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DYNAMIC ASPECTS OF INTERACTION: PANTOGRAPH – CATENARY

DYNAMICZNE ASPEKTY WSPÓŁPRACY PANTOGRAFU Z SIECIĄ TRAKCYJNĄ

Abstract

There are many existing models for the interaction of a pantograph–catenary system. The authors propose a specific model, which is amended by taking into account the dynamic part of the motion equation. This model was compared to another attempt and conclusions were drawn. A numerical study was performed using the Matlab Simulink Environment.

Keywords: pantograph, catenary, pantograph-catenary model, traction system

Streszczenie

W artykule przedstawiono zagadnienie modelowania układu drgającego odbierak prądu–sieć trakcyjna jako modelu fizycznego z masą skupioną sieci trakcyjnej. Do celów symulacyjnych spośród wielu istniejących modeli pantografów wybrano model WBL 85 3 kV, w który m.in. wyposażone są lokomotywy EU11 oraz EU43. Zaprezentowano wyniki symulacji dla tego modelu odbieraka prądu przy ruchu postępowym wzdłuż sieci jezdnej. Zaprezentowane wyniki pochodzą z symulacji wykonanej dla dwóch modeli matematycznych sieci trakcyjnej. Modele te różniły się od siebie jednym czynnikiem, który zazwyczaj jest pomijany w badaniach nad opisywanym układem. Następnie wyniki te porównano.

Słowa kluczowe: trakcja elektryczna, tunele, profil trasy kolejowej, zużycie energii

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1. Introduction

With an increase in the speed potential of traction vehicles, there is also an increase in the quality requirements related to their electrical energy consumption. The energy is transferred by a moving contact point between the pantograph and the catenary [1]. It is difficult to maintain a constant clamping force of the pantograph to the catenary in order to ensure a continuous and uninterrupted supply of energy to the traction vehicle [1].

Modeling of the described system in terms of dynamic properties is a complex task that is affected, among other things, by: vehicle speed and parameters of the OCS system (from Over Contact System). Other influential factors also include interfering factors related to the environment of the OCS system, such as [1]: geometrical characteristics of the track and their changes that occur during the operation, atmospheric conditions like air temperature and its humidity, icing of traction networks, wind speed and its direction changes, and others. As can be seen, because of the multiplicity of various aspects, the optimization of the parameters of the pantograph – catenary system – requires comprehensive research and analysis. Simulations of the OCS system are presented *inter alia* in publications: [2–5].

2. Model of the pantograph and catenary

Figure 1 shows the physical model of interaction in the OCS. The pantograph in this model has four degrees of freedom and consists of a lower frame, an upper frame and a panhead. The model of the pantograph consists of four effective moving masses, which represent the components listed above. They are interconnected by spring and damping forces [3]. The effective masses, springs and dampers are matched to the actual behavior of the pantograph in dynamic conditions.

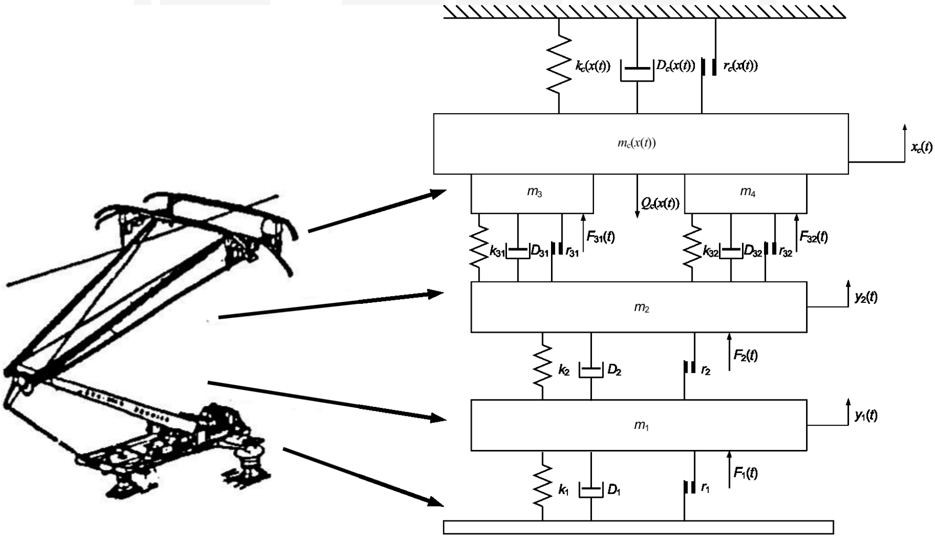

Fig. 1. The pantograph model (illustration on the left side from [4])

The dynamic properties of the pantograph are determined by parameters, such as a concentrated mass (reduced to the contact point with the catenary), static force of the pantograph on the contact wire, aerodynamic force, electrodynamic force and other factors.

The described model of interaction in the OCS also comprises the model of the catenary system, taking into account the instantaneous value of the reduced mass as a function given by the relation (3a).

The authors formulated the assumptions for the catenary model in the following way [3]:
- the system only performs vertical oscillations,
- the catenary system is symmetric,
- the catenary span length is fixed and determined,
- the catenary is made of a homogeneous material,
- the materials used for the construction of the catenary are isotropic,
- the equivalent mechanical parameters of the catenary change periodically and meet the conditions of Dirichlet,
- the catenary affects masses m_3 and m_4 in the same way, at any time,
- the loss of contact between masses m_3 and m_4 and the catenary element $m_c(x(t))$ is not possible,
- an equivalent center of gravity was assumed for the elements with masses m_3, m_4, $m_c(x(t))$, whose displacement and velocity coordinates were described by the coordinates of the state $x_c(t)$ and $\frac{dx_c(t)}{dt}$.

The following parameters were assumed in the description of the discrete catenary model:

$m_c(x(t))$ – function of the reduced mass of the catenary,
$D_c(x(t))$ – function of viscous damping of the catenary,
$k_c(x(t))$ – function of the stiffness of the catenary,
$r_c(x(t))$ – function of dry damping of the catenary,
$Q_c(x(t))$ – the sum of the instantaneous value of the weight acting on the panhead of the pantograph and the weight of the masses m_3, m_4,
L – span length,
$x(t)$ – instantaneous distance measured along the catenary from the nearest pole to the pantograph,
$v(t)$ – instantaneous velocity of the translational motion of the pantograph, measured in the longitudinal direction of the catenary,
a – acceleration of the translational motion of the pantograph, measured in the longitudinal direction of the catenary,
V_0 – initial velocity of the translational motion of the pantograph, measured in the longitudinal direction of the catenary,
x_0 – initial distance measured along the catenary from the nearest pole to the pantograph.

The model takes into account, inter alia, the impact of the aerodynamic force on the pantograph, which is usually skipped in less complex models. No information about the values of the coefficients of air resistance K_{31}, K_{32}, K_2 for the pantograph model caused that they were assumed as constants. However, the assumed values may not correspond to the actual values of these coefficients.

At the same time, the model presented in Fig. 1 does not take into account the impact of vibration of the traction vehicle on the vibration of the pantograph base frame nor the possibility of loss of contact of masses m_3 and m_4 with the catenary element $m_c(x(t))$. In fact, the vibrations of a locomotive body are directly transferred on the pantograph base frame, as well as the mentioned loss of contact is possible.

In the description of the system vibrations corresponding to masses m_1 and m_2 the displacement coordinates were assumed relative to their equilibrium positions.

The mathematical model for the model of the pantograph-catenary interaction assumed in the article is presented by (1). The coefficients and variables in this equation are explained above and in Fig. 1.

$$
\begin{gathered}
\frac{d}{dt}\left[\left(m_c\left(x(t)\right)+m_3+m_4\right)\cdot\frac{dx_c(t)}{dt}\right] \\
=-k_c\left(x(t)\right)\cdot x_c(t)-D_c\left(x(t)\right)\cdot\frac{dx_c(t)}{dt}-r_c\cdot \mathrm{tg}h\left(\kappa\cdot\frac{dx_c(t)}{dt}\right)-\left(k_{31}+k_{32}\right)\cdot\left(x_c(t)-y_2(t)\right) \\
-\left(D_{31}+D_{32}\right)\cdot\left(\frac{dx_c(t)}{dt}-\frac{dy_2(t)}{dt}\right)-\left(r_{31}+r_{32}\right)\cdot \mathrm{tg}h\left(\kappa\cdot\left(\frac{dx_c(t)}{dt}-\frac{dy_2(t)}{dt}\right)\right)+F_{31}(t)+F_{32}(t)-Q_c\left(x(t)\right) \\
m_2\cdot\frac{d^2y_2}{dt^2}=\left(k_{31}+k_{32}\right)\cdot\left(x_c(t)-y_2(t)\right)+\left(D_{31}+D_{32}\right)\cdot\left(\frac{dx_c(t)}{dt}-\frac{dy_2(t)}{dt}\right)+\left(r_{31}+r_{32}\right)\cdot \mathrm{tg}h\left(\kappa\cdot\left(\frac{dx_c(t)}{dt}-\frac{dy_2(t)}{dt}\right)\right)-k_2\cdot\left(y_2(t)-y_1(t)\right) \\
-D_2\cdot\left(\frac{dy_2(t)}{dt}-\frac{dy_1(t)}{dt}\right)-r_2\cdot \mathrm{tg}h\left(\kappa\cdot\left(\frac{dy_2(t)}{dt}-\frac{dy_1(t)}{dt}\right)\right)+F_2(t) \\
m_1\cdot\frac{d^2y_1}{dt^2}=k_2\cdot\left(y_2(t)-y_1(t)\right)+D_2\cdot\left(\frac{dy_2(t)}{dt}-\frac{dy_1(t)}{dt}\right)+ \\
r_2\cdot \mathrm{tg}h\left(\kappa\cdot\left(\frac{dy_2(t)}{dt}-\frac{dy_1(t)}{dt}\right)\right)-k_1\cdot y_1(t)-D_1\cdot\frac{dy_1(t)}{dt}-r_1\cdot \mathrm{tg}h\left(\kappa\cdot\frac{dy_1(t)}{dt}\right)+F_1
\end{gathered}
\tag{1}
$$

The mathematical model of the time derivative of the product of the mass of the catenary $m_c(x(t))$ and the vertical velocity $\frac{dx_c(t)}{dt}$ for the model assumed in the article is given by equation (2a) or (2b). Equation (2b) includes the impact of the product of the time derivative of the mass and the vertical velocity $\frac{dx_c(t)}{dt}$ on the behavior of the OCS system. In the present paper, the model of the OCS interaction, which uses equation (2a) in the system of equations (1), is called the model without the mass time derivative term, while the model of the OCS interaction, which uses equation (2b) in the system of equations (1), is referred to as the model taking into account the time derivative of the mass.

$$
\frac{d}{dt}\left[m_c\left(x(t)\right)\cdot\frac{dx_c(t)}{dt}\right]=m_c\left(x(t)\right)\cdot\frac{d^2x_c(t)}{dt^2}
\tag{2a}
$$

$$\frac{d}{dt}\left[m_c\left(x(x)\right)\cdot\frac{dx_c(t)}{dt}\right]=\frac{dm_c\left(x(t)\right)}{dx(t)}\cdot\frac{dx(t)}{dt}\cdot\frac{dx_c(t)}{dt}+m_c\left(x(x)\right)\cdot\frac{d^2x_c(t)}{dt^2} \tag{2b}$$

Where:

$$m_c\left(x(t)\right)=m_{co}+\sum_{i=1}^{3}m_{ci}\cdot\cos\left(\frac{2\pi i}{L}\cdot x(t)\right) \tag{3a}$$

$$D_c\left(x(t)\right)=D_{co}+\sum_{i=1}^{3}D_{ci}\cdot\cos\left(\frac{2\pi i}{L}\cdot x(t)\right) \tag{3b}$$

$$k_c\left(x(t)\right)=k_{co}+\sum_{i=1}^{3}k_{ci}\cdot\cos\left(\frac{2\pi i}{L}\cdot x(t)\right) \tag{3c}$$

Distance as a function of time:

$$\begin{aligned} x(t)&=0.5\cdot a\cdot t^2+v_0\cdot t+x_0\\ v(t)&=a\cdot t+v_0 \end{aligned} \tag{4}$$

Forces acting on the catenary:

$$Q_c\left(x(t)\right)=\left(m_c\left(x(t)\right)+m_3+m_4\right)\cdot g \tag{5}$$

$$\begin{aligned} F_{31}(t)&=K_{31}\cdot v^2(t)\\ F_{32}(t)&=K_{32}\cdot v^2(t)\\ F_2(t)&=K_2\cdot v^2(t) \end{aligned} \tag{6}$$

3. Simulation results

From among many existing models of pantographs, the type WBL 85 3kV was selected as the simulation model. Its parameter values were taken from [3].

This paper presents the analysis of movement for the chosen pantograph type interacting with a catenary in which the derivative of the product of catenary mass and vertical velocity $\frac{dx_c(t)}{dt}$ is represented by equation (2a) or (2b). Simulations were carried out under the following assumptions [3]:

- the system performs only vertical oscillations,
- the damping in kinematic pairs of the pantograph is a dry damping, as well as viscous damping,
- elements connecting the reduced masses of the pantograph arms m_1, m_2 have linear characteristics with constant coefficients k_2, D_2, r_2,
- elements connecting the lower mass m_1 with the pantograph's base frame have linear characteristics with constant coefficients k_1, D_1, r_1,
- elastic elements connecting the reduced masses of pantoheads m_3, m_4 with the upper pantograph arm m_2 have a nonlinear characteristics,
- the value of the static force $F_1(t)$ being the impact of the pantograph lifting system on the catenary is the same at any speed and does not depend on the position of the body with mass m_1,

- the velocity $v(t)$ of the moving pantograph is constant,
- the oscillations of the system for the elements m_1 and m_2 take place around the static equilibrium positions, which result from the action of the force of gravity coming from these masses,
- the simulations were performed with initial conditions assumed as zero.

The following parameters of the pantograph were assumed in calculations of the model of the OCS [3]:

Mass [kg]:

$m_1 = 10.15$;
$m_2 = 8.73$;
$m_3 = 7.93$;
$m_4 = 7.93$;

Stiffness [N/m]:

$k_1 = 13\ 500$;
$k_2 = 13\ 500$;

Viscous damping [kg/s]:

$D_1 = 60$;
$D_2 = 0$;
$D_{31} = 0$;
$D_{32} = 0$;

Dry damping [N]:

$r_1 = 2{,}5$;
$r_2 = 2{,}5$;
$r_{31} = 2$;
$r_{32} = 2$;

Drag coefficient [kg/m]:

$K_2 = 0.36 \cdot 0.7/3$;
$K_{31} = 0.36 \cdot 0.7$;
$K_{32} = 0.36 \cdot 0.7$;

The value of the static force $F_1(t)$ was assumed as [3]:

$F_1 = 110$ N;

Figure 2 shows the nonlinear characteristics of the spring $F_{k3j} = f(y_2(t) - x_c(t))$ (where $j = 1, 2$) [3], corresponding to the magnitudes k_{31}, k_{32} (Fig. 1) at specified intervals of the displacement difference $y_2(t) - x_c(t)$.

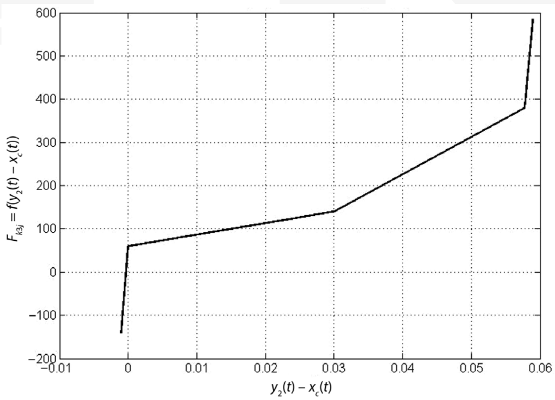

Fig. 2. The nonlinear characteristics $F_{k3j} = f(y_2(t) - x_c(t))$ (where $j = 1, 2$) of a spring [3] referring to the values k_{31}, k_{32} (Fig. 1) in specific intervals of differences of displacements $y_2(t) - x_c(t)$

The catenary parameters assumed as in [5]. They have the following values:

Mass [kg]:	Stiffness [N/m]:	Viscous damping [kg/s]:
$m_{c0} = 195$;	$k_{c0} = 7000$;	$D_{c0} = 240$;
$m_{c1} = 100$;	$k_{c1} = 3360$;	$D_{c1} = 240$;
$m_{c2} = 20$;	$k_{c2} = 650$;	$D_{c2} = 50$;
$m_{c3} = 5$;	$k_{c3} = 160$;	$D_{c3} = 12$;

Dry damping [N]:	Span length [m]:
$r_c = 0$;	$L = 65.52$;

It was assumed that the pantograph moved with uniform motion along the catenary:

$a = 0$ m/s^2;
$x_0 = 0$ m;
$V_0 = 20$ m/s.

The value of acceleration due to gravity was taken as $g = 9.81$ m/s^2.

Figure 3 shows the function of the reduced mass depending on time for velocity $V_0 = 10$, 20 and 40 m/s and simulation time equal to 6.552 s. This time refers to traveling, respectively, once, twice and four times the assumed distance L at given speeds V_0. The line denoted by corresponds to the speed $V_0 = 10$ m/s, the solid line corresponds to the speed $V_0 = 20$ m/s, and to the line denoted by corresponds to the speed $V_0 = 40$ m/s. The functions in Fig. 3 are drawn for the condition of the initial pantograph position $x_0 = 0$ m.

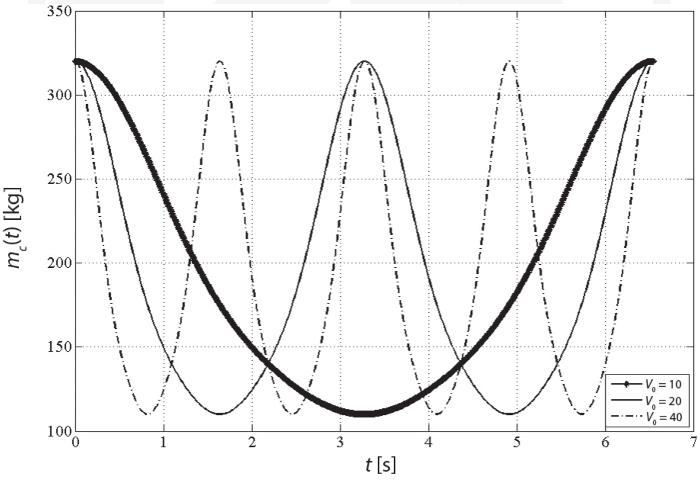

Fig. 3. The function of the reduced mass versus time for velocity $V_0 = 10$, 20 and 40 m/s of the pantograph moving with respect to the catenary

The next part of this paper presents the results corresponding to the speed V_0 = 20 m/s (72 km/h) for the simulation time from 0 s to 3·6.552 s. The simulation time was increased due to the occurrence of an aperiodic component. The simulation was performed for the condition of the initial pantograph position x_0 = 0 m.

Figure 4 shows the timeline of the vertical displacement $x_c(t)$ calculated from the model of the system without the time derivative of the mass component. This displacement is called $x_c(t)$ in the graph. Figure 4 also shows the timeline of the vertical displacement $x_c(t)$ calculated from the model, taking into account the time derivative of the mass. This displacement is called *x*cpoch(t) in the figure.

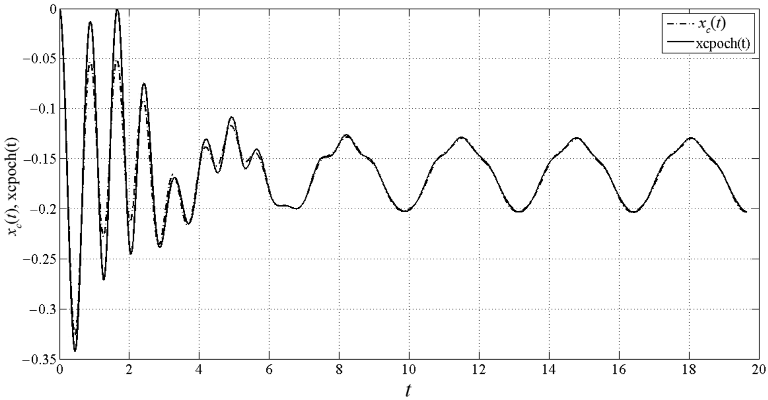

Fig. 4. Time courses of vertical displacement $x_c(t)$ calculated for the discussed models of the OCS for movement of the pantograph along the catenary with speed V_0 = 20 m/s. The solid line – denotes the model taking into account the time derivative of mass; the dashed line – is for the model without the time derivative of the mass

Figure 5 depicts the time course of the instantaneous difference in the vertical displacement $x_c(t)$ calculated for the compared models of the OCS for movement of the pantograph along the catenary with speed V_0 = 20 m/s. The difference was calculated as the difference between the displacement $x_c(t)$ from the model, taking into account the time derivative of mass and the displacement $x_c(t)$ from the model without the time derivative of mass component.

Figure 6 shows time courses of vertical velocity $\frac{dx_c(t)}{dt}$ calculated for the discussed models of the OCS for movement of the pantograph along the catenary with speed V_0 = 20 m/s.

Figure 7 gives the time course of the instantaneous difference in the vertical velocity $\frac{dx_c(t)}{dt}$ calculated for the compared models of the OCS for movement of the pantograph along the catenary with speed V_0 = 20 m/s. The difference was calculated as the difference between the velocity $\frac{dx_c(t)}{dt}$ calculated from the model, taking into account the time derivative of mass and the displacement $\frac{dx_c(t)}{dt}$ from the model without the time derivative of mass component.

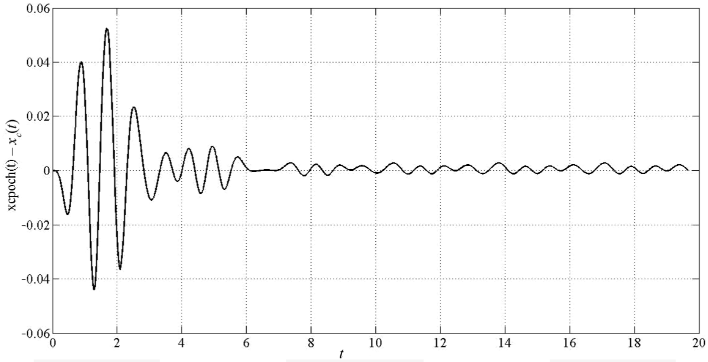

Fig. 5. Time course of the instantaneous difference in the vertical displacement $x_c(t)$ calculated for the compared models of the OCS for movement of the pantograph along the catenary with speed $V_0 = 20$ m/s

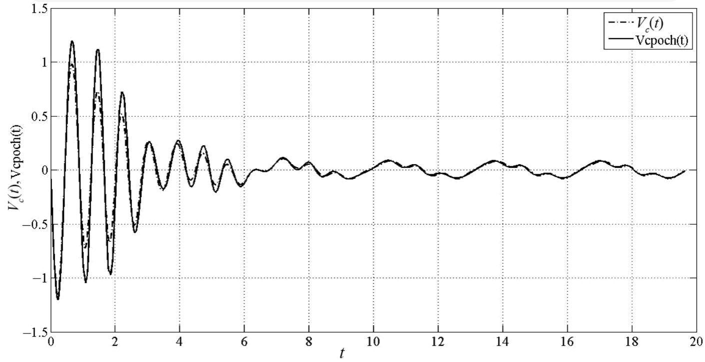

Fig. 6. Time courses of vertical velocity $\frac{dx_c(t)}{dt}$ calculated for the discussed models of the OCS for movement of the pantograph along the catenary with speed $V_0 = 20$ m/s
Vcpoch(t) – velocity calculated for the model including the time derivative of mass; $V_c(t)$ – is for the model without the time derivative of the mass component

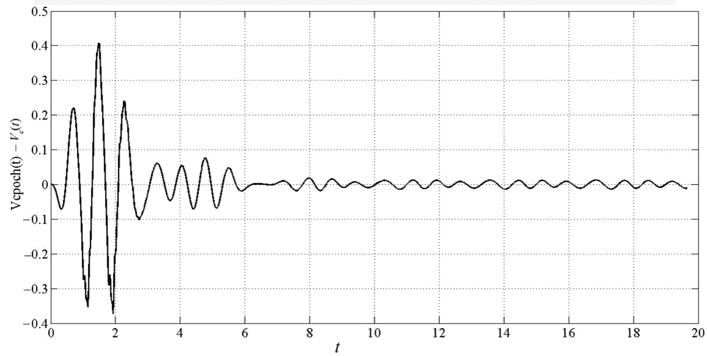

Fig. 7. Time course of the instantaneous difference in the vertical velocity $\frac{dx_c(t)}{dt}$ calculated for the compared models of the OCS for movement of the pantograph along the catenary with speed $V_0 = 20$ m/s

4. Conclusions

The presented study takes into account the effect of aerodynamic forces acting on the pantograph. Modeling of the pantograph was simplified by omitting the impact of vehicle vibrations on the vertical movements of the pantograph. In the model interaction OCS, it was assumed that the loss of contact between masses m_3 and m_4 and the catenary element $m_c(x(t))$ is not possible (Fig. 1).

Simulations were performed in Matlab using method ode23. The simulation results indicate that, for the chosen zero initial conditions, the calculated displacements $x_c(t)$ and velocity $\frac{dx_c(t)}{dt}$ after certain time reach a steady state for the considered models of the OCS for the movement of the pantograph along the catenary with speed $V_0 = 20$ m/s.

Based on the obtained simulation results, it can be concluded that the difference between the values of the instantaneous vertical displacements $x_c(t)$ calculated for the discussed models of the OCS, for movement of the pantograph along the catenary with speed $V_0 = 20$ m/s, with given initial conditions, for the time interval without the aperiodic component of the vertical displacement $x_c(t)$, can reach values larger than 0.002 m.

The simulation results also indicate that the difference between the values of the instantaneous vertical velocities $\frac{dx_c(t)}{dt}$ calculated for the discussed models of the OCS, for movement of the pantograph along the catenary with speed $V_0 = 20$ m/s, with given initial conditions, for the time interval without the aperiodic component of the vertical velocity $\frac{dx_c(t)}{dt}$, can reach values exceeding 0.01 m/s in absolute value.

It should be noted that the simulation results have not been verified in real conditions and require further work and analysis. The initial conditions were taken as zero, which causes the time courses presented here to have an aperiodic character. Such a state exposes the most significant impact of the product of the time derivative of the mass and velocity $\left(\frac{dm_c\left(x(t)\right)}{dx(t)} \cdot \frac{dx(t)}{dt} \cdot \frac{dx_c(t)}{dt}\right)$ on the OCS interaction· The authors suggest appropriate choice of initial conditions for the analysis of the OCS interaction.

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