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THE INFLUENCE OF SUPPORT WIDTHS OF TRAPEZOIDAL SHEETS ON LOCAL TRANSVERSE RESISTANCE OF THE WEB ACCORDING TO PN-EN 1993-1-3

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A b s t r a c t

This paper comments on the calculations algorithm and presents examples of the determination of local transverse resistance of the web for trapezoidal sheets according to the rules given in the standard PN-EN 1993-1-3 [4]. The calculations were carried out for different support widths and thicknesses of the stiffened and unstiffened webs of trapezoidal sheets. The results obtained in the calculations show a distinctly positive influence of the increase of the support widths on the local transverse resistance of the web of trapezoidal sheets.

Keywords: *trapezoidal sheet*, *local transverse forces*, *local transverse resistance of the stiffened and unstiffened web*, *effective bearing length*

S t r e s z c z e n i e

W artykule przedstawiono wraz z komentarzem algorytm oraz przykłady wyznaczania nośności poprzecznej środnika blachy trapezowej wg PN-EN 1993-1-3 [4]. Obliczenia przeprowadzono dla różnych szerokości podparcia i różnych grubości blach trapezowych z usztywnieniami i bez usztywnień środnika. Otrzymane wyniki pokazały wyraźnie korzystny wpływ zwiększenia szerokości podparcia blachy trapezowej na nośność poprzeczną środnika.

Słowa kluczowe: *blacha trapezowa, obciążenie skupione*, *nośność poprzeczna środnika nieusztywnionego i z usztywnieniem*, *efektywna długość strefy docisku*

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1. Introduction

Technical catalogue prepared by particular producers are usually used in the process of designing trapezoidal sheets. They help to avoid tedious calculations [1] in order to determine their carrying capacities. The choice of a trapezoidal sheet is made through comparison of its uniformly distributed design loads in ULS (ultimate limit state) and characteristic loads in SLS (serviceability limit state) with the limit values of the loading given in the tabular form. While choosing trapezoidal sheets, it is important to pay attention to the minimum width on the support, because the limit load values given in the technical catalogue are given under the condition it is sufficient [2, 3, 7]. However, the technical catalogue may only be a supplementary tool for designing the sheets under consideration. Rather, the standard PN-EN-1993-1-3:2008 [4] should be applied for a roof covering in certain instances. According to its guidelines, in the case of webs of trapezoidal sheets, unstiffened transversally within the cross-section at the end supports as well as at the internal supports, the check of the patch load acting on the webs neglected, though the widths of the supports are of great importance for the local transverse resistance of the sheets.

This paper presents the algorithm and examples of the determination of the local transverse resistance of unstiffened and stiffened webs for a trapezoidal sheet 92 mm high. The influence of support widths changing from 10 mm to 200 mm on the local transverse resistance of the web of trapezoidal sheets with thicknesses from 0.75 mm to 1.5 mm is visible.

2. The resistance of a trapezoidal sheet loaded by local transverse forces

2.1. The local transverse resistance of an unstiffened web of a trapezoidal sheet

To avoid crushing, crippling or buckling in a web subjected to a support reaction or other local transverse force applied through the flange, the transverse force F_{Ed} shall satisfy, according to (6.13) in [4]:

$$F_{Ed} \leq R_{w,Rd} \tag{1}$$

where

- F_{Ed} – the transverse force,
- $R_{w,Rd}$ – the local transverse resistance of the web, according to (6.18) in [4]:

$$R_{w,Rd} = \alpha \cdot t^2 \sqrt{f_{yb} \cdot E}\left(1 - 0.1\sqrt{\frac{r}{t}}\right)\left[0.5 + \sqrt{0.02\frac{l_a}{t}}\right]\left(2.4 + \left(\frac{\phi}{90}\right)^2\right) / \gamma_{M1} \tag{2}$$

where

- r – the internal radius of the corners (see Fig. 1),
- t – the design core thickness of the steel material before cold-forming, exclusive of metal and organic coatings (see Fig. 1),
- ϕ – the angle of the web relative to the flanges [degrees] (see Fig. 1),
- l_a – the effective bearing length for the relevant category,

α – the coefficient for the relevant category,
f_{yb} – the basic yield strength,
γ_{M1} – the partial factor.

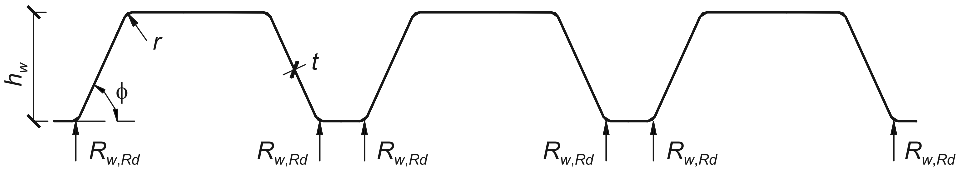

Fig. 1. Example of a cross-section for a trapezoidal sheet with a few unstiffened webs [4]

The local transverse resistance $R_{w,Rd}$ of an unstiffened web should be determined as specified in (2), provided that both of the following conditions are satisfied, according to (6.17) in [4]:

$$c \geq 40 \text{ mm} \tag{3}$$

$$\left(\frac{r}{t}\right) \leq 10 \tag{4}$$

$$\left(\frac{h_w}{t}\right) \leq 200 \sin \phi \tag{5}$$

$$45^{\circ} \leq \phi \leq 90^{\circ} \tag{6}$$

where

c – the distance from the support reaction to a free end (see Fig. 2, 3),
h_w – the web height between the midlines of the flanges (see Fig. 1).

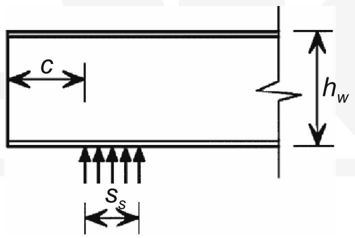

Fig. 2. Category 1 – reaction at the end support with $c \leq 1{,}5h_w$ clear from a free end [4]

The values of l_a and α should be obtained depending on category 1 and 2. The relevant category (l or 2) should be based on the clear distance c from the local load distributed on the distance s_s to a free end (see Fig. 2, 3).

The maximum value for l_a = 200 mm. When the support is a cold-formed section with one web or a round tube, a value of 10 mm should be taken for s_s. The value of the effective bearing length l_a should be obtained as follows, after formulas 6.19 in [4]:

for category 1:

$$l_a = 10 \text{ mm} \tag{7}$$

for category 2:

$$l_a = s_s \quad \text{for} \quad \beta_v \leq 0.2 \tag{8}$$

$$l_a = 10 \text{ mm} \quad \text{for} \quad \beta_v \geq 0.3 \tag{9}$$

l_a = interpolated linearly between the values of l_a for 0.2 and 0.3 for $0,2 < \beta < 0,3$ (10)

where

s_s – the length of stiff bearing,

and

$$\beta_v = \frac{|V_{Ed,1}| - |V_{Ed,2}|}{|V_{Ed,1}| + |V_{Ed,2}|} \tag{11}$$

in which $|V_{Ed,1}|$ and $|V_{Ed,2}|$ are the absolute values of the transverse shear forces on each side of the local load or the support reaction, and $|V_{Ed,1}| \geq |V_{Ed,2}|$.

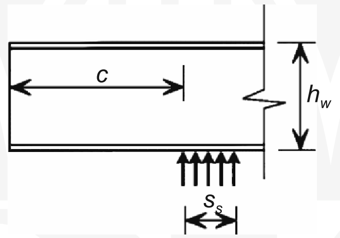

Fig. 3. Category 2 – reaction at the end support with $c > 1,5h_w$ clear from a free end [4]

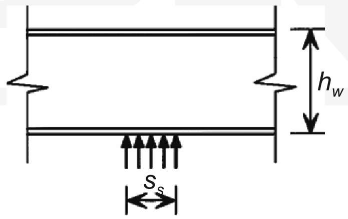

Fig. 4. Category 2 – reaction at the internal support [4]

The value of coefficient α should be obtained as follows:

for category 1:

$$\alpha = 0,075 \text{ for sheeting profiles} \tag{12}$$

for category 2:

$$\alpha = 0,015 \text{ for sheeting profiles} \tag{13}$$

The analytical calculations were carried out with use of Mathcad 14 [5] and the graphs were presented with the use of Microsoft Excel 2010 [6].

Example 1

Calculate the local transverse resistance of the unstiffened web of a 92 mm high trapezoidal sheet. The cross-section of the trapezoidal sheet is presented in Fig. 5.

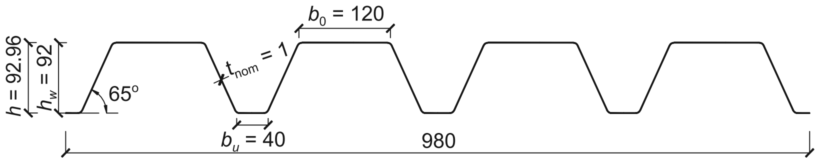

Fig. 5. The cross-section of a trapezoidal sheet 92 mm high

Calculation of the design thickness of the trapezoidal sheet

After formula (3.3c) in [4]:

$$t = t_{\text{cor}} = t_{\text{nom}} - t_{\text{metallic}} = 1.0 - 0.04 = 0.96 \text{ mm}$$

where:

t – design core thickness of steel material before cold forming,

t_{nom} – nominal sheet thickness after cold forming inclusive of zinc and other metallic coating, t_{nom} = 1.0 mm,

t_{cor} – the nominal thickness minus zinc and other metallic coating,

t_{metallic} – the thickness of the metallic coating (for the usual Z275 zinc coating, t_{zinc} = 0.04 mm).

Verification of the condition of the geometrical proportions of the trapezoidal sheet

After Table 5.1 in [4]:

For the upper flange:

$$\frac{b}{t} = \frac{120}{0.96} = 125 < 500$$

where according to Fig. 5: $b = b_0$ = 120 mm, as calculated: t = 0.96 mm.

For the bottom flange:

$$\frac{b}{t} = \frac{40}{0.96} = 41.67 < 500$$

where according to Fig. 5: $b = b_u$ = 40 mm, as calculated: t = 0.96 mm.

For the web:

$$45^{\circ} \leq \phi = 65^{\circ} \leq 90^{\circ}$$

$$\frac{h}{t} = \frac{92.96}{0.96} = 96.83 < 500 \cdot \sin(\phi) = 453.15$$

where according to Fig. 5: h = 92.96 mm, ϕ = 65°, as calculated: t = 0.96 mm.

Conclusion: Appropriate geometrical proportions a trapezoidal sheet allow the use of standard PN-EN-1993-1-3 [4].

Verification (obtained from 2–5) of the condition for a trapezoidal sheet during calculation of the local transverse resistance of the web

$$c \geq 40 \text{ mm}$$

$$\frac{r}{t} = \frac{6.5}{0.96} = 6.77 \leq 10$$

$$\frac{h_w}{t} = \frac{92}{0.96} = 95.83 \leq 200 \sin\phi = 181.26$$

$$45^\text{o} \leq \phi = 65^\text{o} \leq 90^\text{o}$$

where after Fig. 5: $h_w = 92$ mm.

Calculation of the local transverse resistance of the web

Category 1 – local load applied with $c \leq 1.5h_w = 1.5\cdot 92 = 138$ mm clear from a free end (see Fig. 2).

After formula (2):

$$R_{w,Rd} = 0.075 \cdot 0.96^2 \sqrt{320 \cdot 2.1 \cdot 10^5}\left(1 - 0.1\sqrt{\frac{6.5}{0.96}}\right)\left[0.5 + \sqrt{0.02\frac{10}{0.96}}\right]\left(2.4 + \left(\frac{65}{90}\right)^2\right)\Big/ 1.0$$

$$R_{w,Rd} = 1.17 \text{ kN}$$

where

$\alpha = 0.075$ – for sheeting profiles (category 1),
$t = 0.96$ mm,
$f_{yb} = 320$ MPa,
$E = 210$ GPa,
$r = 6.5$ mm,
$l_a = 10$ mm – for category 1,
$\phi = 65$ deg,
$\gamma_{M1} = 1.0$.

Category 2 – reaction at the end support with $c > 1.5h_w = 1.5\cdot 92 = 138$ mm clear from a free end (see Fig. 3) and reaction at the internal support (see Fig. 4), $\beta_v \geq 0.3$.

After formula (2):

$$R_{w,Rd} = 0.15 \cdot 0.96^2 \sqrt{320 \cdot 2{,}1 \cdot 10^5}\left(1 - 0.1\sqrt{\frac{6.5}{0.96}}\right)\left[0.5 + \sqrt{0.02\frac{10}{0.96}}\right]\left(2.4 + \left(\frac{65}{90}\right)^2\right)\Big/ 1.0$$

$$R_{w,Rd} = 2{,}343 \text{ kN}$$

where:

a = 0.15 – for sheeting profiles (category 2),
$l_a = 10$ mm – for category 2 and $b_v \geq 0.3$; the other values as above.

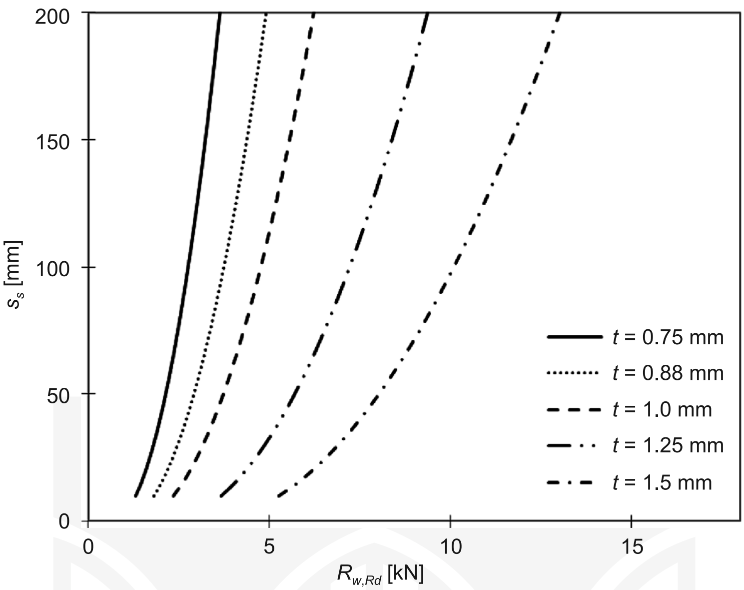

Fig. 6. Relation $s_s - R_{w,Rd}$ for the trapezoidal sheet 92 mm high, with an unstiffened web of thickness t = 0.75 mm – 1.5 mm

Figure 6 presents the way in which support widths s_s = 10 mm – 200 mm influence on the local transverse resistance of the unstiffened web of the trapezoidal sheet 92 mm high (category 2) of thickness from 0.75 mm to 1.5 mm.

2.2. The local transverse resistance of stiffened web of the trapezoidal sheet

For cross-sections with stiffened webs, the local transverse resistance of a stiffened web may be determined by multiplying the corresponding value for a similar unstiffened web by the factor $\kappa_{a,s}$, according to (6.22) in [4]:

$$F_{Ed} \leq R_{w,Rd} \cdot \kappa_{s,a} \tag{14}$$

where

$$\kappa_{a,s} = 1.45 - 0.05 \cdot \frac{e_{\max}}{t} \tag{15}$$

but

$$\kappa_{a,s} \leq 0.95 + 35000 \cdot t^2 \cdot \frac{e_{\min}}{(b_d^2 \cdot s_p)} \tag{16}$$

where (see Fig. 7):

- $e_{\max}$ – the larger eccentricity of the folds relative to the system line of the web,
- $e_{\min}$ – the smaller eccentricity of the folds relative to the system line of the web,
- b_d – the developed width of the loaded flange,
- s_p – the slant height of the plane web element nearest to the loaded flange.

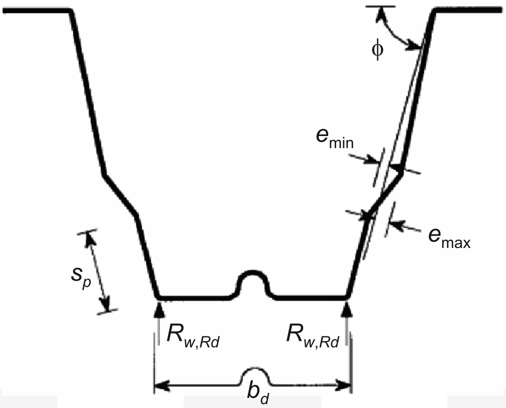

Fig. 7. Stiffened webs [4]

The local transverse resistance of a stiffened web may be determined as specified in (2) for cross-sections with longitudinal web stiffeners folded in such a way that the two folds in the web are on opposite sides of the system line of the web joining the points of intersection of the midline of the web with the midlines of the flanges (see Fig. 7), that satisfy the condition:

$$2 < \frac{e_{\max}}{t} < 12 \tag{17}$$

Example 2

Calculate the local transverse resistance of stiffened web of trapezoidal sheet 92 high. The cross-section of trapezoidal sheet are presented in Fig. 8.

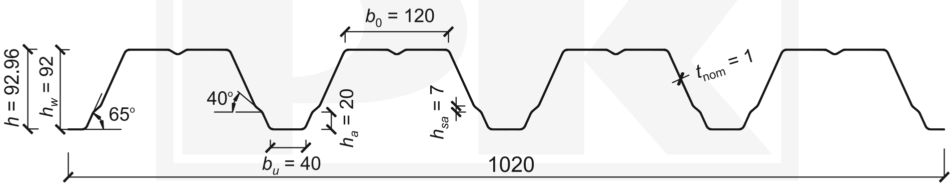

Fig. 8. The cross-section of a 92 mm high trapezoidal sheet

Calculation of the design thickness of the trapezoidal sheet

According to example 1.

Verification of the condition of the geometrical proportions of the trapezoidal sheet

According to example 1.

Verification (obtained from 2–5) of the condition of the trapezoidal sheet during calculation of the local transverse resistance of the web

According to example 1.

Calculation of the local transverse resistance of the unstiffened web

Category 1 – local load applied with $c \leq 1.5h_w = 1.5 \cdot 92 = 138$ mm clear from a free end (see Fig. 2).

According to example 1: $R_{w,Rd} = 1.17$ kN

Category 2 – reaction at the end support with $c > 1.5h_w = 1.5 \cdot 92 = 138$ mm clear from a free end (see Fig. 3) and reaction at internal support (see Fig. 4), $\beta_v \geq 0.3$.

According to example 1: $R_{w,Rd} = 2.343$ kN

Calculation of the local transverse resistance of the stiffened web

After formula (17):

$$2 < \frac{e_{\max}}{t} = \frac{3.24}{0.96} = 3.375 < 12$$

After formula (15):

$$\kappa_{a,s} = 1.45 - 0.05 \cdot \frac{e_{\max}}{t} = 1.45 - 0.05 \cdot \frac{3.24}{0.96} = 1.281$$

where according to Fig. 7: $e_{max} = 3.24$ mm, as calculated: $t = 0.96$ mm.

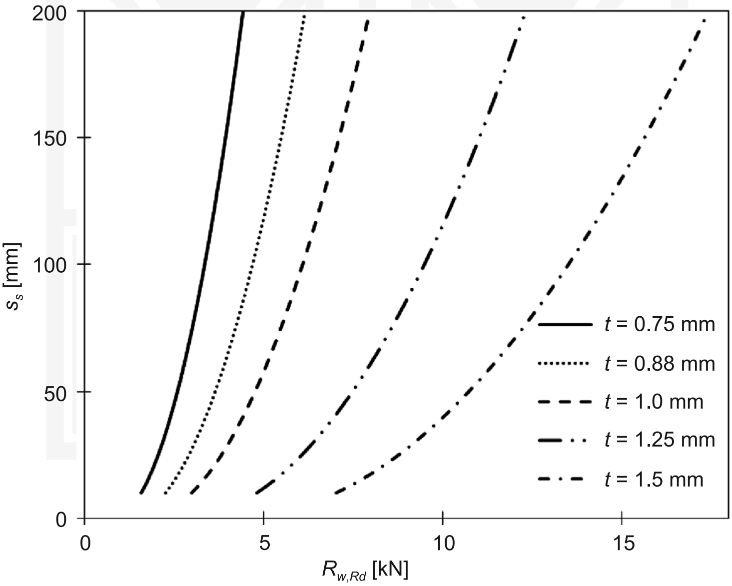

Fig. 9. Relation s_s - $R_{w,Rd}$ for the trapezoidal sheet 92 mm high, with an stiffened web from thickness $t = 0.5$ mm – 1.5 mm

After formula (16):

$$\kappa_{a,s} \leq 0.95 + 35000 \cdot t^2 \cdot \frac{e_{\min}}{(b_d^2 \cdot s_p)} = 0.95 + 35000 \cdot (0.96)^2 \cdot \frac{1.0}{(40^2 \cdot 22.07)} = 1.864$$

$$\kappa_{a,s} = 1.281 \leq 1.864$$

where according to Fig. 7: $e_{\min}$ = 1.0 mm, $b_d = b_u$ = 40 mm, s_p = 22.07 mm, as calculated: t = 0.96 mm.

For cross-sections of a trapezoidal sheet 92 mm high with stiffened webs, the local transverse resistance of the stiffened web should be determined from:

$$R_{w,Rd} \cdot \kappa_{a,s} = 1.17 \cdot 1.281 = 1.5 \text{ kN} \quad \text{(category 1)}$$

$$R_{w,Rd} \cdot \kappa_{a,s} = 2.343 \cdot 1.281 = 3.0 \text{ kN} \quad \text{(category 2)}$$

Figure 9 presents the path in which support widths s_s = 10 mm – 200 mm influence on the local transverse resistance of the stiffened web of trapezoidal sheet 92 mm high (Category 2) of thickness from 0.75 mm to 1.5 mm.

3. Conclusions

The analysis of the influence of the support width s_s for the 92 mm high trapezoidal sheet showed an increase of the local transverse resistance of the web in the case of the modification of the support width s_s from 10 mm to 200 mm. The values and increase [%] of the local transverse resistance of the unstiffened and stiffened webs for different thickness t and support width s_s are given in Table 1.

Table 1

The values of the local transverse resistance Rw,Rd [kN]of the unstiffened and stiffened webs for the 92 mm high trapezoidal sheet

t [mm]	Web without stiffening			Web with stiffening		
	s_s = 10 mm	s_s = 200 mm	Increase $R_{w,Rd}$ [%]	s_s = 10 mm	s_s = 200 mm	Increase $R_{w,Rd}$ [%]
0.75	1.302	3.629	179	1.591	4.435	179
0.88	1.808	4.908	171	2.273	6.169	171
1.0	2.343	6.224	166	3.001	7.973	166
1.25	3.663	9.367	156	4.821	12.327	156
1.5	5.257	13.022	148	7.039	17.436	148

By comparison of the relation s_s – $R_{w,Rd}$ for the 92 mm high trapezoidal sheet with the unstiffened web (see Fig. 6) and with the stiffened web (see Fig. 9), it is possible to determine the influence of the stiffening of the web on its local transverse resistance. The stiffening of the web of the 92 mm high trapezoidal sheet results in an increase of transverse resistance in relation to the unstiffened web by about 22% for t = 0.75 mm, 26% for t = 0.88mm, 28% for t = 1.0 mm, 32% for t = 1.25 mm and 34% for t = 1.5 mm.

Moreover, the thicknesses of both the unstiffened and stiffened webs of the trapezoidal sheet have an important influence on its transverse resistance. An increase of the thickness t from 0.75 mm to 0.88 mm causes an increase the local transverse resistance of the web by about 35% (with the unstiffened web) and by about 39% (the stiffened web). A further increase from 0.75 mm to 1.0 mm results in an increase the local transverse resistance of the web at about 71% (unstiffened web) and at about 80% (stiffened web). With an increase of thickness from 0.75 mm to 1.25 mm, the increase is approx. 158% (unstiffened web) and approx. 179% (stiffened web), and from 0.75 mm to 1.5mm it is approx. 259% (unstiffened web) and at about 293% (stiffened web).

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References

[1] Bródka J., Broniewicz M., Giżejowski M., *Kształtowniki gięte. Poradnik projektanta*, Polskie Wydawnictwo Techniczne, 2007.
[2] Pięciorak E., *Elementy z kształtowników i blach profilowanych na zimno*, XXVII Ogólnopolskie Warsztaty Pracy Projektanta Konstrukcji, tom II, Szczyrk, 7–10 marca 2012,147-208.
[3] Pięciorak E., Piekarczyk M., *Determination of Effective Cross-Section for Trapezoidal Sheet in Bending According to PN-EN 1993-1-3 Standard*, Czasopismo Techniczne, Budownictwo, R. 109, z. 3-B, 2012, 113-137.
[4] PN-EN 1993-1-3:2008 Eurocode 3. Design of Steel Structures. Part 1-3. General Rules. Supplementary Rules for Cold-Formed Members and Sheeting.
[5] Program Mathcad 14.
[6] Program Microsoft Excel 2010.
[7] Winter G., Pian R.H.J., *Crushing Strength of Thin Steel Webs*, Cornell University Engineering Experimental Station Bulletin, No 35, Part 1, 1946.