TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE
AUTOMATIC CONTROL | AUTOMATYKA

4-AC/2013

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# ROBOT PATH PLANNING MAP PRE-OPTIMIZATION BY MULTILAYER FUZZY SITUATIONAL MAPS

# WSTĘPNA OPTYMALIZACJA PLANOWNIA RUCHU ROBOTA ZA POMOCĄ WIELOWARSTWOWYCH ROZMYTYCH MAP SYTUACYJNYCH

Abstract

Intelligent robot path planning require have very complex decision-making and computational processes. Collecting and calculating a high amount of data is one of the weakest points of a such system. In addition, it is necessarily processed in real-time on a limited computational capacity. In this paper, we propose some novel algorithms for coping with these problems and give some information about Fuzzy Situational Maps and their use as a multidimensional extension of Fuzzy Signatures. An example takes to the field of path planning map pre-optimization by Fuzzy Situational Map.

*Keywords*: *fuzzy situational maps, mobile robotics*

Streszczenie

Zadania inteligentnego planowania ruchu robota obejmują bardzo złożone procesy decyzyjne oraz obliczeniowe. Agregacja i przetwarzanie dużej ilości danych są jednym z najsłabszych punktów tego typu systemów. Dodatkowo, przetwarzanie konieczne jest w czasie rzeczywistym przy ograniczonej mocy obliczeniowej. W artykule zaproponowano kilka nowych algorytmów, które radzą sobie z tymi problemami oraz przedstawiono informacje na temat rozmytych map sytuacyjnych oraz wielowarstwowych rozmytych map sytuacyjnych jako wielowymiarowego rozwinięcia rozmytych sygnatur. Przedstawiony przykład dotyczy metody wstępnej optymalizacji planowania ruchu robota z użyciem rozmytych map sytuacyjnych.

*Słowa kluczowe*: *rozmyte mapy sytuacyjne, roboty mobilne*

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## 1. Introduction

Intelligent mobile robot path planning tasks are a challenging and very perspective research field. Optimal path planning in a mainly autonomous manner involves many difficult decision making processes and consumption of calculation. Our research is focused on a special case of map based robot path planning systems, where intelligent map pre-optimization takes place before the path planning process. One of the main problems in a map based path optimization system is the processing of very complex data structures, where the data are often noisy, distorted or simply absent (therefore unmeasurable).

In this paper, we propose a novel approach for describing and processing high-scale multidimensional structured data in a simple way. This method is called Fuzzy Situational Map (FSM), a multidimensional geometric extension of Fuzzy Signatures.

First, we give a short overview of Fuzzy Signatures as the 'ancestors' of Fuzzy Situational Maps. Then, the theory of FSM and Multilayer FSM will be presented. Finally, we will see an example of a path planning map pre-optimization task by Multilayer Fuzzy Situational Map.

## 2. Fuzzy Signatures

The original definition of fuzzy sets [1] was $A$: $X \rightarrow [0, 1]$, and was soon extended to *L-fuzzy sets* by Goguen [2].

This definition is $A_L$: $X \rightarrow L$, $L$ being an arbitrary algebraic lattice. A practical special case, *Vector Valued Fuzzy Sets* was introduced in [3], where $A_{V,k}$: $X \rightarrow [0, 1]^k$, and the range of membership values was the lattice of $k$-dimensional vectors with components in the unit interval. A further generalization of this concept is the introduction of fuzzy signatures and signature sets, where each vector component is possibly another nested vector (Fig. 1).

## 3. Fuzzy situational maps

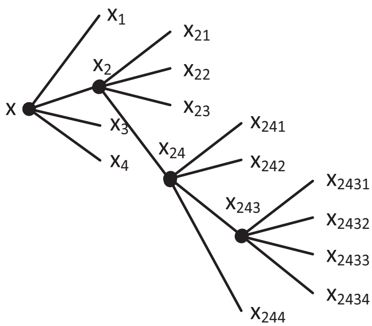
We propose a novel approach to support difficult decision-making and to depict situational or contextually dependent structured data. A special form of fuzzy signatures [4–7] with a spatial structure is used, namely, the Fuzzy Situational Map (FSM).

Fuzzy situational maps as multidimensional extended fuzzy signatures (FS) are suitable for describing complex multidimensional system conditions in cases where the information is fragmented, distorted or noisy [8–11].

The FSM can be two-, three- or even n-dimensional. Let us see the simplest case, two-dimensional fuzzy situational map (Fig. 2).

### 3.1. Two-dimensional Fuzzy Situational Map

Two-dimensional FSM may be considered as a geometric lattice, where each node has a fuzzy value or a whole fuzzy set, in extended case.




$$x = \begin{bmatrix} x_1 \\ \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \\ \begin{bmatrix} x_{241} \\ x_{242} \\ \begin{bmatrix} x_{2431} \\ x_{2432} \\ x_{2433} \\ x_{2434} \end{bmatrix} \\ x_{244} \end{bmatrix} \end{bmatrix} \\ x_3 \\ x_4 \end{bmatrix}$$

Fig. 1. Example of a Fuzzy Signature Structure

$$\begin{array}{cc|c|c|c|c|}
 & & \multicolumn{4}{c}{w} \\
 & & 1 & 2 & \ldots & n \\
\hline
 & 1 & x_{11} & x_{12} & \ldots & x_{1n} \\
\hline
 & 2 & x_{21} & x_{22} & \ldots & \ldots \\
\hline
h & \vdots & \ldots & \ldots & \ldots & \ldots \\
\hline
 & m & x_{m1} & \ldots & \ldots & x_{mn} \\
\hline
\end{array}
\qquad
\begin{array}{l}
x_{hw} = [0, 1] \\
h = 1 \ldots m \\
w = 1 \ldots n
\end{array}$$

Fig. 2. A Fuzzy Situational Map

FSM can be represented as lattices or in matrix form as in Fig. 2 and in .

$$\mathrm{FSM} = \begin{bmatrix} x_{011} & x_{012} & x_{013} & \begin{bmatrix} x_{111} & x_{112} \\ x_{121} & \begin{bmatrix} x_{211} & x_{212} & x_{213} \\ x_{221} & x_{222} & x_{223} \\ x_{231} & x_{232} & x_{233} \end{bmatrix} \end{bmatrix} \\ x_{021} & x_{022} & x_{023} & x_{024} \\ x_{031} & x_{032} & x_{033} & x_{034} \\ x_{041} & x_{042} & x_{043} & x_{044} \end{bmatrix} \tag{1}$$

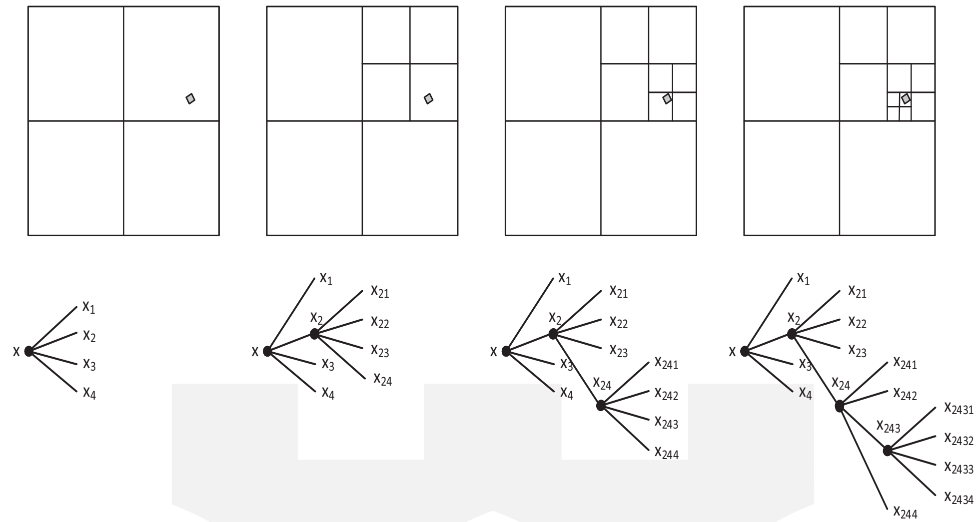



Fig. 3. FSM as multidimensional Fuzzy Signature

The values in the individual nodes can be interpreted as elements of a fuzzy signature, so a fuzzy situational map can be described as a multidimensional spatially structured fuzzy signature, see Fig. 3.

In the example, a very simple case of ‘refining’ the situational map by 2 × 2 grids is presented.

Following this interpretation, it can be said that each node in a FSM can be a further nested FSM and continued iteratively, this extension may go to depth *z* (applying increased resolution). The approach can lead to a fine structured FSM in each node as Fig. 4 shows. The resolution of nodes are independent of each other.

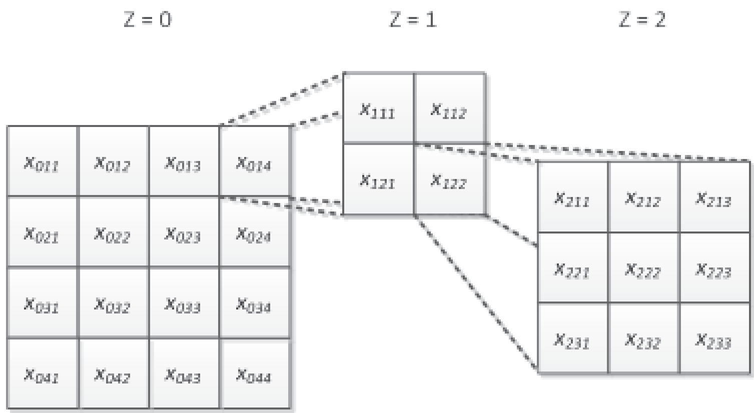



Fig. 4. Sub-lattices of a Fuzzy Situational Map

The individual nodes and the corresponding sub-lattices (high-resolution lattices) are related in the sense that the subgroups of sub-lattices jointly determine the features of the higher (parent) level. This structure, where each node can store significant amount of additional information which is processed only in the necessary resolution depth, can greatly reduce the computational requirements. FSM can describe hierarchically structured multidimensional data in a more concise way than simple fuzzy signatures.

## 4. *n*-dimensional fuzzy situational map

In the previous section, the properties of the two-dimensional fuzzy situational map were presented. Easily conceivable, these properties can be extended from the two-dimensional fuzzy situational map to a three-dimensional situational map, which may be used extensively in practice (eg. as a description of 3D robotic tasks).

Theoretically, there is no limit to increasing the dimension of a fuzzy situational map. Mathematically, any $n$-dimensional fuzzy situational map can be prepared, but from a practical point of view, increasing the dimension of a situational map exponentially increases the complexity of the inference on the situational map.

Consider a two-dimensional fuzzy situational map with a $h \times w$ node ($h$ and $w$ are the height and the width of the map, respectively), where each node is a leaf, so this situational map can be used in an inference system without any reduction or aggregation.

The computational complexity of a rule in such an inference system that is based on a fuzzy situational map can be calculated in a similar way to how Kóczy calculated it on a classic fuzzy inference system in [8, 9]. The $C_{\text{time}}$ computational complexity is proportional to the resolution of the fuzzy situational map.

$$C_{\text{time}} = O(h \cdot w) \tag{2}$$

Let us increase the dimension of the situational map to $n$. In this case, the computational complexity turns into:

$$C_{\text{time}} = O(h \cdot w)^{n-1} \tag{3}$$

It can be concluded that the complexity increases exponentially, which can quickly result in an unacceptable level of computation time.

Of course, if the number of rules are increased in a FSM based inference system, then the computational complexity grows exponentially too [8, 9].

From a practical point of view, such a growth rate of requirements of computational capacity greatly reduce, the usability of the $n$-dimensional fuzzy situational maps. Therefore, we developed a new method for implementing the complex structured data description capacity of n-dimensional situational maps, whilst bypassing such computational capacity problems. The new algorithm is a multilayer fuzzy situational map.

## 5. The multilayer fuzzy situational map (MFSM)

The multilayer fuzzy situational map can be considered as several two-dimensional situational maps which are superimposed to each other, as Fig. 5 shows. Each individual layer can be handled as a two-dimensional situational map.

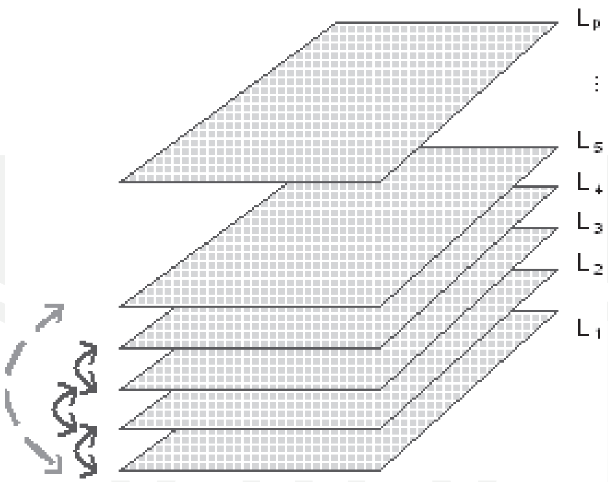



Fig. 5. Multilayer Fuzzy Situational Map

A multilayer fuzzy situational map always has a master-layer (or map), which contains the main situational map and carries the other layers. The sub-layers can be sorted over this master-layer for adding special information or modifying the data in the main layer. The sub-layers are called layer-maps.

The structure of a multilayer fuzzy situational map is always defined by the master-layer. It means that the basic structure of the master-layer and the layer-maps must be the same, that is, the master-layer structure can always be obtained from layer-maps by some reduction or aggregation.

In this case, the relationship between the layers and effects to each other are interpreted only on the identically indexed nodes . Of course, indirect effects may occur at other nodes.

The nodes with identical index in different layer may be connected in three type of relation:
– directed dependency,
– interdependency,
– independency.

*Directed dependency* means the nodes of one layer have influence over the nodes of another layer, but the second do not have any influence over the first mentioned layer. The directed dependency is not mutual. For example, in a robot navigation tasks the layer-map of the obstacles influences some nodes on the path planning layer, but the path planning layer does not influence the obstacle sub-layer.

*Interdependency* means mutual interaction between the layers. In this case, a more complex relationship must be described than in the other two cases. Often, these layers should be decomposed to additional directionally dependent layers for an acceptably practical result.

In the previous example, if multiple layers are used for route optimization processes, e.g. one for time optimization and one for energy consumption optimization, then these layers consist of interdependent nodes which require complex calculation and high computational capacity.

The layers are *independent* or *partly independent* if there is not any effective relation between the nodes of layer-maps, or the layers are relationship by only a few points.

### 5.1. Node connections between the layers

After the restructure of the layers of the situational map, the leaf nodes of the layer-maps are located in the same lattice points in each layer.

The node connections and effects between the layers vary depending on the task. From simple weighting and scaling relations, to complex functions, everything is conceivable. The weighting and other fuzzy operators are used on fuzzy situational maps actually.

The value of $l_\mathrm{p}$ leaf of the *MA* multilayer fuzzy situational map is obtained by the *Con* function which calculates the connection between the $l_\mathrm{p}$ leaf nodes of $A_i$ and $A_j$ layer-maps . These leaves may be the result of some reduction or aggregation.

$$\forall l_\mathrm{p}\ \text{leaf}\ MA\left(l_\mathrm{p}\right) = Con\left(A_i\left(l_\mathrm{p}\right), A_j\left(l_\mathrm{p}\right)\right), \quad i = 1, \dots, n;\ \ j = 1, \dots, n;\ \ i \neq j \tag{4}$$

where $n$ is the number of layers and $Con(x_\mathrm{p}, y_\mathrm{p})$ function calculates the modified value of connected leaves.

## 6. The benefits of multilayer fuzzy situational maps

The multilayer fuzzy situational maps with an $n$-layer have many practical advantages over $n$-dimensional non-layered fuzzy situational maps.

The information in the $n$-layered fuzzy situational map can be processed as $n$ two-dimensional fuzzy situational maps, thus the demand on the computational capacity is dramatically reduced. In addition, the operation can be parallelized layer by layer on multi-processor or multi-core systems.

The layered structure of the situational map can be designed in a modular manner, which means that the individual layers can be removed, replaced or adapted in the multilayered situational map. Thus a complex information descriptor can be created that is as flexible as a classical fuzzy rule base. Deleting or modifying individual layers does not cause the destruction of the whole system, the situational map remains viable even in the absence of certain layers.

The modularity feature of the multilayer fuzzy situational map makes very flexible its applicability and allows for continuous development, in such autonomous and adaptive cases too. Modular multilayered fuzzy situational maps are good base for machine learning systems. These are ensured by the definition of independency-dependency between the individual sub-layers, each layer affects only certain other layers in certain cases.

## 7. Route map pre-optimization with Fuzzy Situational Maps

In robotics, logistics and other disciplines, it plays an important role in finding the optimal routes, in other words, the minimization of one or more cost functions. There are a lot of methods to resolve these problems [12–17], but these procedures need high computational capacity, so in a real-time system, the size and the dimension of the function input data and the number of iteration steps to find the optimum solution makes big difference.

We have developed an algorithm based on multilayer fuzzy situational maps, which pre-optimizes the input data for the map based path planning functions. This algorithm reduces the amount of required input data (the size of the map) compared to the traditional procedures, so that the quality of path planning does not deteriorate.

The essence of this method is that more FSM layers are used to depict the separable system parameters. In our case, three FSMs are used: the target position FSM, the robot actual position FSM and the descriptive map of obstacles. The robot position FSM is a dynamic map, with resolution on the basis of the actual refining of the robot sensors (Fig. 6a). The target FSM is a rough situational map, where only the target area has a finer resolution, due to the expected accuracy of positioning (Fig. 6b). These two map layers may be combined and virtually, only the positions are important to us in terms of optimization. If the obstacles are not taken into account, then this map will be used to plan the route. Fig. 6c shows an example for the planned route of the robot which is limited only to horizontal and vertical movements. In this paper, this limitation will be assumed.

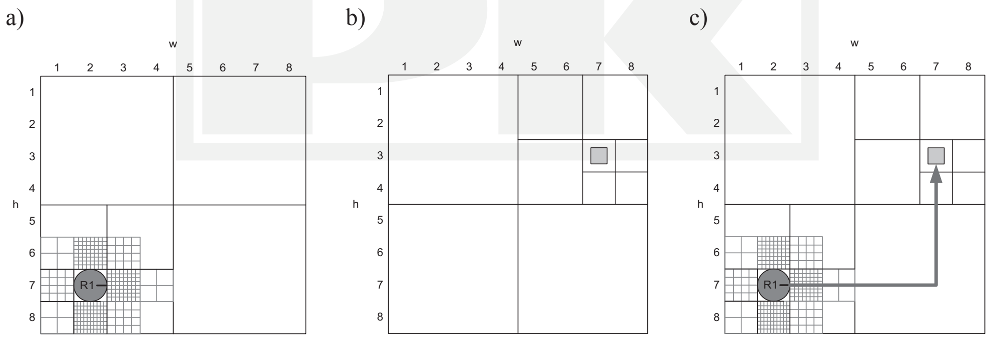



Fig. 6a) robot position FSM, b) goal position FSM, c) planed route

The situational map of obstacles has a similar structure as the preceding (Fig. 7), where the obstacles are represented by fuzzy values in the lattice. In the actual algorithm the map structure is taken in account, and here the fuzzy values have only secondary roles.

The obstacles FSM, as a new layer, is superimposed on the top of the position situational map.

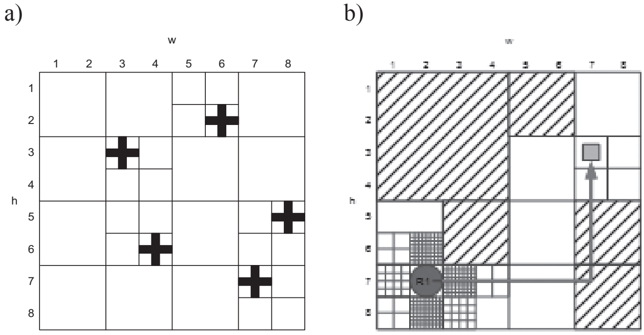



Fig. 7a) Situational map of obstacles, b) multilayered FSM

In order to make the initial contacts between the layers, the new layer should be converted into the common structure of the master layer. In this case, the master layer is the robot and goal position situational map and the additional layer is the obstacles situational map, so the structure of the obstacles situational map has to be converted. The resulting formed structures of layers are illustrated in Fig. 7.

The marked locations indicate the nodes where obstacles are located in the planned route. In these nodes the resolution must be refined, but only in these nodes. The refining steps are iterated until the accessible nodes, which map can be used to a final route optimization, they are used in as rough resolution as possible.

The next pseudo-code writes down the essential steps of this algorithm.

```
procedure
    while struct(robot_FSM) <> struct(goal_FSM)
        if struct(robot_FSM) > struct(goal_FSM) then      // > means higher resolution
            robot_FSM = reduce(robot_FSM)
        else
            goal_FSM = reduce(goal_FSM)
        endif
    endwhile
    pathmap_FSM = add(robot_FSM, goal_FSM)
    while struct(obstacle_FSM) > struct(pathmap_FSM)
        obstacle_FSM = reduce(obstacle_FSM)
    endwhile
    pathmap_MLFSM = addlayer(pathmap_FSM, obstacle_FSM)
    path = pathplanning(pathmap_MLFSM)
endproc
```

This way, the prepared FSM may contain far less grid points (Fig. 8a, b) than a traditional route map (Fig. 8c), so the path planning function can get results with a smaller amount of data and less iteration steps.

a) b) c)

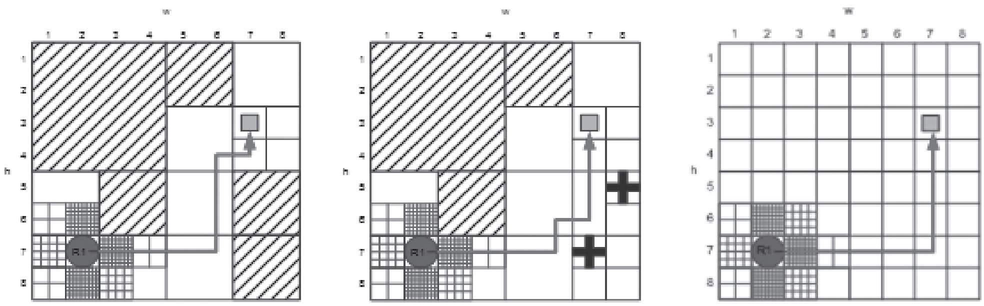


Fig. 8a) Fast route planning, b) route in a zoomed FSM, c) traditional route planning map

## 8. Conclusion

In this paper, we presented the essential idea of Fuzzy Situational Maps and the use of FSM algorithms in the field of mobile robotics map based route planning. These methods open a new way for complex decision-making. With Fuzzy Situational Maps, the complex structured and multidimensional data can be described in a compact and manageable manner. Thus, the difficult data management processes are becoming a more easily preparable system.

Here, we illustrated information processing by Fuzzy Signature Map might lead to an effective pre-optimized route planning map. In this example, the FSM allows optimization of the route with 15% less data than in the case of the same structured classical grid or map.

*The research was supported by TÁMOP-4.2.2.A-11/1/KONV-2012-0012, Hungarian Scientific Research Fund Grant OTKA K108405 and OTKA K105529.*

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