression and the book on infinity by the French mathematician Renè de Clèrè encouraged the bishop to define the concept of infinity for himself. The bishop had an excellent education in philosophy and a naturally insightful mind that helped him to perceive the existence of two infinities – the actual one and potential one. The reasoning went as follows: "If we label actual infinity "*a*", nothingness "0" and infinity "∞", then these three characters will mean three different spheres, not only separate and having nothing in common, but also not coming in contact with each other at all. Between actual infinity and nothingness there lies the sphere of infinite potential $p = \infty$. Also, between actual infinity and infinity there is a sphere of infinite potential. It leads to the conclusion that actual infinity does not come into contact either with infinity "∞" or with zero, which means nothingness. Every existence (apart from God) is limited from all sides and all aspects by limitless potential" [3]. From this we can see that Antanas Baranauskas distinguished the actual infinity, in other words – the infinity in itself and infinite sequence of numbers, and the potential infinity, which we understand in terms of a finite process.

In addition, the bishop remarked that comprehending the contact of the mentioned spheres means that "the creation from nothingness is the creation from potential: the potential is born from nothingness and the existence is born from potential". The Rev. Dambrauskas commented on this by saying "that Baranauskas had a deeper understanding of the problem of infinite creation than many other theologians" [6, p. 10].

6. Closing Remarks

The works in various fields of mathematics, such as number theory, geometry and linear order theory were based on thorough and sound calculations as well as on considerations of philosophical concepts. His accomplishments show that Antanas Baranauskas had a talent in mathematics. On the other hand, the bishop used calculation as a very important tool of mathematics, not only to develop his mind but also to solve the problems of morality and faith.

He was one of those Lithuanian amateur mathematicians who encouraged others to pursue the study of mathematics in the international environment. We can also say with full confidence that the bishop's mathematical works proclaimed the unity of science and faith.

Translated by Jūratė Marchertaitė, Lithuanian University of Educational Sciences

Appendix. Math works of bishop A. Baranauskas

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