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COOLING OF HOT WATER WITH NON-UNIFORM INITIAL TEMPERATURE

CHŁODZENIE GORĄCEJ WODY O NIEJEDNORODNEJ TEMPERATURZE POCZĄTKOWEJ

A b s t r a c t

A mathematical model of water cooling in a vertical tank with non-uniform initial temperature has been presented in this paper. This kind of process is important in solar installations. Calculation results for various cases comprising, among others, two or three layers of water with different initial process temperatures as well as various combinations of heat resistance of the side wall, the tank bottom and the tank cover, have been given as well.

Keywords: thermal destratification, heat conduction equation, renewable energy sources

S t r e s z c z e n i e

Przedstawiono model matematyczny chłodzenia wody o niejednorodnej temperaturze początkowej w zbiorniku o osi pionowej. Taki proces ma praktycznie znaczenie w instalacjach solarnych. Przedstawiono wyniki obliczeń dla różnych przypadków obejmujących m.in. dwie lub trzy warstwy wody o różnych temperaturach na początku procesu oraz rozmaite kombinacje pomiędzy wielkościami oporów cieplnych ściany bocznej oraz dna i pokrywy zbiornika.

Słowa kluczowe: destratyfikacja termiczna, równanie przewodzenia ciepła, odnawialne źródła energii

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1. Introduction

A temporary, a twenty-four-hour, as well as a seasonal periodical operation of renewable energy sources require the employment of heat storage systems [1, 2]. The storage of thermal energy in solar systems considerably improves their power efficiency – the energy delivery can be continuous and proportional to the energy demand. Both the collection of heat from the water and the delivery of heat to the water in a tank cause a differentiation of water temperatures called "thermal stratification": the hot water is located in the top space of tank, while the cooler in the bottom. The thermal stratification is a desired phenomenon, which improves the power efficiency of the installation. A phenomenon opposite to thermal stratification is spontaneous equalisation of temperatures (destratification). The intensity of this process depends on the vertical temperature gradient: the greater the vertical temperature gradient, the greater the intensity. Destratification appears when a break in the delivery and collection of heat occurs in a tank with different water temperature layers. A destratification process usually is overlapped by a heat transfer process through the tank wall. The process of thermal stratification was the subject of many experimental works. An overview of issues related to the thermal stratification in water storage tanks was presented recently in [3].

In this paper, a mathematical model and the results of simulation calculations of the temperature equalisation process in a tank with several different water temperature layers has been presented. The model also takes into account heat losses to the surroundings through the side wall, the tank bottom and the tank cover. A simplified form of this model, only taking into account heat losses through the side wall and its experimental verification, was given in [4].

2. Calculational relations

A vertical axis tank full of different water temperature layers has been considered. The temperature of each layer is higher than the temperature of the surroundings. As a result of both the heat conduction and convection, equalisation of temperatures in the tank appears. At the same time, the tank contents are cooled as a result of simultaneous heat losses to the surroundings through the side wall, the bottom and the cover.

Heat transport in water has been described with the use of the heat conduction equation with an effective heat conduction coefficient also taking into account the convection.

For high values of the tank height to the tank diameter ratio, the radial water temperature changes can be neglected. Heat losses through the tank side wall can be described with a source term (not with the boundary conditions). This allows for a mathematical description of both the heat transfer inside the tank and the heat transfer through the side wall with the source term, simultaneous to heat transfer through the tank bottom and the tank cover with boundary conditions. For this reason, the s height tank has been assumed in the model as an s thick infinite plate.

For transient heat conduction in an infinite plate with a heat source, the following relation is valid [5, 6]:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} + \frac{q_v}{\rho c} \tag{1}$$

where:

T – temperature, K,
t – time, s,

$a = k/(\rho c)$ – thermal diffusivity coefficient, m²/s,
x – position coordinate, m,
k – effective heat conduction coefficient, W/(m·K),
q_v – capacity of heat source, W/m³,
c – specific heat, J/(kg·K),
ρ – density, kg/m³.

Heat losses through the side wall can be calculated using the following relation:

$$d\dot{Q} = U(T - T_a)dA \tag{2}$$

where:

U – overall heat transfer coefficient between water and surroundings, W/(m²·K),
T_a – temperature of the surroundings, K,
A – heat transfer surface area, m².

In the presented model, q_v describes heat transport through the side wall. The heat source capacity is thermal power generated in a unitary volume of solid. In the case of heat loss, q_v is negative. Because $q_v = -d\dot{Q}/dV$, one can obtain:

$$q_v = \frac{dA}{dV} U(T_a - T) \tag{3}$$

The quantity of q_v is a function of the position in tank, $q_v(x)$, because of the temperature variability in the tank. When $T > T_a$, the contents of tank lose the heat: the greater the T, the greater the heat losses. But the ratio dA/dV is constant. For a vertical axis and D diameter cylindrical tank, the following relation is valid:

$$\frac{dA}{dV} = \frac{4}{D} \tag{4}$$

therefore:

$$q_v = \frac{4U}{D}(T_a - T) \tag{5}$$

There are two different temperature water layers in the simplest case of the equalisation of temperatures. Let's assume that, at the beginning, the water temperature T_0 is in the range of height $(0, s_m)$, whereas T_1 (usually $T_1 > T_0$) is in the range of height (s_m, s).

Then, the initial condition is as follows (Fig. 1):

$$t = 0 \qquad \begin{cases} \text{for } 0 < x \le s_m & T = T_0 \\ \text{for } s_m < x \le s & T = T_1 \end{cases} \tag{6}$$

The mean water temperature in the tank at the beginning of the process can be calculated as a weighted average:

$$T_m = \frac{s_m}{s} T_0 + \left(1 - \frac{s_m}{s}\right) T_1 \tag{7}$$

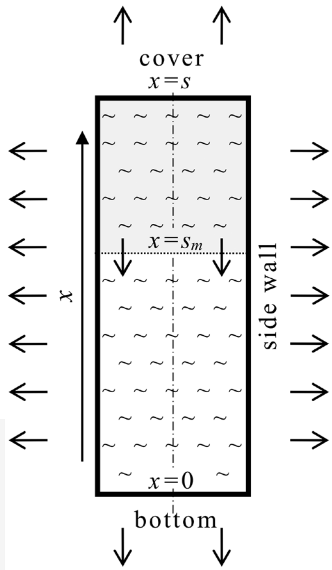

Fig. 1. Vertical axis tank

The boundary conditions concern the tank bottom and the tank cover. For the tank bottom:

$$x = 0 \qquad k\frac{\partial T}{\partial x} = \frac{T - T_s}{R_0} \tag{8}$$

For the tank cover:

$$x = s \qquad -k\frac{\partial T}{\partial x} = \frac{T - T_s}{R_1} \tag{9}$$

R_0 and R_1 are the thermal resistances of the tank bottom and the tank cover, respectively, while T_s is the external temperature of the surroundings near the tank bottom/the tank cover. The temperature T_s can be different to T_a.

In the case of a high thermal resistance of the tank bottom and the tank cover, R depends mainly on the insulation conduction resistance s_i/k_i. Generally, the resistances depend also on the partial heat transfer coefficients h_0 and h_1 outside the tank:

$$R_0 = \frac{1}{h_0} + \frac{s_{0i}}{k_{0i}} \qquad R_1 = \frac{1}{h_1} + \frac{s_{1i}}{k_{1i}} \tag{10}$$

If the tank bottom and the tank cover are perfectly insulated ($R \to \infty$), the boundary conditions are as follows:

$$x = 0 \qquad \frac{\partial T}{\partial x} = 0 \tag{11}$$

$$x = s \qquad \frac{\partial T}{\partial x} = 0 \tag{12}$$

Relations (11) and (12) denote non-permeability of heat flux through the tank bottom and the tank cover, respectively.

The model equations can be solved easily with the method of finite differences.

3. Results of calculations

In all of the calculations, the initial temperature in the top tank space is equal to 90°C, whereas in the bottom, it is 70°C. The total tank height equals 1.8 m. The upper layer ranges from 1.1 m upwards. The assumption $T_a = T_s = 20$°C has been made.

Fig. 2 presents the temperature profiles in a totally insulated tank. The value of effective heat conduction coefficient equal to $k = 0.5$ W/(m·K), approximate to the water conductivity, has been assumed in the calculations. Each individual course concerns a specific process time length. It can be observed that the equalisation of temperatures is slow and the system temperature tends to be a value of $T_m = 77.8$°C, calculated according to the relation (7).

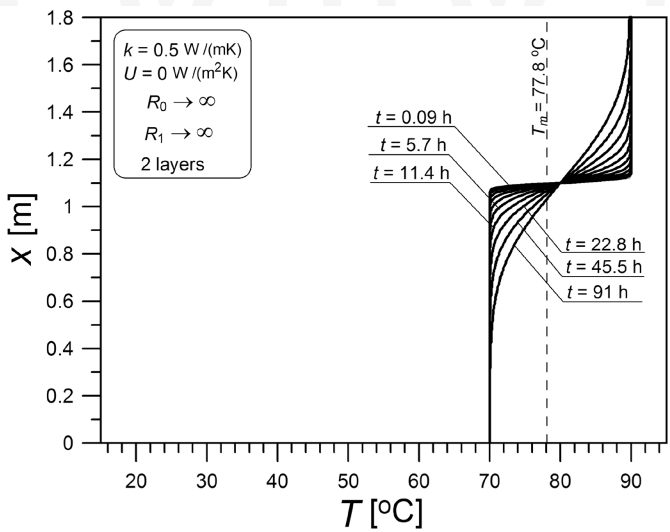

Fig. 2. Temperature profiles in tank filled with water; side wall, bottom and cover perfectly insulated; two water layers; $k = 0.5$ W/(m·K)

The temperature profiles in a tank initially containing three different water temperature layers – upper (90°C), intermediate (80°C) and bottom (70°C) – have been also determined. The height of the intermediate layer ranges from 0.9 to 1.1 m. As it was previously, the tank is totally insulated and the effective heat conduction coefficient equals 0.5 W/(m·K).

The courses of temperature profiles, presented in Fig. 3, result from the existence of three different water temperature layers at the beginning of the process. The system tends to reach a temperature of T_m, resulting from the following calculation:

$$T_m = \frac{0.9}{1.8}\cdot 70 + \frac{0.2}{1.8}\cdot 80 + \frac{0.7}{1.8}\cdot 90 = 78.9°C \tag{13}$$

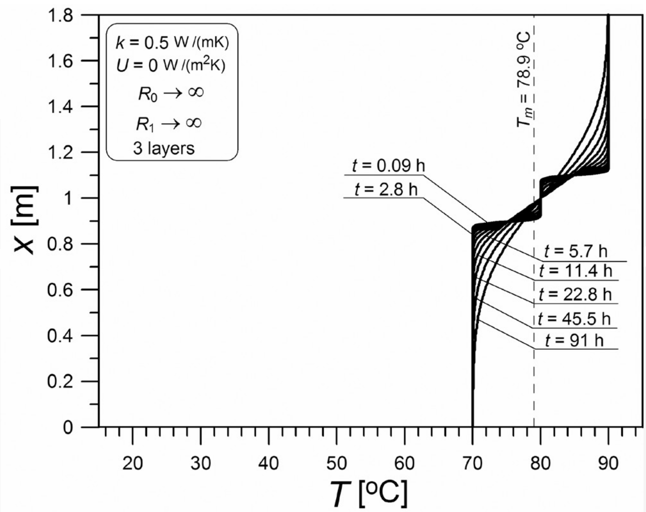

Fig. 3. Temperature profiles in tank filled with water; side wall, bottom and cover perfectly insulated; three water layers; $k = 0.5$ W/(m·K)

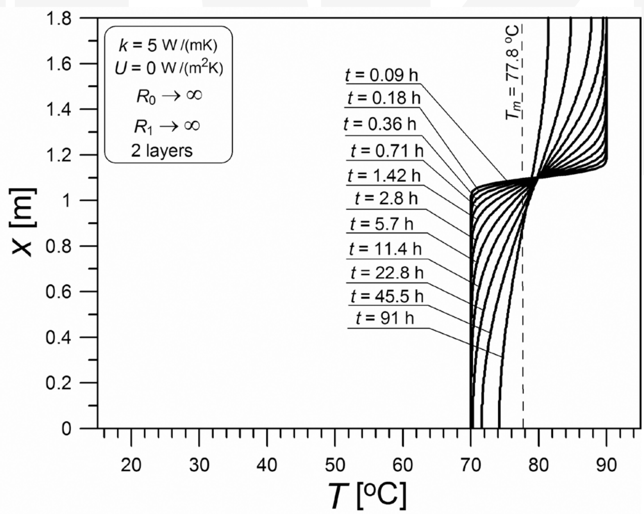

Fig. 4. Temperature profiles in tank filled with water; side wall, bottom and cover perfectly insulated; two water layers; $k = 5$ W/(m·K)

More realistic shapes of temperature profiles have been obtained under the assumption of a value of the effective heat conduction coefficient equal to k = 5 W/(m·K), i.e. as much as 10 times greater than a value of the ordinary coefficient, independent of the convection. The profiles are presented in Fig. 4. An assumption of the total tank insulation also has been made. The system tends to reach an equilibrium temperature of T_m = 77.8°C faster. In real conditions of the destratification process, a considerable amount of heat is transferred by the convection accelerating the equalisation process of temperatures in relation to the conduction, which practically does not take place in pure form within fluids.

The actual temperature profiles in a tank generally have a different shapes resulting from heat losses to the surroundings. If heat losses exist, a cooling process of the tank contents overlaps the process of the equalisation of temperatures. It should be taken into account that the rate of the cooling process varies both in time and along the tank height. The higher the water temperature, the greater the cooling rate. Figs. 5, 6 and 7 present profiles of various combinations of thermal resistances values of different tank elements: the side wall, the bottom and the cover.

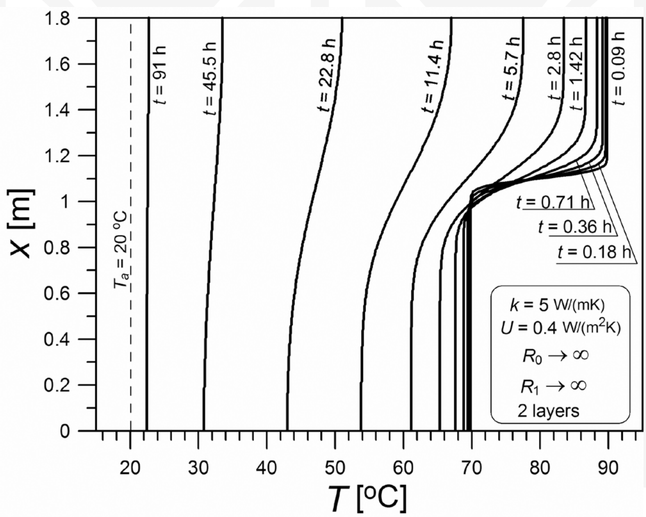

Fig. 5. Temperature profiles in tank filled with water; bottom and cover perfectly insulated; two water layers; k = 5 W/(m·K)

Fig. 5 depicts the temperature profiles in the process of the equalisation of temperatures as well as the process of the cooling of the tank content, if the tank bottom and the tank cover are perfectly insulated. The value of the overall heat transfer coefficient of U = 0.4 W/(m^2·K) has been assumed. Because of the non-permeability of heat through the tank bottom and the tank cover, the temperature profiles near x = 0 and x = s are vertical, in accordance with the boundary conditions (11) and (12). The longer the process time, the more flat the temperature profiles. The system tends to reach a temperature of 20°C, i.e. the temperature of the surroundings.

The presented model gives the possibility to take into account both the heat losses through the side wall as well as the heat losses through the tank bottom and the tank cover (it is possible to assume that the temperature of the surroundings near the tank cover and near the tank bottom T_s is different than the temperature near the side wall T_a). Fig. 6 depicts the temperature profiles in the case of a perfectly insulated tank cover only. A difference in courses in Figs. 5 and 6 can be observable in the bottom figure part: near $x = 0$ the profiles are vertical (Fig. 5) or are deflected (Fig. 6).

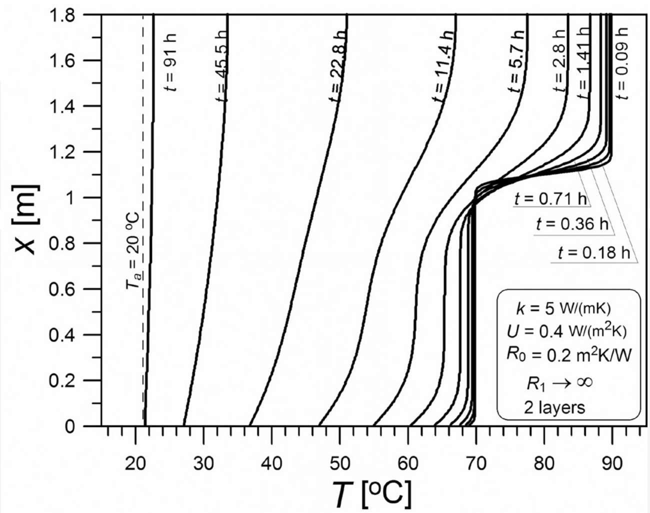

Fig. 6. Temperature profiles in tank filled with water; tank cover perfectly insulated; two water layers; $k = 5$ W/(m·K)

Fig. 7 presents all of the possible directions of heat loss from water: a horizontal direction through the tank side wall, and a vertical direction downwards through the tank bottom, and upwards through the tank cover. The heat resistance of the tank cover has been assumed to be considerably lower than the heat resistance of the tank bottom. This assumption causes a strong deflection of temperature profiles in the upper tank space.

A comparison of profiles in Figs. 5, 6 and 7 leads to a conclusion about the rate of heat losses in the considered cases. Under the conditions described in Fig. 5, the heat losses are the lowest, whereas under conditions described in Fig. 7 – the highest. This situation is reflected by the water temperature ranges in the tank. By way of example, for the process time length $t = 22.8$ hr and for the existence of heat losses only in the side wall (Fig. 5), the water temperature in the tank ranges from 43°C to 51°C. If the heat losses exist also in the tank bottom, the water temperature in the tank ranges from 37°C to 50°C (Fig. 6). However, if the heat losses exist also in the tank bottom, the water temperature in the tank after 22.8 hr ranges from 31°C to 37°C (Fig. 7). In the last case, the water temperature near the tank cover is lower than the temperature near the tank bottom (in spite of an opposite relation at initial conditions) because of a small value of the tank cover heat resistance causing a considerable heat loss in this location.

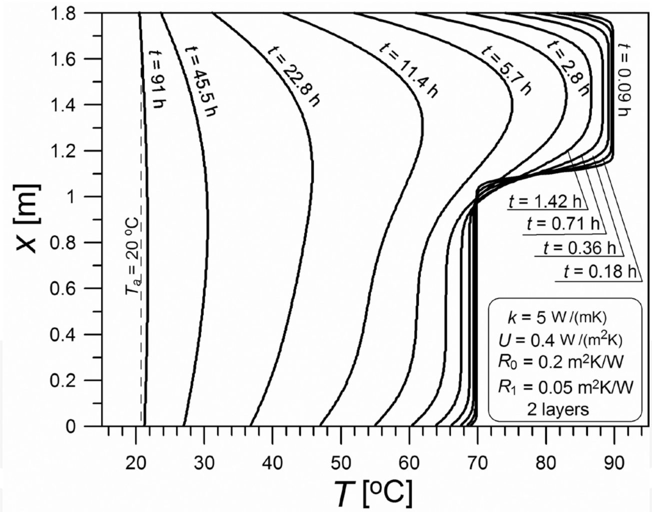

Fig. 7. Temperature profiles in tank filled with water; two water layers; k = 5 W/(m·K)

4. Conclusions

The presented mathematical model is based on a one-dimensional equation of heat conduction, describing simultaneous heat transfer inside of a vertical axis tank and heat losses through the side wall, the tank bottom, and the tank cover, to surroundings.

The model has been verified experimentally and can be applied for the simulation of thermal processes in storage tanks filled with hot water.

The model is valid for both the heat transfer resulting from the internal temperature gradients as well as for the heat transfer resulting from the temperature differences between the tank interior and the surroundings.

Model parameter values can be determined on the basis of a comparison between the values obtained in the calculations and in the measurements. The model parameters are: an effective heat conduction coefficient and the thermal resistances through the side wall, the bottom and the cover of the tank.

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