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INNOVATIVE ANALYSIS METHODS OF REINFORCED CONCRETE STRUCTURES: THE STRUT AND TIE METHOD

NOWOCZESNE METODY ANALIZY KONSTRUKCJI ŻELBETOWYCH: METODA STRUT AND TIE

Abstract

The paper is devoted to the strut-and-tie method (STM) which is used for analysis and design of reinforced concrete structures. The most important and valuable information about the STM has been gathered and the example that applies to the method has been presented.

Keywords: strut-and-tie method, corbels, reinforced concrete structures

Streszczenie

Metoda strut-and-tie służy do lokalnych analiz konstrukcji żelbetowych dla obszarów z silnymi nieciągłościami, w których założenie o liniowym rozkładzie naprężeń nie jest spełnione. Innowacyjność metody polega na wykorzystaniu zasad teorii plastyczności oraz założeń odnoszących się do transferu sił z prętów zbrojeniowych na beton. W artykule zaprezentowano aspekty teoretyczne metody oraz przedstawiono przykład jej zastosowania do wymiarowania krótkich wsporników. Otrzymane rezultaty porównano z wynikami analizy alternatywną metodą belkową.

Słowa kluczowe: metoda strut-and-tie, krótkie wsporniki, konstrukcje żelbetowe

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1. Introduction

The strut-and-tie method (STM) is nowadays considered to be a powerful tool for the analysis by the limited stages of reinforced concrete structures with non-linear stress distribution. Although STM can be applied to all parts of reinforced concrete structures, it is usually applied to regions of stress discontinuities (*D*-Regions). *D*-regions which are not governed by Saint-Venant's principle are located near abrupt changes in geometry, openings, concentrated forces, anchorage zones etc.
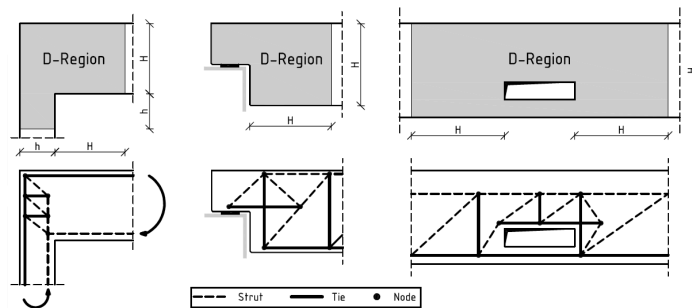

Fig. 1. Typical D-Regions and corresponding strut-and-tie models

The innovation of the method is the usage of plasticity principles and assumptions regarding the flow of forces in reinforced concrete structures. The STM is based on the lower-bound theory of the limit analysis. The strut-and-tie model is a statically admissible stress field in lower bound (static) solutions. The total capacity of a strut-and-tie model is always smaller or equal to the real capacity of the construction and, as a result, ensures a certain safety margin.

The essence of the method is changing the stress trajectories in an element into compressive or tensile forces and linking them in nodes. The strut-and-tie model is a truss-analogy of the construction in which following elements can be identified: struts, ties and nodes.

The role of the struts is bearing the compressive stresses. The struts are modelled as straight axial-compressed bars, although the real stress state is multi-axial. On the basis of the flow of the compressive strains, we can define three strut shapes: bottle-, fan- or prismatic-shaped. The natural tendency of struts to extend between the nodes can be suppressed by creating a reinforcement in the perpendicular line distributed over the length, as well as by the reduction of the compressive strength of concrete in struts.

Ties are modelled as linear axial tension elements which represent reinforcement that is smeared in layers. The centroid of the reinforcement corresponds to the center of the tie. It is crucial to ensure adequate anchorage of reinforcement out of the nodal zones, where it is not required.

Nodes can be classified according to the type of the forces which meet in the node, where *C* (compression) and *T* (tension) are as follows: CCC, CCT, CTT.

To develop strut-and-tie models, several methods, like elastic stress distribution or load path, can be used. When choosing the ST model, the practical aspects of the reinforcement location should be taken into consideration. If the location of the struts varies significantly

from the tension stress trajectory, it can cause the initial cracking of the construction. Another criterion that should be taken into account when choosing the model is whether it fulfils the optimization problem of minimum strain energy for linear elastic behavior of the struts and ties after cracking.

The strut-and-tie method has a vast range of usages. It can be applied in constructions such as: single corbel from column, half joint in beam, corners in a frame structures, deep beams, shields etc. The strut-and-tie method is recommended by Eurocode 2 [1] in the [2–4] numerous examples of the applications have been presented.

2. Design of corbels

One of the most popular usages of the STM is in designing corbels. In short corbels, the location of the vertical force (F_{Ed}) from the face of the supporting element (a_c) is smaller than the total height of the corbel (h_c). Through the corbels the load is transferred from beams (crane girders or joists) to columns. Apart from the vertical force, the corbel bears the horizontal load which results from non-mechanical influences (shrinkage, thermal effect) or mechanical (e.g. braking of gantry crane). When analyzed, the horizontal force (H_{Ed}) cannot be smaller than $0.2F_{Ed}$.

2.1. Beam analogy method

The main stage of the design of a corbel is calculating the requirements of reinforcement in the top part of the element. To achieve this, the beam analogy method has been commonly used. In this model, the effective span of the beam equals $0.15b_w$, where b_w is the width of the cross-section.
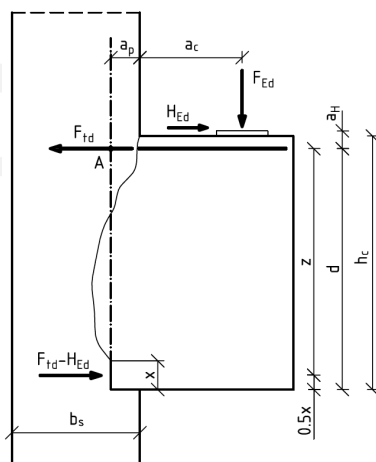

Fig. 2. Schematics to measure reinforcement in corbel using beam analogy method

Stresses in the compression area in concrete are assumed to be equal to f_{cd} (design value of concrete compressive strength). From the condition of equilibrium: the sum of the moments of all external forces in the point A is zero; the following formula (1) for the dimensions of the compressive area is obtained:

$$x = d - \sqrt{d^2 - \frac{2M_A}{b_w \cdot f_{cd}}} \ , \tag{1}$$

where:

$$M_A = F_{Ed} \cdot \left(a_c + a_p\right) + H_{Ed} \cdot a_H \ .$$

2.2. Truss analogy method (STM)

According to the European standard – Eurocode 2, short corbels may be designed using strut-and-tie model which is shown in the following figure (Fig. 3). Apart from the main tension reinforcement, by using STM, the requirement for additional reinforcement (closed horizontal stirrups) can be obtained.
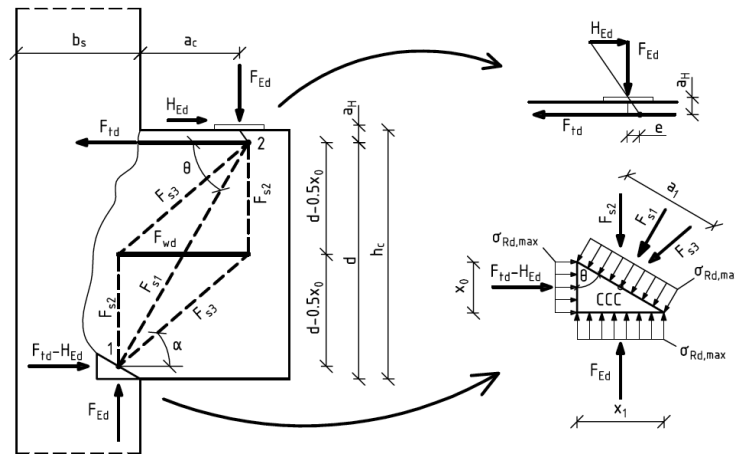

Fig. 3. The strut-and-tie model of the short corbel

The inclination of the strut (θ) is limited by $1.0 \leq \tan\theta \leq 2.5$.

In the recommended strut-and-tie model, node 2 is located in the point where the sum of moments of the vertical and horizontal forces is zero. Consequently, the distance equals

$$e = a_H \cdot \frac{H_{Ed}}{F_{Ed}} \ . \tag{2}$$

Forces in the bars of the strut-and-tie model can be calculated in accordance with the following formulas (3)–(7).

$$F_{Td} = \frac{F_{Ed}(a_c + 0.5x_1) + H_{Ed}(d + a_H - 0.5x_0)}{d - 0.5x_0} \quad (3)$$

$$F_{s1} = \frac{2(F_{Td} - H_{Ed})\sin\alpha - F_{Ed}\cos\alpha}{2\cos\theta\sin\alpha - \cos\alpha\sin\theta} \quad (4)$$

$$F_{s3} = \frac{F_{Ed}\cos\theta - (F_{Td} - H_{Ed})\sin\theta}{2\cos\theta\sin\alpha - \cos\alpha\sin\theta} \quad (5)$$

$$F_{s2} = F_3 \sin\alpha \quad (6)$$

$$F_{wd} = F_3 \cos\alpha \quad (7)$$

The formula (3) is obtained through the equilibrium condition: the sum of moments of the forces in the point one is zero. Dimensions of the nodal zone – x_1 and x_0 are calculated by taking into account limited stresses in the CCC node and geometrical relationships (similarity of triangles). Formulas (4), (5), (6) and (7) are derived from the system of equilibrium equations of forces in the nodes of the truss.

2.3. Example

In this paragraph, the short corbel which is shown in the following figure (Fig. 4) has been analyzed. The vertical load F_{Ed} from 200 kN to 1000 kN with 50 kN step, and horizontal force equal $0.2F_{Ed}$ have been established. According to these assumptions and formulas (3)–(7), the change of forces in bars of the strut-and-tie models is linear.
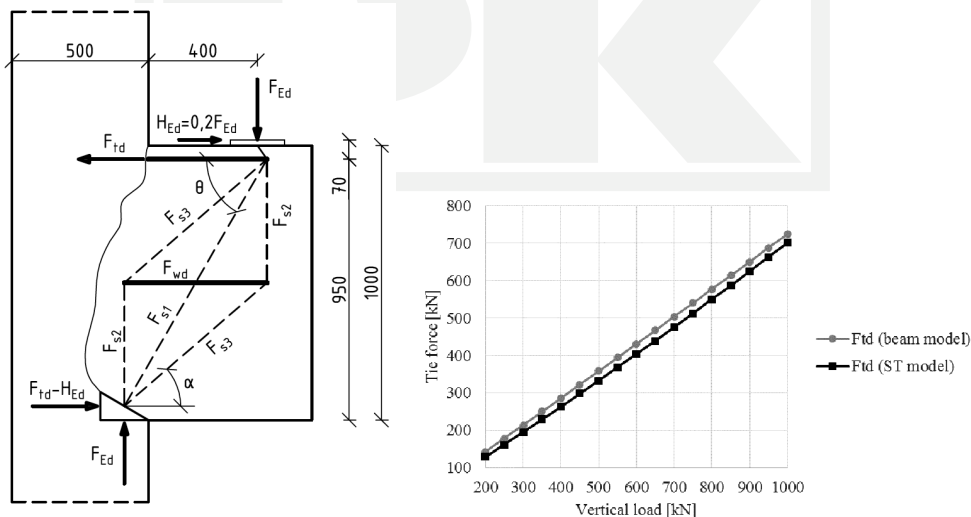

Fig. 4. The geometry of the corbel and the change of forces in the main reinforcement of the corbel for both of the methods

In the analysis, two of the methods have been employed: beam analogy method and STM. In the following graph (Fig. 5), the change of the force in the main reinforcement, which has been obtained from both of the methods for every load case, has been shown.

The results of the two methods are similar. However, those from the beam analogy method are higher in every load case. In the context of the lower bound theory in the limit analysis, the safety margin of the traditional beam analogy method is considerable.
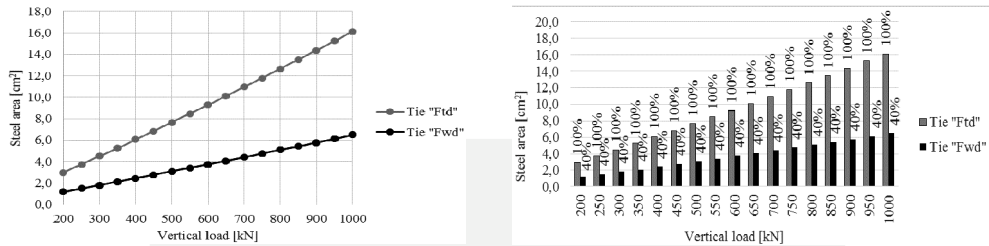

Fig. 5. The graph of requirement area of the main and additional reinforcement of the corbel

Short corbels, which satisfy the additional condition: $a_c < 0{,}5h_c$, work similarly to a shield. This is necessary to provide closed horizontal links that carry significant loads and are distributed over the height of the corbel. According to the recommendation in the original Eurocode 2 [1], the area of horizontal stirrups should not be smaller than 25% of the main tension reinforcement. This restriction was corrected in the Polish National Annex to 50%, which gives an impression to be proper, based on the analyzed example. In every case, the force in the horizontal tie equals 40% (Fig. 5) of the force in the main tie (F_{Td}).

The innovative approach to the analysis of reinforced concrete structures and usage rules of plasticity, expand design fields to the regions with non-linear stress distribution. For elements that are commonly designed using alternative methods, the results from the strut-and-tie method are similar which unveils its accuracy and effectiveness.

R e f e r e n c e s

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