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THE MODEL OF DISPERSION OF PARTICLES DURING THEIR FLOW FROM CHIPPING THE SURFACE

MODELOWANIE ROZKŁADU WIELKOŚCI CZĄSTEK PO UDERZENIU O PŁASZCZYZNĘ

A b s t r a c t

We propose a method for forming differential of the distribution functions of the number of particles of bulk materials on the angle of reflection from bumper (baffle) plate.

Keywords: mixing, brush elements, the bumper, the granular mixture, model

S t r e s z c z e n i e

Zaproponowano metodę wyznaczania różniczkowych funkcji rozkładu wielkości cząstek stałych w zależności od kąta odbicia od płaszczyzny.

Słowa kluczowe: mieszanie, elementy szczotkowe, zderzak, mieszanina cząstek, model

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1. Introduction

The problem of obtaining a homogeneous particle mixture containing unequal volume fractions of particulate components, for example, in routine ratio 1:10 or more, remains relevant and can be successfully achieved using a gravity type mixer [1]. In the designed device of special purpose for the formation sparse streams offered to apply mixing drums with brush elements above the tray, which serves the working materials. In order to improve the quality of the granular mixture obtained in the gravity apparatus, bumper plates (baffle plates) may be used, which are established after drums with brush elements. Shock interaction between the baffle plate and the arrive flows of loose components that are gripped by end of brush elements, resulting in the formation of reflected flow of mixture, which is sliding on the tray located under the drum.

The development of engineering methods of calculating gravity mixer involves choosing the most efficient ranges for changing its design and operational parameters. In connection with this, a special interest is had in the evaluation of the angles of reflection streams of loose components, which are formed after the shock interaction with the flat baffle plate. It is believed that these reflection angles are angles in which the average speed of particle of *i*-th component (i = 1, ..., n_k) is directed perpendicular to the plane of impingement. Such modelling of the dispersion of particles in the process of reflection stream of *i*-th material from the baffle plate can be made on the basis of a stochastic approach [2, 3].

2. Geometric features of ways of moving sparse flows of loose components in the working volume of a gravity mixer

The tray, which is under the mixing drum (with brush elements) and on which there are vertically fed n_k particle ingredients in a volume ratio that given by the technological regulations, is at an angle μ_0 to the vertical direction. In the gap, which is had by the height h_0, between this tray and rotating drum, there is a deformation of brush elements and gripping by them portions of layers working loose materials. The number of deformed brush elements – n_b. Baffle plate is located behind the drum at an angle ψ to it tray. Arrive on the baffle plate, flow of bulk component i formed after gripping of particles by brush elements (j = 1, ..., n_b), mounted on a cylindrical surface of the mixing drum along helical lines in opposite directions, starting from its ends. Step of helical winding –h_s; Length of brush element – l_b; the radius and the length of the mixing drum – r_b and L_b; the angular speed of rotation ω. The expressions for the relationship between constructive-regime parameters of the mixing drum and characteristic angles λ_{ij}, α_{ij}, φ_{ij}, β_{ij} and averaged by the number of deformed brush elements values λ_i, α_i, φ_i, β_i for the arriving on the baffle plate flow of *i*-th component after gripping by brush element j according to [4] are of the form

$$\lambda_{ij} = \alpha_{ij} + \varphi_{ij} - \beta_{ij}, \quad \lambda_i = \frac{\alpha_i}{n_b} + \varphi_i - \beta_i, \quad \alpha_i \equiv \sum_{j=1}^{n_b} \alpha_{ij} \tag{1}$$

$$\beta_{ij} = \operatorname{arctg}\left[\frac{2\cdot h_s\cdot(l_b-h_0)}{L_b\left\{r_b+h_0+(l_b-h_0)\cdot\left[1-\frac{2\cdot h_s\cdot(\alpha_{ij}+\varphi_{ij})}{L_b}\right]\right\}}\right],\quad \beta_i \equiv n_b^{-1}\sum_{j=1}^{n_b}\beta_j \qquad (2)$$

$$\varphi_{ij} = \frac{\pi}{2}+\alpha_{ij}-\frac{\arcsin\left(\left\{(r_b\cdot\sin\sigma_{ij})^2+(r_b+h_0)^2\right\}\cdot\frac{\cos\sigma_{ij}}{r_b+h_0}\right)}{1+\frac{d_0\cdot\left\{2\cdot h_s\cdot(l_b-h_0)/L_b\right\}^2}{d_1^{1/2}}},\quad \varphi_i \equiv n_b^{-1}\sum_{j=1}^{n_b}\varphi_{ij} \qquad (3)$$

where

- λ_{ij}, α_{ij} – angle spreading by brush element j for material *i*, respectively, from the vertical and the horizontal directions;
- φ_{ij} – central angle relative to the drum rotation axis, describing the bending of brush element that have number *j* depending on the position on it occupied portions of component *i*;
- β_{ij} – characteristic angle between the direction of the material flow rate *i* and vector tangent to the helical path of end of brush element *j* in section plane of the drum;
- σ_{ij}, d_0, d_1 – constants depending on the values of these parameters of the mixing element.

By introducing the concept of the coefficient of restitution (k_{Vi} = $\sin\gamma_{1i}$ / ($\sin\gamma_{2i}$)) for the average velocities of the move of arriving on the baffle plate, loose flow of *i*-th component and reflected from it respectively at angles γ_{1i} and γ_{2i}, can similarly [5] to obtain the functional dependence $k_{Vi} = f_i\,(\mu,\ \psi,\ L_i,\ h_i)$, where μ – characteristic angle between the perpendiculars to the baffle plate and to the tray under the mixing drum; ψ – angle between the tray and the baffle plate; L_i – spreading width of the *i*-th material along a tray; h_i – height between tray and baffle plate to shock point for average speed of arriving flow of component *i*. Then, the average value of spreading angle α by the number of deformed brush elements from (1) with considering (2) and (3) and the expression $\psi = \psi_1 + \mu_0 - \pi/2$ takes the form of functional dependence

$$\alpha_i(\gamma_{2i}) = n_b\cdot\left\{\pi-\left[\frac{\psi_1+\mu_0+\gamma_{2i}}{f_i(\mu,\psi,L_i,h_i)}\right]-\varphi_i+\beta_i\right\} \qquad (4)$$

where

- ψ_1 – the angle between baffle plate and horizontal direction;
- μ_0 – the angle of the tray under the drum to the vertical direction.

3. Determination of differential distributions of particles of flow reflected from baffle plate depending on the angle of reflection

Stochastic modelling of process dispersion of particles of mixed components after reflection from the rectilinear baffle plate inclined at a predetermined angle to the vertical can be accomplished using complete differential distribution function of particles of each material on the angle of spreading after interaction with brush elements [6]

$$S_i^{(np)}\left(\alpha_{ij}\right)=\prod_{j=1}^{n_b} S_{ij}^{(np)}\left(\alpha_{ij}\right) \tag{5}$$

$$\begin{aligned} S_{ij}^{(np)}\left(\alpha_{ij}\right)=\varsigma_{ij}\cdot\Bigg\{\Bigg[\operatorname{erf}\left(\frac{\eta_{ij}\left(\tilde{\alpha}_{ij}\right)\cdot\left[1+\varepsilon_0\cdot\varepsilon_{3i}\cdot\left(\alpha_{ij}+\varphi_{ij}\right)\right]^2}{\varepsilon_{3i}}\right)-\operatorname{erf}\left(\frac{\eta_{ij}\left(\tilde{\alpha}_{ij}\right)}{\varepsilon_{3i}}\right)\Bigg]\times \\ \times\left[\eta_{ij}\left(\tilde{\alpha}_{ij}\right)\right]^{-1}\cdot\exp\left\{-\frac{\left[\eta_{ij}\left(\tilde{\alpha}_{ij}\right)\right]^2\cdot\varepsilon_4\cdot\varepsilon_0^{\,2}\cdot\left(\alpha_{ij}+\varphi_{ij}\right)^2}{\varepsilon_{1i}\cdot\varepsilon_{2i}}\right\}\Bigg\} \end{aligned} \tag{6}$$

These functions are non-equilibrium and correspond to the kinetic equations for the energetically open macrosystem with macroscale fluctuations of states [1] caused by the influx of outside material (energy) and leading to the ordering of macrosystem in unstable conditions. For example, such as fluctuations in [6] are collisions between particles arriving streams of loose components, which are formed after the gripping by symmetrically arranged brush elements in the form of helical winding with different ends of the mixing drum. Wherein this modelling of stochastic energy $E_{ij}^{(np)}$ for a particle i of bulk material when gripped by brush element j, which is part of the expression to determine the number of particles of component i

$$\mathrm{d}N_{ij}^{(np)}=A_{ij}^{(np)}\cdot\exp\left[-\frac{E_{ij}^{(np)}}{E_{0ij}^{(np)}}+\frac{\left(E_{ij}^{(np)}\right)^2}{2\cdot E_{fij}^{\,2}}\right]\mathrm{d}\Omega_{ij} \tag{7}$$

in the element of phase volume $\mathrm{d}\Omega_{ij} = -\,\omega^2 r_{ij}\mathrm{d}r_{ij}\mathrm{d}\Theta_{ij}$ for polar coordinate system in the cross-section of the drum with centre on its axis of rotation allows for translational movement of bulk materials particles, the random nature of their angular momentum when gripping by deformed brush element j and the interaction with the brush element having angular stiffness. Parameters $E_{0ij}^{(np)}$ and $E_{fij}^{(np)}$ included in the expression (7) have the meaning, respectively, of a macrosystem energy at the moment of its randomisation and the energy loss of the macrosystem at macroscale fluctuations of its condition; $A_{ij}^{(\mathrm{np})}$ – normalization parameter.

Geometric relationship (1) in (4) between the angles of spreading flows of particles arriving on the baffle plate and the angle of inclination according to expressions (5) and (6) allows to create full differential distribution function of particles of loose components $\chi_{\mathrm{ij}}^{(\mathrm{np})}(\gamma_{2i})$ on the angle of reflection from the baffle surface taking into account the coefficient of restitution $k_V = k_V(\gamma_{2i})$

$$\chi_i^{(np)}(\gamma_{2i}) = \prod_{j=1}^{n_b} \xi_{ij}^{(np)}(\gamma_{2i}) \tag{8}$$

$$\begin{aligned}\xi_{ij}^{(np)}(\gamma_{2i}) = \varsigma_{ij} \cdot \Bigg\{ \Bigg[\operatorname{erf}\Bigg(\frac{\eta_{ij}(\tilde{\alpha}_{ij}) \cdot \{1 + \varepsilon_0 \cdot \varepsilon_{3i} \cdot [\alpha(\gamma_{2i}) + \varphi_{ij}]\}^2}{\varepsilon_{3i}} \Bigg) - \operatorname{erf}\Bigg(\frac{\eta_{ij}(\tilde{\alpha}_{ij})}{\varepsilon_{3i}} \Bigg) \Bigg] \times \\ \times [\eta_{ij}(\tilde{\alpha}_{ij})]^{-1} \cdot \exp\Bigg\{ - \frac{[\eta_{ij}(\tilde{\alpha}_{ij})]^2 \cdot \varepsilon_4 \cdot \varepsilon_0{}^2 \cdot [\alpha(\gamma_{2i}) + \varphi_{ij}]^2}{\varepsilon_{1i} \cdot \varepsilon_{2i}} \Bigg\} \Bigg\}\end{aligned} \tag{9}$$

where the functional dependence of $\alpha_i(\gamma_{2i})$ has the following form

$$\alpha_i(\gamma_{2i}) = n_b \cdot \left(\beta - \varphi_i + \frac{B_1 \cdot B_3 - B_2 + \left[(B_1 \cdot B_3 + B_2)^2 - 4 \cdot B_3 \cdot \gamma_{2i} \right]^{1/2}}{2 \cdot B_3} \right) \tag{10}$$

$$B_1 = \frac{\pi}{2} - \psi + \mu_0, \quad B_2 = P_{1i} \cdot [(\pi - \psi) \cdot \cos\mu - P_{2i}], \quad B_3 = (\pi - \psi) \cdot P_{1i} \cdot \sin\mu \tag{11}$$

$$P_{1i} = \frac{1 - (\pi - \psi) \cdot \left(\dfrac{L_i}{h_i} - \operatorname{ctg}\psi \right)}{2 \cdot \left(\dfrac{L_i}{h_i} - \operatorname{ctg}\psi \right) \cdot \left(1 + \dfrac{L_i}{h_i} - \operatorname{ctg}\psi \right)}, \quad P_{2i} = \left(\pi - \psi + \frac{L_i}{h_i} - \operatorname{ctg}\psi \right) \cdot \operatorname{ctg}\psi \tag{12}$$

4. The main results of modelling and conclusions

We illustrate the results obtained for functions $\chi ij^{(np)}(\gamma_{2i})$ and $\xi_{ij}^{(np)}(\gamma_{2i})$ from expressions (8) and (9) on the example of the reflection natural sand GOST 8736-93 (i = 1) from the baffle plate. The dependence presented by the graphs in Fig. 1 allow us to estimate the maximum value of the reflection angle γ_{21} of the flow of loose ingredient (i = 1), after interaction with brush elements j = 1, 2, 3. In particular, the sparse stream of particles arriving on the baffle plate, formed after gripping of natural sand by brush elements, which first was captured by the layers of material from the clearance with the tray (Fig. 1, curve *3*, j = 3, with n_b = 3), is discarded from the baffle plate on angle greater by 2.6 times than a similar flow formed after interaction with brush elements (Fig. 1, curve *1*, j = 1), which came out of the gap tray-drum last. The maximum value of the angle of reflection that characterises the dispersion of the full arriving flow of natural sand flow after hitting the baffle plate (Fig. 1, curve 4), close to the value of the angle for the flow relating to gripping by brush elements with a number j = 2 (Fig. 1, curve 2).

Thus, hereinafter depending $\chi_{ij}^{(np)}(\gamma_{2i})$ and $\xi_{ij}^{(np)}(\gamma_{2i})$ from the expressions (8) and (9) may be used for calculating the weight fraction of the mixed components of the resulting mixture, and for assessing it quality for selected criteria.

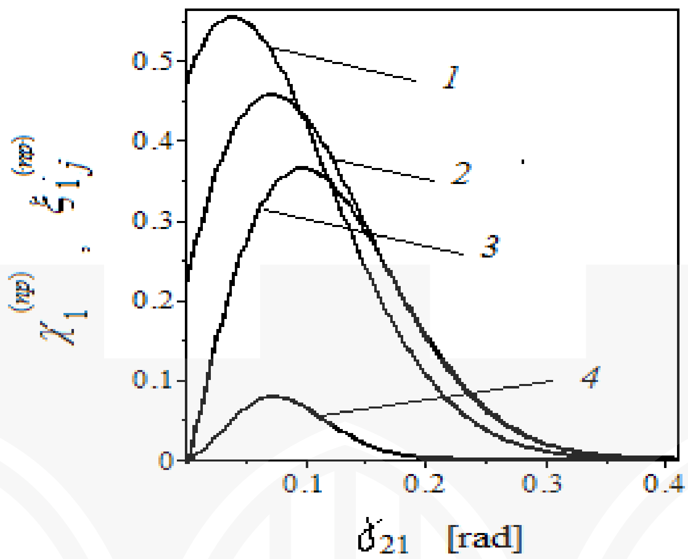

Fig. 1. Dependencies $\chi_1^{(np)}(\gamma_{21})$ and $\xi_{1j}^{(np)}(\gamma_{21})$ for differential non-equilibrium distribution functions of the number of particles of natural sand GOST R 8736-93 (i = 1) in the angle of reflection γ_{21} from the baffle plate: $L_b = 1{,}85\ 10^{-2}$ m; $l_b = 4{,}5\cdot10^{-2}$ m; $r_b = 3{,}0\cdot10^{-2}$ m; $h_s = 1{,}6\cdot10^{-2}$ m; $h_0 = 3{,}0\cdot10^{-2}$ m; $\omega = 52{,}36\ s^{-1}$; $n_b = 3$; $\mu_0 = 1{,}3089$ rad; $\Psi_1 = 0{,}9599$ rad; $\mu = 0{,}7071$ rad; $L_1 = 2{,}8\cdot10^{-1}$ m; $h_1 = 8{,}0\cdot10^{-2}$ m; 1-3 – $\xi_{1j}^{(np)}(\gamma_{21})$; 1 – j = 1; 2 – j = 2; 3 – j = 3; 4 – $\chi_{1(}^{np)}(\gamma_{21})$

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