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DIFFERENT OPTIMISATION TASKS
USED IN WATER QUALITY MANAGEMENT

WYBRANE ZADANIA OPTYMALIZACYJNE
DLA POTRZEB ZARZĄDZANIA JAKOŚCIĄ WÓD

Abstract

This study presents a methodology for the management of surface water quality which guarantees that the concentrations of pollutants remain within acceptable levels. The paper presents different optimisation tasks that can be used for managing the quality of surface waters. The paper presents calculations for a hydrographic network with both point and non-point pollution sources. The calculations include the total BOD and the dissolved oxygen concentrations. The author shows how different types of optimisation tasks affect the chemical profiles of the receiving water.

Keywords: management, water quality, optimisation, pollution transformation models

Streszczenie

Celem pracy jest przedstawienie metodyki zarządzania jakością wód powierzchniowych umożliwiającą zagwarantowanie stężeń zanieczyszczeń na dopuszczalnym poziomie. Przedstawiono różne typy zadań optymalizacyjnych umożliwiających zarządzanie jakością wód powierzchniowych. Zaprezentowano przykładowe obliczenia dla sieci hydrograficznej z punktowymi i obszarowymi źródłami zanieczyszczeń. W obliczeniach uwzględniono całkowite biochemiczne zapotrzebowanie tlenu i ilość rozpuszczonego w wodzie tlenu. Pokazano, jak różne typy zadań optymalizacyjnych wpływają na profile chemiczne wody w ciekach.

Słowa kluczowe: zarządzanie, jakość wód, optymalizacja, modele przekształcania zanieczyszczeń

1. Introduction

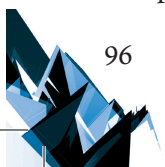
This study presents a methodology for the management of surface water quality which guarantees that the concentrations of pollutants remain within acceptable levels. Frequently, as a result of pollution loads coming from various sources, substances identified as quality indicators exceed their allowable values in the aquatic environment. To reduce their levels of concentration, the decision to reduce their discharge to the environment should be made. Such decisions should be optimised and be a compromise between the amount of pollutants produced by the source and the amount of pollutants that may be released into surface water; thus, it is necessary to perform optimisation calculations to help improve water quality. Different classes of optimisation tasks are solved in optimisation calculations. These can be tasks in which the objective functions are single-criterion [1, 2] and which concern, for example, costs of wastewater treatment or multicriteria [3] and take into account costs of wastewater treatment, frequency or magnitude of violation of the allowable water quality values, water-quality parameters, costs of wastewater treatment and pollutant loads [4]. The optimisation task with a single objective function can be accomplished by finding extreme points of the objective function against properly defined constraints. In the case of multi-objective tasks, the extreme points are sought in a number of functions while satisfying the properly set constraints. There are many methods used for solving optimisation tasks; they have been described in detail in publications such as: [5–9].

Another approach for the management of water quality is fuzzy programming—this is not analysed in this paper [10–12].

Pollutants present in surface waters can originate from single sources, so-called point sources, or they can originate from sources distributed along the watercourse. Since the pollutant discharge along the watercourse may occur by means of the flow of groundwater to the river, runoff, or a series of concentrated point source discharges, in the following paragraphs, sources in the form of multiple discharges distributed along a stream are referred to as non-point sources.

2. Optimal water-quality management

To determine the discharge of pollutants to surface waters by both individual point sources and non-point sources, specific optimisation tasks have to be solved. Each optimisation task consists of the objective function and the constraint functions (constraints) [5], [8]. The objective function represents a certain criterion, due to which, water quality should be improved. Constraints define the so-called area of feasible solutions in a multi-dimensional space – this includes the independent parameters of the objective function. Optimisation is based on the search for extremes (maximum or minimum) of the objective function in the area of feasible solutions. Optimisation tasks may refer to an entire hydrographic network or just a part of it for which the discharge of pollutants into receiving water from point or non-point sources has to be limited (Fig.1).



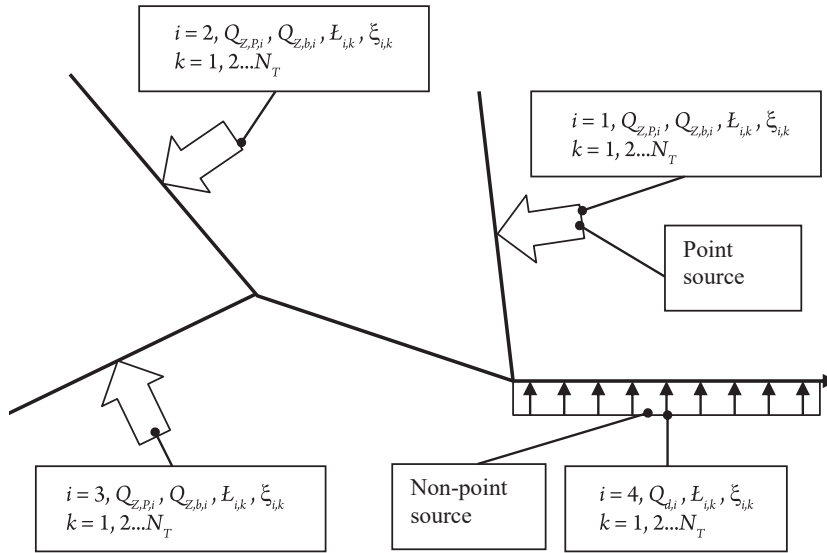


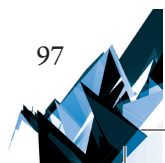
Fig. 1. A hydrographic network with point and non-point pollution sources. (Q_{z,p_i} – capacity of the plant associated with i -th source, Q_{z,b_i} – balance output of i -th source as a flow difference between points upstream and downstream from i -th source, $E_{i,k}$ – k -th pollutant load from i -th source, N_T – number of types of pollutants, $\xi_{i,k}$ – coefficient of level of pollution reduction for i -th source and k -th pollutant).

If water quality is managed, the objective function can be defined in different ways, such as the function of wastewater treatment costs and/or the function of the cost of water treatment, the total pollutant load, the sum of the coefficients of levels of pollution reduction and the sum of the coefficients of concentration variations [1–4, 13–16]. Depending on the objective function its maximum or minimum value is searched within the area of feasible solutions. The constraints are equalities and inequalities developed for concentrations of substances – these are water quality indicators, and decision variables, e.g. coefficients of levels of pollution reduction and efficiency of wastewater treatment. Constraints for concentrations may be associated with one or more water monitoring points i.e. cross sections of the watercourse where measurements are taken. Constraints functions for decision variables define the acceptable range of values for these variables.

3. Optimisation tasks

3.1. The objective function : Total wastewater treatment costs

The optimisation task (OT) with a cost function attempts to minimise this function against some constraints [1, 2, 13, 15, 16]. The task has the following form:



$$ZO : \min K = \sum_{i=1}^{N_Z} K_i(\xi_i) \quad (1)$$

$$C_j \begin{cases} \leq \\ = \\ \geq \end{cases} C_{\lim,j} \quad j=1,2,\dots,N_p \quad (2)$$

$$\xi_{\min,i} \leq \xi_i \leq \xi_{\max,i} \quad i=1,2,\dots,N_Z \quad (3)$$

where:

- i – index for pollution source number
- K – total treatment costs [PLN/year]
- K_i – treatment costs for i -th source [PLN/year]
- N_Z – number of pollution sources
- j – index for cross section number
- C – concentration in j -th cross section [g/m³]
- $C_{\lim,j}$ – limiting concentration in j -th cross section [g/m³]
- N_p – number of cross sections
- ξ_i – level of pollution reduction from i -th pollution source
- $\xi_{\min,i}$ – minimum level of pollution reduction for i -th pollution source
- $\xi_{\max,i}$ – maximum level of pollution reduction for i -th pollution source

The optimisation task allows performing calculations for practically one type of pollutant. The general formula of the cost function for i -th source (associated with a wastewater treatment plant) can be presented as the following relationship [1]:

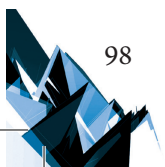
$$K_i = aQ_{Z,P,i}^b \left(\frac{1-\xi_i}{\xi_i} \right) = aQ_{Z,P,i}^b \left(\frac{\eta_i}{1-\eta_i} \right) \quad (4)$$

where:

- a, b – parameters
- $Q_{Z,P,i}$ – capacity of the plant associated with i -th pollution source [m³/s]
- ξ_i – level of pollution reduction for i -th pollution source
- η_i – efficiency of wastewater treatment for i -th source ($\eta_i = 1-\xi_i$)

The costs associated with a reduction of discharged pollutants vary and sometimes can be difficult to determine because they depend on the magnitude of level of pollution reduction ξ_i or treatment efficiency $\eta_i = 1-\xi_i$ and treatment plant capacity $Q_{Z,P,i}$. Reduction of pollutant discharges can also be achieved in other ways, by, for example:

- ▶ change of production technology at the plant discharging the pollutants
- ▶ closed water circuits
- ▶ elimination of pollution sources (closure of the industrial plant), etc.



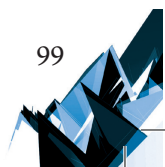
The costs related to pollution discharges also concern non-point sources and the structure of sewage systems; therefore, they are affected by:

- ▶ less intense fertilisation of farmland (compensation for farmers)
- ▶ reduction of breeding and grazing (compensation for farmers)
- ▶ construction or modernisation of sewage network
- ▶ construction of sewage pumping stations, etc.

Therefore, the costs will be generally determined by the method used to reduce pollutant discharge. With regard to the pollution source, it is possible to use one or more methods of pollution control; thus, for some sources, the cost estimations may be very complicated.

Complications arise also in situations in which the cost function for a number of pollution indicators has to be determined. Indeed, if there is a need to determine the treatment cost for two parameters (e.g. BOD_t and ammonia nitrogen) then a function that combines both BOD_t removal and nitrification efficiencies has to be defined. In the biological process, the reduction of BOD_t is accompanied by the growth of microorganisms. These microorganisms assimilate ammonium nitrogen, which is both oxidised and absorbed by nitrifying bacteria in the same biological reactor where oxidation of organic compounds takes place. In addition, the BOD_t removal efficiency is closely related to the efficiency of nitrification. In this situation, some increases of the BOD_t removal efficiency increases the nitrification efficiency too, and if so, only one efficiency will impact the costs. There may be, however, a situation when the nitrification efficiency is not satisfactory and has to be increased due to the required ammonia nitrogen removal. In this case, the cost will depend on the nitrification efficiency, which should be recalculated with respect to BOD_t efficiency; however, in more complex cases, the costs may depend on efficiencies of many other processes (biochemical oxidation, nitrification, denitrification, phosphorus removal). In extreme cases, the higher efficiency of one process (e.g. nitrification) may suppress the other unit processes (e.g. biochemical oxidation of organic compounds due to an ammonia nitrogen shortage for protein synthesis in heterotrophs organisms cells.). Thus, the use of the cost function for a number of indicators is possible, but sometimes it can be very difficult. Furthermore, estimation of operating costs for the treatment plant may be difficult since they depend on unknown parameters, such as treatment efficiency when the technological system has yet to be selected (the system is designed once the required process efficiency is known).

Therefore, the cost function, as the objective function, should preferably be formulated only for the point sources related to a given treatment plant and a single pollution parameter. A number of constraints functions (2) concerning concentrations may be high (many cross section) – such instances make the optimisation calculations much more complicated. Conditions (2) may include some restrictions for oxygen. The oxygen concentration is unlikely to affect the value of the objective function; however, taking its impact into account is possible.



3.2. The objective function: total pollution load

In the optimisation task (OT) where the objective function takes a form of a total pollution load the function is maximised against some constraints. The task may have the following form:

$$ZO: \max L = \sum_{i=1}^{N_z} \xi_i L_i \quad (5)$$

$$C_j \left\{ \begin{array}{l} \leq \\ \\ \geq \end{array} \right\} C_{\text{lim},j} \quad j = 1, 2, \dots, N_p \quad (6)$$

$$\xi_{\text{min},i} \leq \xi_i \leq \xi_{\text{max},i} \quad i = 1, 2, \dots, N_z \quad (7)$$

where:

L – total pollution load discharged to receiving water [g/s]

L_i – total pollution load discharged to water by i -th source [g/s]

(other abbreviations are as defined above)

The task (5, 6, 7) allows carrying out a calculation for only one type of pollutant. It is not possible to add different types of loads, e.g. g O₂ BODt / s and g N^{NH₄⁺} / s. Obviously, such a sum would be a value of unspecified unit; however, the coefficients of level of pollution reduction obtained after maximisation of objective function would depend on sensitivity of the sum (5) on changes of pollution loads. To some extent, this problem may be solved through the introduction of weighting factors $w_{i,k}$ for each type of pollutant. The objective function in this case assumes the form:

$$ZO: \max A = \sum_{i=1}^{N_z} \sum_{k=1}^{N_T} w_{i,k} \xi_{i,k} A_{i,k} \quad (8)$$

where:

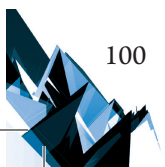
k – index for type of pollutant

$w_{i,k}$ – weighting factor for i -th source and k -th pollutant

N_T – number of pollutant types

(other abbreviations are as defined above)

The determination of the weighting factors can be a very subjective process; consequently, the obtain optimisation result may be not objective. As with the task with the cost function, task (5, 6, 7) can be comprised of a number of constraints (6) for the concentrations calculated in many cross sections. Maximisation of the sum of the coefficients of levels of the pollution reduction tends to lower the treatment efficiency, thereby lowering treatment costs and all other costs incurred while reducing pollution loads discharged to water.



3.3. The objective function: total level of pollution reduction

In the optimisation task (OT) where the objective function takes the form of a sum of levels of pollution reduction, the function is maximised against some constraints. The task may have the following form:

$$ZO: \max \Xi = \sum_{i=1}^{N_Z} \sum_{k=1}^{N_T} \xi_{i,k} \quad (9)$$

$$C_{j,k} \begin{cases} \leq \\ = \\ \geq \end{cases} C_{\lim,j,k} \quad j = 1, 2, \dots, N_p, k = 1, 2, \dots, N_T \quad (10)$$

$$\xi_{\min,i,k} \leq \xi_{i,k} \leq \xi_{\max,i,k} \quad i = 1, 2, \dots, N_Z, k = 1, 2, \dots, N_T \quad (11)$$

where:

- Ξ – sum of coefficients $\xi_{i,k}$
- $\xi_{i,k}$ – coefficient of level of pollution reduction for i -th source and k -th pollutant
- $C_{j,k}$ – concentration of k -th pollutant in j -th check point [g/m^3]
- $C_{\lim,j,k}$ – limiting concentration of k -th pollutant in j -th check point [g/m^3]

(other abbreviations are as defined above)

Task (9, 10, 11) has been developed for many water-quality parameters. This procedure is possible because $\xi_{i,k}$ is dimensionless and can be summed (9). The weighting factors can also be introduced to the objective function if necessary, then:

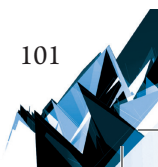
$$ZO: \max \Xi = \sum_{i=1}^{N_Z} \sum_{k=1}^{N_T} w_{i,k} \xi_{i,k} \quad (12)$$

Just as with the preceding tasks, task (9, 10, 11) may be comprised of many constraints functions (10) for the concentrations calculated in numerous cross sections. Maximisation of the sum of the coefficients of pollution reduction levels tends to lower the treatment efficiency, thereby reducing treatment costs and all other costs incurred while reducing pollutant loads discharged to water.

3.4. The objective function: sum of coefficients of variation (CVs)

In the optimisation task (OT) where the objective function takes the form of a sum of CVs, the function is minimised against some constraints [11]. The task may have the following form:

$$ZO: \min v = \sum_{k=1}^{N_T} v_k (\xi_{1,k}, \xi_{2,k}, \dots, \xi_{N_Z,k}) \quad (13)$$



$$C_k \left\{ \begin{array}{l} \leq \\ = \\ \geq \end{array} \right\} C_{\text{lim},k} \quad k=1,2,\dots,N_T \quad (14)$$

$$\xi_{\text{min},i,k} \leq \xi_{i,k} \leq \xi_{\text{max},i,k} \quad i=1,2,\dots,N_Z, k=1,2,\dots,N_T \quad (15)$$

where:

- v – sum of coefficients of variation $v_k(\xi_{1,k}, \xi_{2,k}, \xi_{N_Z,k})$
- $v_k(\xi_{1,k}, \xi_{2,k}, \xi_{N_Z,k})$ – CVs for concentrations for k -th pollutant
- C_k – concentration of k -th pollutant in the lowest cross section [g/m³]
- \bar{C}_k – the limiting concentration of k -th pollutant in the lowest cross section [g/m³]
- $C_{\text{lim},k}$ – limiting concentration of k -th pollutant in the lowest cross section [g/m³]

(other abbreviations are as defined above)

Coefficients of concentration variation v_k are dimensionless and therefore can be summed. They are determined for the entire hydrographic network. By definition, the CVs are the ratio of the concentration σ_k deviation from the average value and the average \bar{C}_k value:

$$v_k = \frac{\sigma_k}{\bar{C}_k} \quad (16)$$

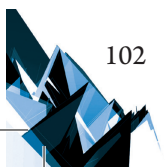
The average and the standard deviation for the concentration of the k -th pollutant is calculated from the formulas:

$$\bar{C}_k = \frac{1}{\sum_{jj=1}^{N_L} L_{jj}} \sum_{jj=1}^{N_L} \int_0^{L_{jj}} C_{k,jj}(x) dx \quad (17)$$

$$\sigma_k = \sqrt{\frac{1}{\sum_{jj=1}^{N_L} L_{jj}} \sum_{jj=1}^{N_L} \int_0^{L_{jj}} (C_{k,jj}(x) - \bar{C}_k)^2 dx} \quad (18)$$

where:

- jj – index of segment number
- N_L – number of segments of hydrographic network
- L_{jj} – length of segment jj [m]
- $C_{k,jj}(x)$ – concentration function related to a distance coordinate x for k -th pollutant and segment jj [g/m³]



In this optimisation task, it is possible to significantly reduce the number of constraints functions for concentrations; they simply require defining for the most distant (lowest) cross section in the hydrographic network. Such a procedure is acceptable because minimisation of v_k promotes a concentration equalisation in the hydrographic network. Moreover, minimisation of the sum of v makes the average concentrations rather high (is in a denominator); furthermore, the sum of the pollutant loads discharged into water remains high. Accordingly, the coefficients of levels of pollution reduction are as high as possible so the costs of limiting pollution load discharged to water are minimised.

3.5. Modification of the objective function in optimisation tasks

Objective functions in optimisation tasks can be defined in different ways. Also, the optimisation problem itself may be modified in numerous ways. Three different tasks (OT) with a modified objective function are shown below:

OT 1:

$$ZO: \min v^* \quad (19)$$

$$v_k^*(\xi_{1,k}, \xi_{2,k}, \dots, \xi_{N_z,k}) \leq v^* \quad k=1, 2, \dots, N_T \quad (20)$$

$$v_k^*(\xi_{1,k}, \xi_{2,k}, \dots, \xi_{N_z,k}) = \frac{v_k(\xi_{1,k}, \xi_{2,k}, \dots, \xi_{N_z,k})}{v_k(\xi_{1,k}=1, \xi_{2,k}=1, \dots, \xi_{N_z,k}=1)} \quad k=1, 2, \dots, N_T \quad (21)$$

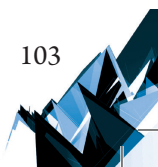
$$C_k \begin{cases} \leq \\ = \\ \geq \end{cases} C_{\text{lim},k} \quad k=1, 2, \dots, N_T \quad (22)$$

$$\xi_{\text{min},i,k} \leq \xi_{i,k} \leq \xi_{\text{max},i,k} \quad i=1, 2, \dots, N_z, k=1, 2, \dots, N_T \quad (23)$$

$v_k(\xi_{1,k}=1, \xi_{2,k}=1, \dots, \xi_{N_z,k}=1)$ – CVs for k -th pollutant for an initial state (all coefficients of levels of pollution reduction for the i -th pollution source and the k -th pollutant $\xi_{i,k}=1$)

(other abbreviations are as defined above)

In task OT 1, coefficients of variation are normalised. A new coefficient v^* was introduced, the values of which vary in the range of $\langle 0, 1 \rangle$. The coefficient v^* is an additional and unknown variable. It was assumed that once the optimisation task was solved, fluctuations of the concentrations of the water quality parameters around their average values were lower than the ones at the initial state i.e. for the primary water quality. Although the CVs are dimensionless and can therefore be compared with each other, their range of variation can be



very large – theoretically, they can be between $< 0, +\infty >$. The CVs for different water quality parameters can take a different order of magnitude. Minimising the sum of these factors does not lower concentration fluctuations of all parameters due to solving the optimisation task. The sum of the coefficients may indeed decrease; however, some of them may take values greater than those observed at the initial state. If one wants to assure lower fluctuations for all parameters, a normalisation is necessary and then all v_k^* stay in the range $< 0, 1 >$; therefore, the coefficients of variation are $v_k(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k}) \leq v_k(\xi_{1,k} = 1, \xi_{2,k} = 1 \dots \xi_{N_z,k} = 1)$.

OT 2:

$$ZO: \min v^* = \sum_{k=1}^{N_T} v_k^*(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k}) \quad (24)$$

$$v_k^*(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k}) = \frac{v_k(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k})}{v_k(\xi_{1,k} = 1, \xi_{2,k} = 1 \dots \xi_{N_z,k} = 1)} \quad k = 1, 2 \dots N_T \quad (25)$$

$$C_k \begin{cases} \leq \\ = \\ \geq \end{cases} C_{\lim,k} \quad k = 1, 2 \dots N_T \quad (26)$$

$$\xi_{\min,i,k} \leq \xi_{i,k} \leq \xi_{\max,i,k} \quad i = 1, 2 \dots N_z, k = 1, 2 \dots N_T \quad (27)$$

(other abbreviations are as defined above)

The task OT 2 is similar to the task OT 1. This task also uses the standardised CVs but their sum is minimised. Despite the significant similarities of the tasks, the results of calculations may vary. It is difficult to predict in advance which solution is better or more satisfying for a decision-maker. The decision will ask for additional assumptions e.g. the real values of level of pollution reduction.

$$OT 3: \quad ZO: \min \alpha \quad (28)$$

$$1 - \xi_{i,k} \leq \alpha \quad i = 1, 2 \dots N_z, k = 1, 2 \dots N_T$$

$$v_k^*(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k}) \leq \alpha \quad k = 1, 2 \dots N_T \quad (29)$$

$$v_k^*(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k}) = \frac{v_k(\xi_{1,k}, \xi_{2,k} \dots \xi_{N_z,k})}{v_k(\xi_{1,k} = 1, \xi_{2,k} = 1 \dots \xi_{N_z,k} = 1)} \quad k = 1, 2 \dots N_T \quad (30)$$



$$C_{j,k} \begin{cases} \leq \\ = \\ \geq \end{cases} C_{\text{lim},j,k} \quad j=1,2,\dots,N_p, k=1,2,\dots,N_T \quad (31)$$

$$\xi_{\text{min},i,k} \leq \xi_{i,k} \leq \xi_{\text{max},i,k} \quad i=1,2,\dots,N_Z, k=1,2,\dots,N_T \quad (32)$$

Task OT 3 is a combination of task OT 1 and minimisation of the sum of the levels of pollution reduction, as discussed earlier. Task OT 3 has two objectives and employs two different types of functions: standardised coefficients of variation v_k^* ; complements to unity of the levels of pollution reduction $(1-\xi_{i,k})$. Task OT 3 is basically a multi-criteria task since both the standardised CVs and the expression $(1-\xi_{i,k})$ refer to different parameters of water quality. All the functions of this task must be either expressed in the same units or be dimensionless since α is common for all the criteria. Task OT 3, like many others, enables solving multi-criteria problems in the Pareto efficiency [4, 5].

Solving task OT 3 it is essentially striving to achieve the ideal point in which $\alpha = 0$.

The above examples do not exhaust all the possibilities of optimisation tasks used in water-quality management.

4. Analysis of the results from different optimisation tasks

4.1. Model of transport and transformation in the water used in the optimisation tasks

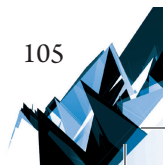
To illustrate the changes of concentrations of some water-quality indicators along the watercourses of the hydrographic network, an advective mass transport model was assumed. The model takes into account the kinetics of changes in the total biochemical oxygen demand and changes in the dissolved oxygen (DO) and has the form [17]:

$$V_x \frac{\partial C_B}{\partial x} = -k_1 C_B \frac{C_T}{K_T + C_T} + R_L (C_{B,d} - C_B) \quad (33)$$

$$V_x \frac{\partial C_T}{\partial x} = k_2 (C_T^* - C_T) - k_1 C_B \frac{C_T}{K_T + C_T} + R_L (C_{T,d} - C_T) \quad (34)$$

where:

- C_B – total biochemical oxygen demand (BODt) [g O₂ BODt / m³]
- C_T – DO concentration [g O₂ / m³]
- C_T^* – saturation concentration of DO [g O₂ / m³]



- $C_{B,d}$ – total biochemical oxygen demand (BODt) for a non-point pollution source [g O₂ BODt / m³]
 $C_{T,d}$ – DO concentration in a saturation state for a non-point pollution source [g O₂ / m³]
 V_x – flow velocity along river (x–axis), (average value for the segment) [m/s]
 k_1 – biochemical oxidation rate constant [1/s]
 k_2 – rate constant of oxygen absorption from the atmosphere [1/s]
 K_T – Michaelis–Menten constant for oxygen [g O₂ / m³]
 R_L – dilution rate constant [1/s]
 x – distance coordinate [m]
 t – time [s]

A non-point pollution source (distributed along the watercourse) results in changes of concentrations in water. A local value of a dilution rate constant is [17]:

$$R_L(x) = \frac{q_L(x)}{A(x)} = \frac{q_L(x)}{Q(x)/V(x)} \quad (35)$$

- $q_L(x)$ – side flow per unit length of watercourse in cross section x [m²/s]
 $A(x)$ – cross section area at a certain water level at cross section x (m²)
 $V(x)$ – average velocity in cross section x [m/s]
 $Q(x)$ – flow in cross section x [m³/s]

With some simplifying assumptions [17], the average value of the dilution rate constant in the whole segment can be calculated with the formula:

$$R_L = \frac{1}{L/V_x} \operatorname{Ln} \left(\frac{Q_0 + q_L \cdot L}{Q_0} \right) = \frac{1}{t_k} \operatorname{Ln} \left(\frac{Q_0 + Q_d}{Q_0} \right) \quad (36)$$

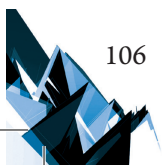
where:

- L – length of segment [m]
 Q_0 – flow at the initial cross section of segment [m³/s]
 q_L – average side flow per unit length of watercourse [m²/s]
 Q_d – total side flow to segment [m³/s]
 t_k – flow time on section of watercourse [s]
 Ln – natural logarithm

(other abbreviations are as defined above)

After the numerical integration of equations (33) and (34) the value of R_L changes along the watercourse, then [17]:

$$R_L(x + \Delta x) = \frac{1}{\Delta x / V_x} \operatorname{Ln} \left(\frac{Q(x) + Q_d \cdot \Delta x / L}{Q(x)} \right) = \frac{1}{\Delta t} \operatorname{Ln} \left(\frac{Q(x) + Q_d \cdot \Delta t / t_k}{Q(x)} \right) \quad (37)$$



Δx – distance step [m]
 Δt – time step [s]
 t_k – flow time along the segment [s]
 $Q(x)$ – flow in the cross section x [m³/s]

(other abbreviations are as defined above)

Assuming that the average side flow per unit length is constant then:

$$Q(x) = Q_0 + q_L \cdot x \quad (38)$$

(other abbreviations are as defined above)

To solve the different optimisation tasks, equations (33) and (34) were integrated numerically by the Newton method with a step $\Delta t = \Delta x / V_x = 0.001d$. The hydrographic network is shown in Figure 2.

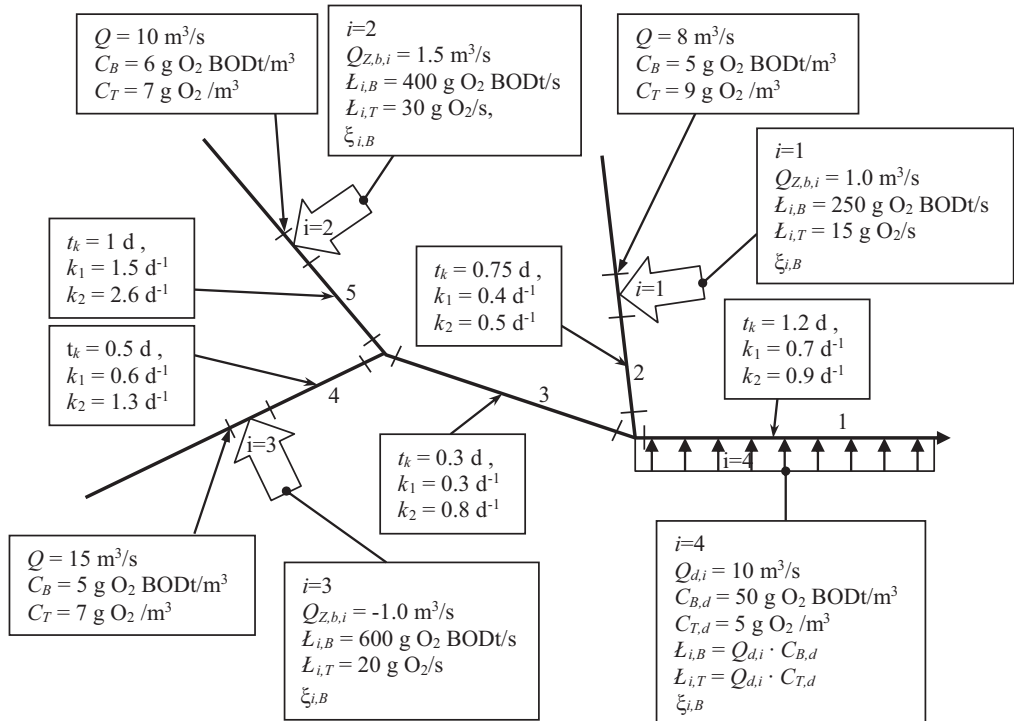
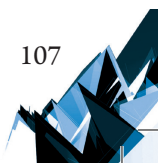


Fig. 2. The hydrographic network with the values describing: point sources; non-point sources; segments of watercourses; cross sections. Parameters for the whole network: the Michaelis-Menten constant for oxygen $K_T = 0.01 \text{ g O}_2/\text{m}^3$; DO at the saturation state $C_T^* = 12 \text{ g O}_2/\text{m}^3$ (from now on a level of pollution reduction $\xi_{z,i,B}$ for BODt is marked as ξ_i)



4.2. Solution of optimisation tasks

Optimisation tasks (5–7), (9–11), (13–15), (19–23), (24–27) and (28–32) were solved and coefficients of the pollution reduction levels were identified in order to compare improvements in the water quality. The coefficients of pollution reduction levels $\xi_{i,B}$ for BODt associated with i -th source were sought in all tasks. It was assumed that $\xi_{i,B}$ would change within the range of 0.05 to 0.95. In real life, the range must be separately specified for each source of pollution. Additionally, it was assumed that in the final cross section of segment 1 $C_B \leq 6g O_2 \text{ BODt} / m^3$ while $C_T \geq 7g O_2 / m^3$

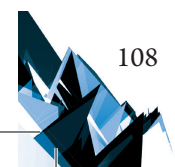
In the particular hydrographic network, the limiting concentration values may be different for different water quality parameters and can be defined for multiple cross sections.

The optimisation calculations are summarised in Table 1. The curves showing the total biochemical oxygen demand (C_B) and the DO concentration (C_T) in the five segments of the hydrographic network for six optimisation tasks are shown in Figs. 3 & 4.

Table 1. Solutions to the optimisation tasks: (5–7), (9–11), (13–15), (19–23), (24–27) and (28–32)

Parameter	$\max \sum L$ r. (5...7)	$\max \sum \xi$ r. (9...11)	$\min \sum v$ r. (13...15)	$\min v^*$ (ZO 1) r. (19...23)	$\min \sum v^*$ (ZO 2) r. (24...27)	$\min \alpha$ (ZO 3) r. (28...32)
$\xi_{1,B} =$	0.0500	0.4709	0.2060	0.4430	0.2878	0.7440
$\xi_{2,B} =$	0.9500	0.9500	0.0500	0.0500	0.0500	0.1944
$\xi_{3,B} =$	0.0500	0.0500	0.2401	0.2456	0.2499	0.2122
$\xi_{4,B} =$	0.8038	0.5193	0.6873	0.5192	0.6175	0.3222
$\sum L = [g O_2 \text{ BODt/s}]$	824.3773	787.4016	559.2351	537.6607	550.6722	552.1565
$v_B =$	0.4515	0.3675	0.1366	0.1319	0.1335	0.1646
$v_T =$	0.1376	0.16104	0.0476	0.0622	0.0523	0.0893
	$\max \sum L = [g O_2 \text{ BODt/s}]$	$\max \sum \xi =$	$\min \sum v =$	$\min v^* =$	$\min \sum v^* =$	$\min \alpha =$
	824.3773	1.9903	0.1842	0.6454	0.9876	0.8056

Solutions of different optimisation tasks, at the same constrains for the decision variables i.e. level of pollution reduction coefficients and the same constrains for BODt and DO showed that different total BODt loads ($\sum L$) would be introduced to the water in the hydrographic network (Table 1). The largest BODt loads are associated with the tasks (5–7) and (9–11) and are 824.38 and 787.40g O_2 BODt/s, respectively (Table 1). These are tasks that attempt to maximise the pollution loads introduced to the network – they allow to utilise the transformation possibilities of the analysed pollutants through the aquatic environment to the highest extent. The solutions of the remaining four tasks: (13–15); (19–23); (24–27); (28–32) show a slightly lower total load, it ranges from 537.66 to 559.24g O_2 BODt / s (Table 1). Tasks (5–7) and (9–11) however, provide an unfavorable distribution of BODt and DO in the hydrographic network. Significant changes in BODt and DO in the network were observed



(Figs. 3 & 4). Deviations of BODt values comparing to the average value reached, in these tasks, respectively 45.15% and 36.75%, while for DO they were respectively 13.76% and 16.10%, (Table 1). Furthermore, these tasks show little sensitivity to pollutants originating at locations far from the lowest check point, for which the limiting conditions for BODt and DO were formulated. The curves obtained for these two parameters are similar to curves for initial conditions observed in the aquatic environment, and thus in all $\xi_{jk} = 1$ (Figs. 3 & 4, segment 5).

The state of the aquatic environment was not substantially improved in segment 5. The BOD_t values are still high and the oxygen concentrations reach the minimum ($0.65 \text{ g O}_2 / \text{ m}^3$), which is only slightly higher than the initial value ($0.25 \text{ g O}_2 / \text{ m}^3$) (Fig. 4, segment 5).

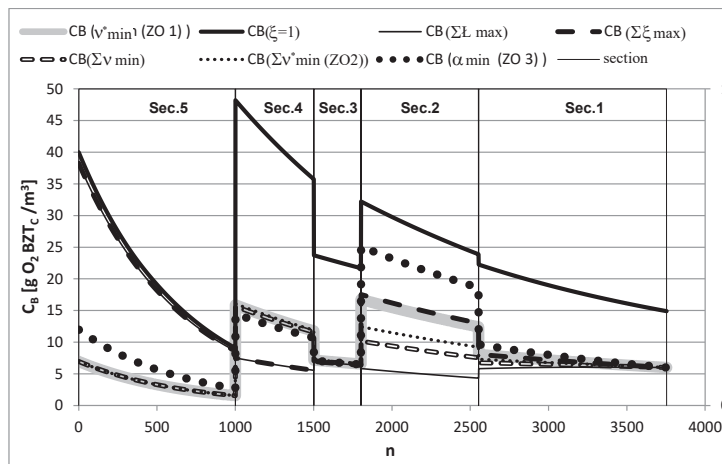


Fig. 3. Changes of BODt along all segments of the hydrographic network resulting from solving different optimisation tasks (Sec. – section of the river)

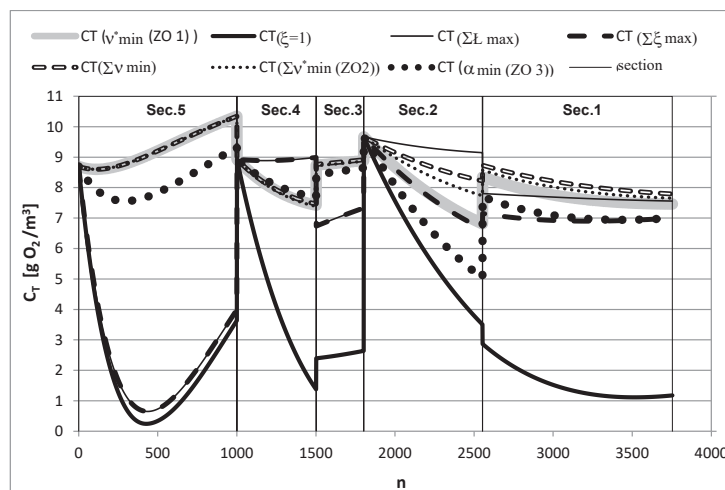
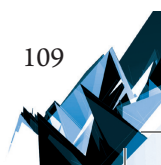


Fig. 4. Changes of the DO concentration along all segments of the hydrographic network resulting from solving various optimisation tasks (Sec. – section of the river)



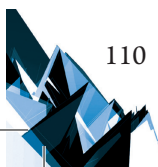
Of course, this disadvantage can be eliminated by setting additional constraints to the optimisation task. It is a simple operation in the case of a small network. In a large hydrographic network, additional constraints in one segment may result in poor concentration distributions in other segments. In extreme cases, the constraints functions for concentrations have to be set in all segments. Such action makes the optimisation task more complicated and the total pollution load lower than that obtained from the optimisation task with a small number of constraints functions for concentrations. Additional constraints for the concentrations may result in a possible conflict of constraints. It may indeed prove that at the assumed ranges of $\xi_{i,k}$, neither sufficiently low pollutant concentrations nor sufficiently high DO concentrations can be achieved. In such cases, the optimisation task cannot be solved.

Tasks (13–15), (19–23), (24–27) and (28–32) show much better concentration distributions. In these tasks, the coefficient of concentration variation contributes to the equalisation of concentrations in the entire network (Figs 3 & 4). Deviations between BOD_t values and the average value varied in the range of 13.19% to 16.46%, while for oxygen, it changed from 4.76% to 8.93% (Table 1). The values were much lower than those found in tasks (5–7) and (9–11). The additional concentration reducing features should be introduced to OT (13–15), (19–23), (24–27) and (28–32) only in exceptional situations. In these tasks, both low concentrations of dissolved oxygen and minimum of this concentration could be eliminated with no additional constraints functions (Fig. 4). It should be noted that in segment 5, the minimum oxygen concentration remained but increased up to the value from range 7.56–8.60g O₂ / m³, depending on the task.

It should also be noted that the minimum shifted in time with respect to the initial minimum – it happens earlier and closer to the upper cross section of segment 5 (Fig. 4). The limiting conditions for oxygen in the upper and lower cross section of segment 5 in tasks (5–7) and (9–11) assure a proper level of the DO but its minimum concentration may be too low for aquatic organisms. Therefore, the minimum oxygen concentration with a variable location along a segment should be sought in optimisation calculations; it should not be lower than the limiting value. Such conditions would solve the problem of extremely low oxygen concentrations; however, looking for location of the minimum oxygen concentration makes the solution more complex. Additionally, it is difficult to determine the actual limiting minimum concentration of DO. In OT, employing the CVs for concentrations, the problem of the acceptable concentration minimum virtually does not exist.

Levelling concentration fluctuations around the average value results in a lower total pollution load discharged.

While solving four tasks (13–15), (19–23), (24–27) and (28–32) similar total pollution loads discharged to the watercourse and similar CVs for water quality parameters are obtained; however, the decision variables $\xi_{i,B}$ may differ significantly (Table 1). The OT (28–32) seems to be the best although it gives the highest CVs and the average total load (Table 1). The distinctive feature of this task is that CVs and levels of pollution reductions are together considered as a separate criterion; such a task becomes more universal and safer due to the maximization of sum of the pollution reduction levels.



The other three tasks do not have such a feature, which may be advantage at the complex hydrographic networks, with large number of different pollution sources and different pollutants. Their results may be beneficial for both the environment users (people, an industrial plant etc.) and the aquatic environment itself.

5. Conclusions

Water quality management requires setting the upper limit load of pollutants that can be discharged into surface water is determined. The acceptable loads may vary slightly depending on the optimisation task. The tasks where the costs of water protection are minimised are difficult to define with respect to one pollutant only. In the case of many pollutants, the cost function can be virtually impossible to determine and be economically unjustified; furthermore, cost assessments may be far from certain. For projects designed to improve water status in the distant future, the solutions may become far from optimal due to changes in economic and administrative conditions within the region.

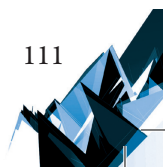
The study proposes a number of alternative optimisation tasks for water quality management.

The optimisation tasks involving the maximisation of total load discharged into watercourses give different solutions (against the same constraints) to those where total levels of pollution reductions are maximised, though similar total loads will be discharged. Such tasks require that a number of constraints functions are considered – this guarantees a proper level of parameters in the water. For a large number of constraints functions, the optimisation task cannot be solved due to the existing conflict and should not be used for equalisation of concentrations in the hydrographic network. The objective function representing the total pollutant load discharged into water can be formulated for one pollutant only. The objective function which is the sum of levels of pollution reduction (the sum of the dimensionless coefficients) can be used for multiple parameters, simultaneously.

The optimisation tasks that use CVs for concentrations ensure a considerable equalisation of concentration distributions along a given watercourse, at a small number of constraints functions for concentrations. As a result, a lower total pollution load is discharged compared to loads from tasks with the objective functions based on the loads or levels of pollution reduction. The tasks with CVs can carry out optimisation calculations simultaneously on many water quality parameters.

All alternative optimisation tasks described in the paper tend to minimise the prospective costs involving water-quality improvement.

The optimisation tasks together with models of pollutant transformations try to utilise the assimilative capacity of a watercourse in a rational manner. If only mass transport models were considered without transformation, the limiting pollution loads would be smaller while the efficiency of treatment would be higher. Such effects are observed while carrying out calculations for low water temperatures which slow down the transformation of pollutants to a significant degree.



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