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A FORMAL NOTATION OF LEGAL RULES APPLICABLE IN THE LOCAL SPACE

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Abstract

In addition to the commonly known stable legal relationships between persons or institutions and fragments of the Earth's surface (real estate), there are widely applied ad hoc relationships to real space, with significantly varying durations. They are a function of acquired rights and remain valid for a strictly defined period of time. They are common in everyday life and take on diverse forms, the basic attribute of which is a reference to a broadly understood fragment of geographical space. Emerging technological achievements, such as digitisation of payments, algorithmisation of procedures, identification of documents and people, autonomous traffic and artificial intelligence - pose a challenge to provide ad hoc relationships to space with formalised notation. The problem of formal notation of legal rules applicable in local space will be presented on the example of the right of entry to the university campus, together with local traffic and parking rules. The space is divided into zones and then into smaller units – parking spaces, according to the hierarchical structure. Social groups that have relationships with these zones are also formed into hierarchical sets. Individual groups have different rights to space – hierarchically – at the zone level. The established hierarchy is the first factor considered. The second factor is variability over time. Relationships to local space change over time, most often in a cyclical form. It can therefore be concluded that in this case the law is a function of time. Parking spaces, as objects in spatial sets, are occupied randomly and subsequent users must start the procedure of searching the remaining objects belonging to the set. This procedure has been written in the form of a flowchart and graph. Objects removed from the set (as they are occupied) may randomly return, so for the purposes of effective searching, this process is presented as repeatable – as a whole or locally (cascading search). The third aspect considered are differences in the way users move around the local road network and in elementary spatial fields. The network structure is represented by intersections, which are noted as node matrices. In order to define how to behave in the local space, a list of rules was prepared and standardised. From a practical point of view, a form of prohibition is advantageous because it can be logically linked to a list (set) of consequences. Following these assumptions, the general form of the behavioural test algorithm was formulated. The presented issues demonstrate complex relationships to space and its integrating rule in relation to legal rules. This approach to law allows for space management using IT methods. The notation of the spatial structure and of legal relationships related to time enable the definition of rules of conduct for diverse groups of local societies. Dissimilarity from equality of law for all and stability over time, occurs in local environments and raises the relationships between community, space and time to a higher level of generality.

Keywords: GIS, hierarchy of space elements, ad hoc relationships to space, variability of rules over time, standardisation of rules, algorithmisation of procedures, formalisation of rule notation

FORMALNY ZAPIS REGUŁ PRAWNYCH OBOWIĄZUJĄCYCH W PRZESTRZENI LOKALNEJ

Abstrakt

Oprócz powszechnie znanych trwałych relacji prawnych osób lub instytucji do fragmentów powierzchni Ziemi (nieruchomości) – istnieją szeroko stosowane doraźne relacje do przestrzeni realnej, o znacznie zróżnicowanym czasie trwania. Są one funkcją nabytych uprawnień i obowiązują w ściśle określonym czasie. Są powszechne w życiu codziennym i przybierają zróżnicowane formy, których podstawowym atrybutem jest odniesienie do szeroko rozumianego fragmentu przestrzeni geograficznej. Wchodzące zdobycze techniki, jak choćby cyfryzacja płatności, algorytmizacja procedur, identyfikacja dokumentów i osób, ruch autonomiczny i sztuczna inteligencja – stanowią w czasach współczesnych wyzwanie do nadania doraźnym relacjom do przestrzeni sformalizowanego zapisu. Problem formalnego zapisu reguł prawnych obowiązujących w przestrzeni lokalnej zostanie przedstawiony na przykładzie reguł związanych z prawem wjazdu na teren kampusu uniwersyteckiego, wraz z lokalnymi regułami ruchu i parkowania. Przestrzeń jest podzielona na strefy, a następnie na mniejsze jednostki – miejsca do parkowania zgodnie ze strukturą hierarchiczną. Grupy społeczne, które posiadają relacje ze strefami, są również uformowane w zbiory hierarchiczne. Poszczególne grupy mają zróżnicowane prawa do przestrzeni – w odniesieniu hierarchicznym – na poziomie stref. Ustalona hierarchia to pierwszy z rozpatrywanych czynników. Drugim czynnikiem jest zmienność w czasie. Relacje do przestrzeni lokalnej zmieniają się w czasie, najczęściej w formie cyklicznej. Można zatem postawić wniosek, że w tym przypadku prawo jest funkcją czasu. Miejsca do parkowania jako obiekty w zbiorach przestrzennych są zajmowane losowo, a kolejni użytkownicy muszą uruchomić procedurę przeszukiwania pozostałych obiektów należących do zbioru. Ta procedura została zapisana w postaci schematu blokowego oraz grafu. Obiekty usunięte ze zbioru (jako zajęte) mogą losowo powracać, dlatego na potrzeby efektywnego szukania przedstawiono ten proces jako powtarzalny – całościowo lub lokalnie (kaskadowo). Trzecim rozpatrywanym aspektem są różnice w sposobie poruszania się użytkowników sieci dróg lokalnych i na elementarnych polach przestrzeni. Strukturę sieci reprezentują skrzyżowania, które zapisano jako macierze węzłów. W celu zdefiniowania sposobu zachowywania się w przestrzeni lokalnej sporządzono listę reguł, którą poddano standaryzacji. Z praktycznego punktu widzenia, korzystna jest forma zakazu, ponieważ może być powiązana logicznie z listą (zbiorem) konsekwencji. Po takich założeniach sformułowano ogólną postać algorytmu testów behawioralnych. Przedstawiona problematyka pokazuje złożone relacje do przestrzeni oraz jej integrujący charakter w stosunku do reguł prawnych. Takie podejście do prawa pozwala na zarządzanie przestrzenią metodami informatycznymi. Zapis struktury przestrzeni i relacji prawnych powiązanych z czasem – umożliwiają określenie reguł postępowania dla zróżnicowanych grup społeczeństw lokalnych. Odmienność od równości prawa dla wszystkich i stałości w czasie występuje w środowiskach lokalnych i stawia relacje między społecznością, przestrzenią i czasem na wyższy poziom ogólności.

Słowa kluczowe: GIS, hierarchia elementów przestrzeni, doraźne relacje względem przestrzeni, zmienność reguł w czasie, normalizacja reguł, algorytmizacja procedur, formalizacja zapisu reguł

LIST OF SYMBOLS USED

(arranged in the order of appearance in the text)

P – area of the university campus developed as parking zones

P_a, P_b, P_c – denotations of parking zones

M_a, M_b, M_c – sets of parking spaces in individual zones

P_d – roads within three parking zones

N_a, N_b, N_c – sets of parking spaces intended for the disabled

W_a – a set of spaces in the P_a zone intended for employees holding rector and dean positions

L_w – a set of employees acting as rector and dean authorities

L – a set of the whole university community

L_a, L_b, L_c – sets of university employees and students who have acquired parking rights in particular zones

P_{al} – additional local parking zone intended for employees of selected units

M_{al} – a set of parking spaces in the additional local zone P_{al}

L_r – a set of rescue service units

L_d – a set of suppliers authorised to enter the campus

L_z – a set of distinguished retired university employees entitled to a limited number of entries to the campus and parking in any zone

ΔT_1 – weekly cycle time, which extends from 0:00 on Monday to 24:00 on Friday
ΔT_2 – weekly cycle time, which extends from 0:00 on Saturday to 24:00 on Sunday
t – time axis denotation in Figures 4 and 5 for the weekly cycle
t_r – time axis denotation in Figure 6 for the annual cycle
P_g – ad hoc parking restriction zone for the core university community in the P_a, P_b, or P_c zones – intended for university guests or for the period of renovation and construction
Δt – time interval for which the P_g zone was temporarily created
d_1, d_2, d_3, d_4 – roads forming the intersection; the denotations are used to note the intersection matrix for a particular access road

1. INTRODUCTION

The ubiquity of spatio-temporal relationships with issues that are not in the traditional scope of geo-information, but are very important in everyday life such as legal conditions, so far has not been presented as a challenge in terms of attempts to formalise them. But emerging technological achievements, such as digitisation of payments, algorithmisation of procedures, identification of documents and people, autonomous traffic and artificial intelligence – pose a challenge to provide ad hoc relationships to space with formalised notation. Data structures and types of algorithms, which are the basis of programming, have been widely discussed in the literature on the subject, e.g. in popular comprehensive books [1–4]. Inherent with the issue of solving problems by means of an algorithm is the issue of its computational complexity, its efficiency, to which the authors pay special attention.

In addition to the commonly known stable legal relationships between persons or institutions and fragments of the Earth's surface (real estate), there are widely used ad hoc relationships to real space, with significantly varying durations. Such relationships to certain fragments of geographical space are a function of acquired rights and remain valid for a strictly defined period of time. Such a right may be considered a civil contract to occupy a certain local space or use it in a different way – having met certain conditions. These types of relationships take on significantly different forms, the basic attribute of which is a reference to a broadly understood fragment of geographical space. The subject of the spatial relations of the law and their formal recording has already been hinted at in the publication [5].

Let us try to analyse these problems and build formal provisions by going through the group of tasks listed below for a selected fragment of geographical space:

- Compilation of legal rules applicable in a selected area.
- Analysis of the area and legal rules in terms of their structure and formulating these rules using formal language.
- Adjusting for changes in rules as a function of time.
- Using such a rule notation method so that the integrating factor is a selected fragment of geographical space.
- Adjusting for the rules regulating behaviour and action in the local geographical space.

Legal language is a specific language. Its logic is not always obvious. Much has already been written on this subject. A description of the concepts and institutions of modern law is contained in the monograph [6], which discusses interpretation and legal reasoning – key issues in the construction of algorithms based on legal rules. Item [7] is a propaedeutic study of the basic conceptual apparatus of legal sciences, the methods used in these sciences and the elementary institutions and principles of law. The theoretical basis of the mathematical theory of formal languages, i.e. mathematical linguistics, is the monographic study [8]. Mathematical linguistics is a branch of linguistics that uses mathematical methods to study languages. It is divided into two directions: synthetic (the study of different types of grammars) and analytical (the study of different types of formal grammars – related to computer science) [8].

As we read in [9], increasingly, access to official information about the law and the efficient exchange of information between actors in the legislative process depend on good functioning of the technical infrastructure and on the way information is tagged in legislative texts. Although modern technologies do not affect the content of the information transmitted, they undoubtedly change the way it is transmitted and thus affect accessibility, selection, perception and interpretation. Traditional media and delivery forms impose many re-

quirements and constraints on the construction of texts that condition their communicative effectiveness. Information technologies offer a whole range of new solutions to those creating and interpreting texts, allowing not only significant improvements to existing activities, but also to perform previously impossible operations on the information contained in the text. These technologies are changing the status of law itself. They have enabled up-to-the-minute access to legal information to an unprecedented extent; their development and their effects and scope of application often elude traditional forms of regulation. Most importantly, however, these technologies have enabled a change in traditional thinking about the production of legal texts and the ways in which legal information is managed.

2. HIERARCHICAL STRUCTURE OF SPACE DIVISION AND CONNECTIONS WITH THE OBJECTS OF THIS STRUCTURE

Effective management of geographic space requires building a hierarchy and adapting the level of detail to subsequent levels. Such rules apply universally in the real world – at the level of countries, lands, cities and local areas. The factor that integrates the management method is uniform law. A certain exception is local law, which applies, for example, after the adoption of the local land use plan (MPZP). But this local law is about giving shape to a fragment of geographical space – within the framework of common law.

Let us analyse the case in which groups of a certain closed society have different rights to the constituent fragments of local space. Let us try to formalise such diverse legal rules. The aim of this procedure is to improve the management of this local space and to use IT methods for management.

For the purpose of this publication, we consider the entitlements of car drivers, an example of legal regulations that can be applied to GIS analyses. Our considerations will be based on the example of a university campus (Fig. 1), which consists of built-up areas, internal roads, a recreational area and areas intended for parking lots P. The traditional hierarchy of the university is visible in the arrangement of parking spaces – they are available to everyone, but adjusted for the position in its structure – from the university authorities and professors to students. In the P area of the campus

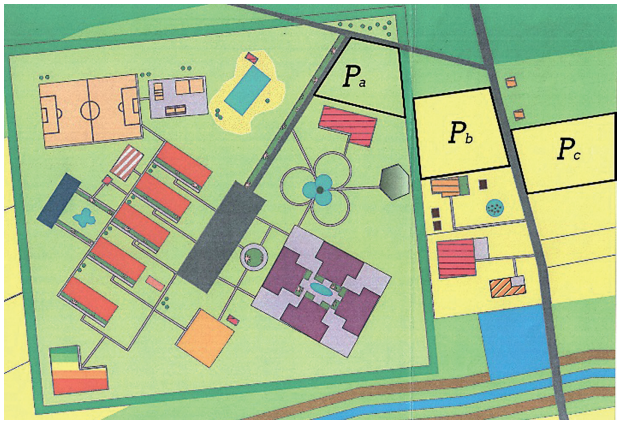

Fig. 1. The area of the university campus including buildings, a road network, recreational areas and areas arranged as a parking lot consisting of three zones P_a, P_b, and P_c. The drawing of the campus is a course project developed by student Aleksandra Stańczak[1]

Ryc. 1. Teren kampusu uniwersyteckiego zawierający zabudowania, sieć dróg, obszary rekreacyjne i tereny urządzone jako parking – składający się z trzech stref P_a, P_b, i P_c. Rysunek kampusu opracowany jako projekt kursowy przez studentkę Aleksandrę Stańczak

intended for parking spaces, three zones have been created: P_a, P_b, and P_c. The basic feature of each zone is its distance from the centre of the campus development | (Fig. 1). The zones are connected by internal roads and with the main entrance to the campus. In parking zones P_a, P_b, and P_c, dedicated parking spaces have been denoted as M_a, M_b, and M_c, respectively.

The analysis of the entire area of the campus and its division into built-up areas, recreational areas, parking areas P and roads P_d, and then moving on to further division into zones and, finally, to individual parking spaces, leads to a conclusion that the division of campus space has a hierarchical structure (Fig. 2). An exemplary visualisation of hierarchical structures and related data in time and space is described in [10].

The internal structure of the zones requires some expansion, which is illustrated in Figure 2: in individual zones, in addition to standard parking spaces, there are parking spaces for the disabled N_a, N_b, and N_c. Moreover, in the P_a zone, a group of parking spaces W_a was created for employees performing the functions of rectors and deans, forming the L_w group.

[1] Aleksandra Stańczak, course project carried out at the AGH University of Science and Technology, Faculty of Mining Surveying and Environmental Engineering.

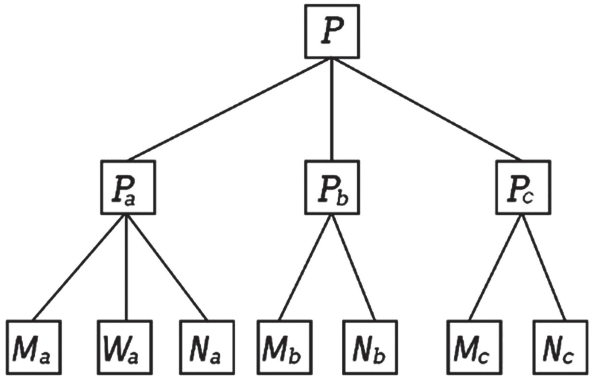

Fig. 2. Hierarchical structure of the campus area arranged as parking lots. Three zones have been distinguished in which there are parking spaces for authorised users M_a, M_b, and M_c and spaces for the disabled N_a, N_b, and N_c. Moreover, in the P_a zone, there are W_a spaces for the rector and dean authorities
Ryc. 2. Struktura hierarchiczna terenu kampusu urządzonego jako parkingi. Zostały wyróżnione trzy strefy, w których znajdują się miejsca parkingowe M_a, M_b, i M_c dla uprawnionych użytkowników i miejsca dla niepełnosprawnych N_a, N_b, i N_c. Ponadto w strefie P_a są zarezerwowane miejsca W_a dla władz rektorskichi dziekańskich

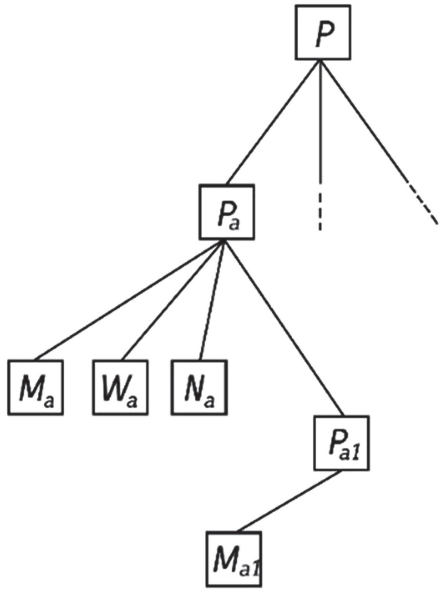

Fig. 3. An example of further expansion of the hierarchical structure – local subzones can be established in the P_a, P_b, and P_c zones for small groups of employees; a set of entitled persons can use M_{a1} spaces in the P_{a1} subzone
Ryc. 3. Przykład dalszej rozbudowy struktury hierarchicznej – w strefach P_a, P_b, i P_c mogą być urządzane lokalne podstrefy dla niewielkich grup pracowników; na rysunku zbiór uprawnionych może korzystać z miejsc M_{a1} w podstrefie P_{a1}

It is possible to further expand the space division hierarchy, as illustrated in Figure 3. Within the P_a zone, a small P_{a1} subzone containing M_{a1} parking spaces was created, intended exclusively for a specific group of employees.

All university employees and students, constituting set L, may apply for the right to enter parking lots located in the P_a, P_b, and P_c zones, however, the hierarchy of positions at the university and different fees apply here. University management staff can apply for parking spaces in the P_a zone, other employees in the, P_b, and P_c zones, and students only in the P_c zone.

Not every member of the university community L applies for the right to park on campus though. Those who apply and meet the condition regarding their professional position or are students are assigned to three groups that form sets L_a, L_b, and L_c.

Members of these sets may apply for the right to enter the P_a, P_b, and P_c zones, respectively, but members of higher-ranking sets have the additional privilege of using zones that are lower in the hierarchy:

- members of the L_a set have the right to use all zones: P_a, P_b, and P_c,
- members of the L_b set have the right to use P_b, and P_c zones,
- members of the L_c set have the right to use only P_c zones.

Although not all members of the University population L apply for a parking space, the number of spaces in each zone M_a, M_b, and M_c is smaller than the number of eligible people and may be smaller than the number of people willing to enter each zone at a given time:

$$L_a > M_a, \quad L_b > M_b, \quad L_c > M_c.$$

It can be concluded therefrom that finding a parking space in particular zones is a random event.

Acquiring the right to enter the appropriate zone, including additional zones, does not guarantee finding a free parking space. The relationships of the sets

L_a and $M_a + M_b, + M_c$ as well as
L_b and $M_b, + M_c$ as well as
L_c and M_c

are not functions because the definition of a function is not met:

$$\bigwedge_{x \in L} \bigvee_{y \in M} x \to y$$

There is only one case of a function, and this is a mutually unambiguous function (bijection): there are spaces in the P_a zone denoted as W_a, intended for the rector and dean authorities, forming the L_w set, where a functional relationship occurs:

for each object belonging to the L_w set, there is a unique and unambiguously assigned space in the | W_a set.

In our considerations so far, we have analysed four sets of persons entitled to occupy parking spaces: L_a, L_b, L_c and L_w. In practice, there are other groups of potential parking lot users:

- rescue services, whose units form the L_r set,
- suppliers included in the L_d set,
- a group of distinguished retired university employees included in the set L_z.

The three additional groups of parking users mentioned above are subject to certain specific temporal relationships.

3. LOCAL LAW AS A FUNCTION OF TIME

In the common civic understanding, law is stable and any changes require appropriate, often complex, procedures. This is different from the example discussed here, where the rules may change over time. It can be concluded that changes in local rules are a function of time. Most frequently, we are dealing with a situation in which changes occur regularly, cyclically, exhibiting certain regularities.

Let us add the time factor to the relationship between sets of permissions and fragments of real space. Such enrichment of relationships and raising them to a higher level is presented in Figure 4. It demonstrates the relationships in a 24-hour, daily cycle, starting at midnight (0:00) and ending at midnight (24:00) as well.

Emergency services L_r have unrestricted access to the campus, to all three parking zones, 24 hours a day. Suppliers L_d have a very limited access time: from 6:00 a.m. to 8:00 a.m. There is a niche presence of staff and students on the campus during this time frame, so the presence of suppliers provides little disruption to parking for regular users.

The blocks illustrated in Figure 4, included in the time interval from 6:00 a.m. to 10:00 p.m., represent users from the sets L_a, L_b, and L_c, according to their rights to access parking zones P_a, P_b, and P_c.

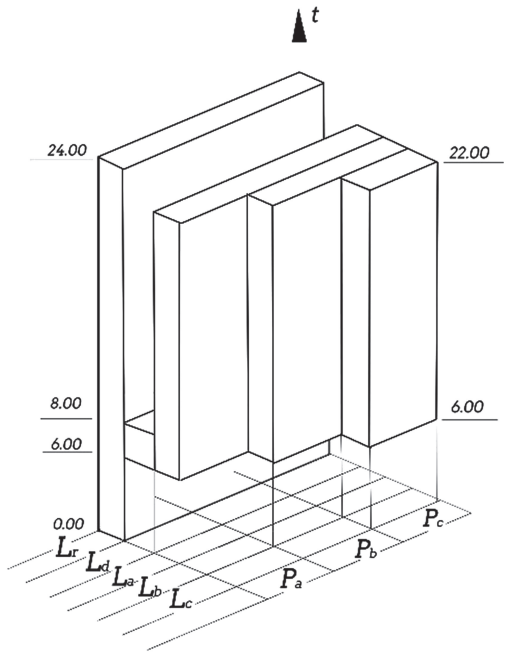

Fig. 4. Relationships of groups (L_a, L_b, L_c) entitled to park in zones P_a, P_b and P_c during the daily cycle. A short parking time has been provided for suppliers L_d, and rescue services L_r benefit from unlimited access and parking time

Ryc. 4. Relacje grup uprawnionych (L_a, L_b, L_c) do parkowania w strefach P_a, P_b, i P_c w godzinach cyklu dobowego. Dla dostawców L_d przewidziano krótki czas parkowania, służby ratownicze L_r korzystają z nieograniczonego czasu dostępu i postoju

We shall now move on to the variability of rules in relation to time. The relationships presented so far are valid during ΔT_1, which extends from 0:00 on Monday to 24:00 on Friday. The period ΔT_2 includes the remaining two days of the week. During this second period, access conditions to the campus remain unchanged only for emergency services L_r. The possibility of entry for suppliers L_d is eliminated.

Due to the small presence of the university community on campus on Saturdays and Sundays, two privileged parking options have been established for groups L_b, and L_c:

- both groups L_b, and L_c gain access to spaces inaccessible to them during working days, to the P_a, and P_b zones,
- groups L_b, and L_c gain the right to park in zones one class closer to the university buildings – group L_b, can park in zone P_a, while group L_c – in zone P_b.

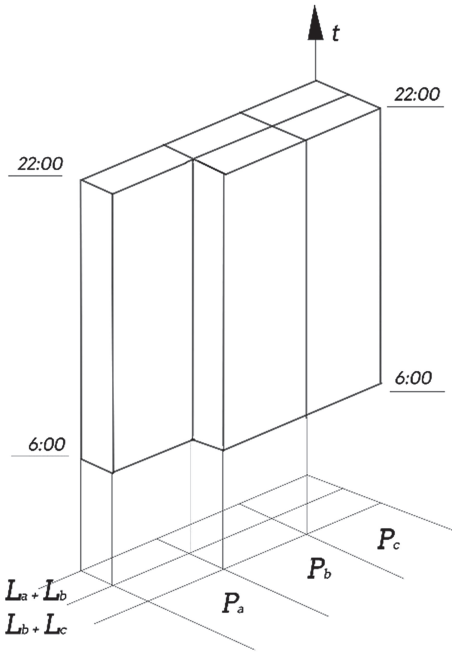

Fig. 5. Example of changing parking rules on non-working days. Groups L_b, and L_c that are lower in the hierarchy gain the right to park in a zone one class higher. Suppliers are not allowed to enter on these days, while rescue services L_r (omitted in this Figure for clarity) continue to benefit from unlimited access and parking time

Ryc. 5. Przykładowa zmiana reguł parkowania w dniach wolnych od pracy. Grupy uprawnionych stojące niżej w hierarchii L_b, i L_c uzyskują prawo do parkowania w strefie o jedną klasę wyższą. Dostawcy w tych dniach nie mają prawa wjazdu, natomiast służby ratownicze L_r (dla przejrzystości opuszczone na tym rysunku) korzystają nadal z nieograniczonego czasu dostępu i postoju

Of course, both groups L_b, and L_c retain full rights to park in their original zones.

Figure 5 illustrates the second variant mentioned above, containing two blocks – for the groups $L_a + L_b$ and for the groups $L_b + L_c$. The P_c zone is, of course, still available during ΔT_2, but the community present at the university at that time can benefit from the privilege of parking closer.

The rule variability presented above is repeated on a weekly basis. But this is not the only time cycle observed. Let us consider the case of an annual cycle for a group of distinguished retired university employees L_z. This group benefits from a limit of 50 entries to any P_a, P_b or P_c zone – in an annual cycle (Fig. 6). This is an example of a long-term cycle with random presence and having no relation to the division of the week into time zones ΔT_1 and ΔT_2.

Other anomalies may also occur in relation to time. When various professional celebrations or conferences are held during the annual cycle, university may allocate ad hoc P_g zones for university guests in the P_a, P_b, or P_c zones for the period Δt. Mobile exhibitions of technical equipment usually require a slightly longer time Δt, and above all, renovations or construction of new facilities, which reduce the range of parking spaces in various zones.

The cases of changeability of legal rules in time described above also occur in certain common legal provisions. An example is the set of rules for vehicular traffic. On an annual basis, the period of obligatory snow chains on wheels in mountainous areas is determined. On a weekly basis, on Saturdays and Sundays, traffic directions on highways near large cities are changed for

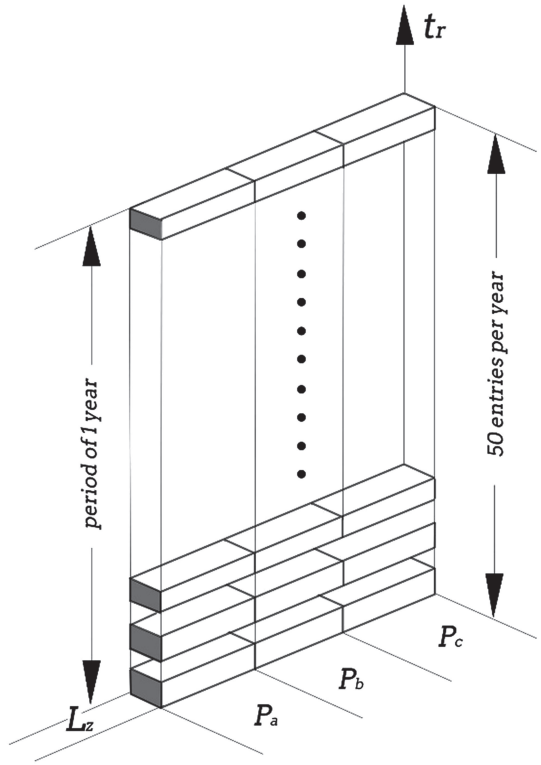

Fig. 6. An example of another form of time reference – a group of distinguished retired university employees L_z enjoys the privilege of parking in any zone 50 times per year

Ryc. 6. Przykład innej formy odniesienia czasowego – grupa zasłużonych pracowników uczelni w stanie spoczynku L_z korzysta z przywileju parkowania w dowolnej strefie 50-krotnie w cyklu rocznym

better flow of the waves of residents leaving the city and returning home. Similarly, on a weekly basis, alternate parking is allowed on city streets – on the left or right side. During the daily cycle, the traffic speed in cities is limited and parking rules change.

4. ALGORITHMS FOR SEARCHING THE SETS

In Chapter 2, it is stated that the number of spaces in each zone M_a, M_b, and M_c is smaller than the number of persons authorised to enter the individual zones – L_a, L_b, and L_c. It is concluded therefore that finding a free space is a random event. Verification of the partial or complete occupation of parking spaces in a given zone is carried out in the process of searching a set of spaces, according to the example algorithm for the P_a zone:

Having entered the Pa zone
Observe another object from the M_a set
IF the object is not occupied THEN – park
OTHERWISE, observe the next object

The above symbolic fragment of the algorithm does not include the examination whether the entire set has been searched. It says nothing about the possibility of moving to other zones, either. Therefore, let us use an algorithm written as a block diagram (Fig. 7), which not only clearly examines the state of parking space occupancy in the set, but also, for those authorised to enter the P_a zone, directly searches the next two zones P_b, and P_c.

It should be emphasised that failure to find a parking space in the original zone (and in other zones, in accordance with the entitlements) does not exclude the possibility of continuing to drive on the internal roads of the zones in order to search the set of spaces again. The block diagram (Fig. 7) presents the possibility of re-searching subsequent P_a, P_b, and P_c zones, starting from the P_a original zone. Let us consider the case of an urgent need to park and repeated searching through a given zone or moving to a higher zone. Such an algorithm for searching sets, which can be called cascading search, is illustrated in Figure 8, in this case using a different formal language – a graph.

The presented algorithms are applicable to the traditional version of organising car parks, but they can also be used to manage the parking lot using IT meth-

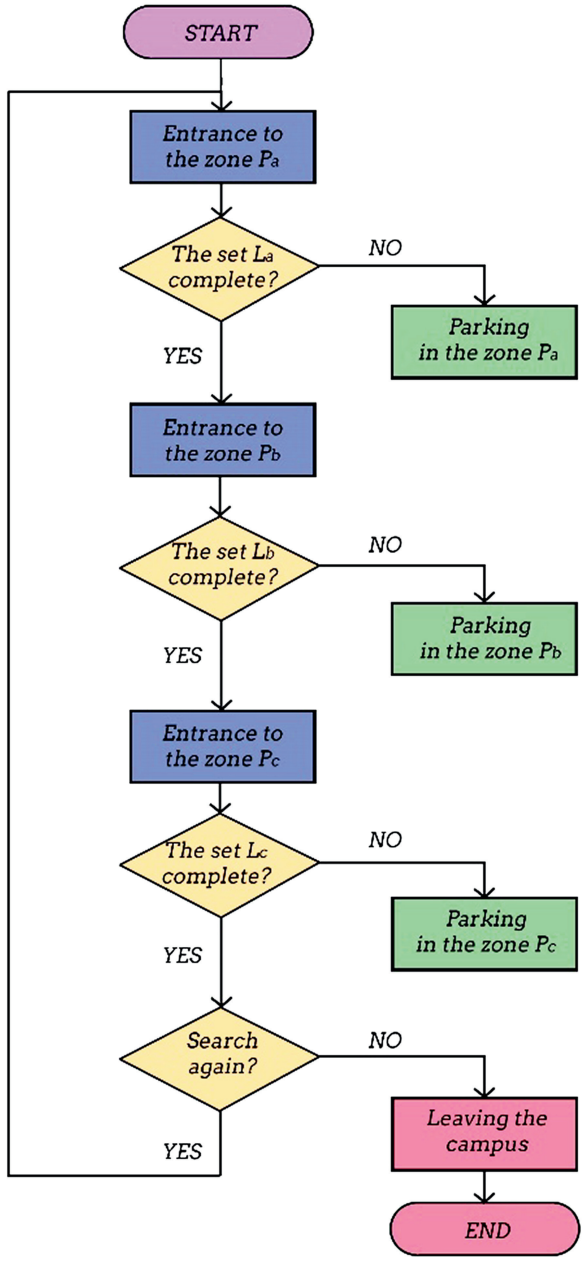

Fig. 7. Block diagram of the procedure for searching subsequent zones P_a, P_b, and P_c in order to find a free space – for the person entitled to enter all three zones. For effective searching, this process is presented as repeatable (search with restart). Repetition was extended throughout the entire range of the hierarchy

Ryc. 7. Schemat blokowy procedury przeszukiwania kolejnych stref P_a, P_b, i P_c w celu znalezienia wolnego miejsca – dla uprawnionego do wjazdu do wszystkich trzech stref. Dla efektywnego szukania przedstawiono ten proces jako powtarzalny (szukanie z restartem). Powtórzenie rozciągnięto na cały zakres hierarchii

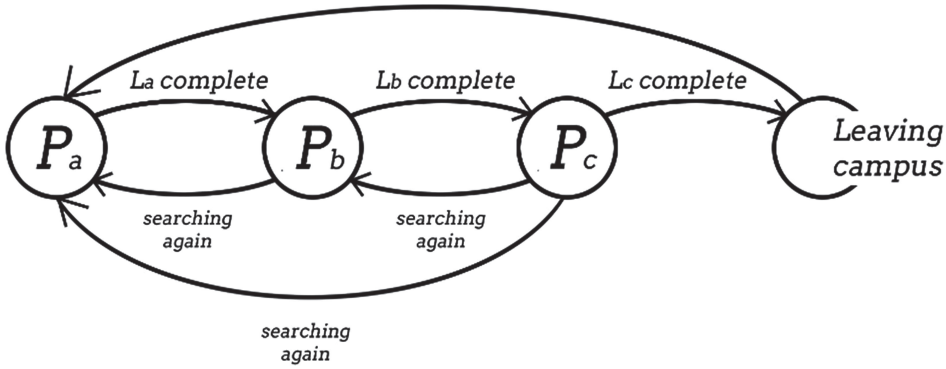

Fig. 8. Searching subsequent zones P_a, P_b, and P_c in order to find a free space, written in a formal graph language. The search process is presented as repeatable, but in this case the current zone is repeatedly searched and then the search is extended to other zones (cascading search)

Ryc. 8. Przeszukiwania kolejnych stref P_a, P_b, i P_c w celu znalezienia wolnego miejsca, zapisane w formalnym języku grafu. Proces przeszukiwania przedstawiono jako powtarzalny, lecz w tym przypadku zastosowano wielokrotnie przeszukiwanie bieżącej strefy, a następnie przejście do dalszych (szukanie kaskadowe)

ods when the identification of all spaces is implemented. Algorithms may also be used in the future to program the traffic of autonomous cars.

5. ANALYSIS OF BEHAVIOURAL PRINCIPLES

On the campus, organised as zones P_a, P_b, and P_c, there is a network of roads P_d. Parking zones have access roads (transit routes) and internal roads leading to a series of parking lots. General road traffic rules apply on all roads. Roads form a geographical network whose nodes are intersections (Fig. 9).

Let us describe the layout of the exemplary intersection illustrated in Figure 9 as a binary matrix that describes the variants of continued driving from the junction. A numerical value of 1 means that the condition specified in the column title is met, a value of 0 means that it is not met. The access roads to the intersection are denoted in Figure 9 as d_1, d_2, d_3, d_4. A separate matrix must be prepared for each approach to the intersection. A binary matrix for access to the intersection using road d_1 will have the following form:

It should be noted that the matrix also includes an access road to continue driving after turning at the intersection. There may be other parameters written in the node matrix, for example determining whether the exit route goes through or ends with a turning field.

In addition to a formal notation of traffic rules, it is also necessary to establish parking rules. In its traditional form, it is most frequently called "parking lot regulations". Let us try to write down selected rules of the regulations using formal language. It is also an attempt to lay down legal rules in the form of sets (lists) and algorithms – as an alternative to traditional legal language.

The presented list is a set of selected rules that apply in the analysed parking lot:

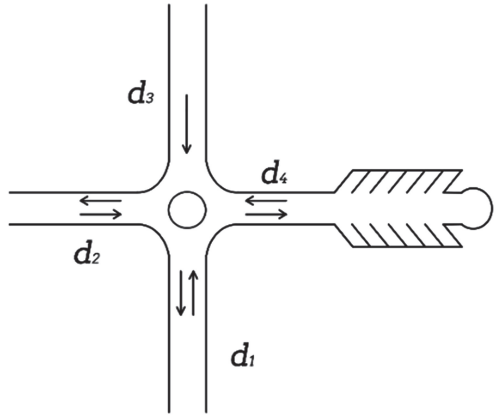

Fig. 9. Roads d_1, d_2, d_3, and d_4 form an intersection. For each access road, a binary node matrix can be written, containing various information, including parking options, traffic direction on the roads beyond the intersection and transit to the campus exit

Ryc. 9. Drogi d_1, d_2, d_3, i d_4 tworzą węzeł skrzyżowania. Dla każdej drogi dojazdowej można rozpisać binarną macierz węzła, zawierającą różne informacje, między innymi o możliwości parkowania, kierunku ruchu na drogach za skrzyżowaniem i tranzycie do wyjazdu z kampusu

Table 1. The binary matrix for access to the intersection using road d_1
Tabela 1. Macierz binarna dla dojazdu do skrzyżowania drogą d_1

Leaving the intersection	Parking on the road after leaving the intersection	Possibility to continue driving after leaving the intersection	Transit to the exit
d_1	0	1	1
d_2	0	1	1
d_3	0	0	0
d_4	1	1	0

. . . . . .

1. The parking lot is open from 6:00 a.m. to 10:00 p.m.
2. Parking rules must be observed in individual zones – in accordance with entitlements.
3. It is prohibited to occupy spaces designated for the university authorities and for the disabled by persons who are not entitled to these | spaces.
4. Parking on escape and fire routes is strictly prohibited.
5. It is obligatory to park in accordance with the lines marking parking spaces (horizontal signs).

. . . . . .

The above list contains unambiguous and clear rules in the traditional sense. However, it should be noticed that the grammar contains three different forms that define behaviour:

– form of information – rule 1 (the phrase "is open"),
– form of order – rules 2 and 5 (the phrase "must/is obligatory"),
– form of prohibition – rules 3 and 4 (the phrase "is prohibited").

To use algorithmic notation, we need to standardise this list to reduce it to one form. In the process of standardisation, we can decide on a form of order, then the form of information (rule 1) would have to be:

– It is obligatory to leave the parking lot before 10 p.m.

For better order and to communicate more clearly with users, it is advisable to use a form of prohibition, i.e. specify prohibited behaviours. The information rules and orders would then take the following form:

. . . . . .

1. Parking is prohibited at night – from 10 p.m. to 6 a.m.
2. Parking in individual zones in violation of the permits is prohibited.

. . . . . .

5. Parking inconsistent with horizontal signs is prohibited

. . . . . .

The form of a prohibition raises the importance of the rules and, above all, makes it possible to link the rules with the consequences of failure to comply with the regulations. Let us now compile an exemplary list of consequences of failure to comply with the prohibition rules:

. . . . . .

– Warning left on the vehicle window.
– Intervention of the university security service.
– Financial penalty for repeated violation of the rules.
– Towing the vehicle away.
– Temporary suspension of parking privileges.

. . . . . .

For the completeness of the procedure for non-compliance with the rules, it is necessary to clearly link the elements of the set of rules with the elements of the set of consequences. Such a relationship can be written in the form of a binary policy violation-consequence matrix (relationship matrix), in which the rows are elements of the policy violation set and the columns represent consequences.

Having a list (set) of rules and a list (set) of consequences of non-compliance, it is possible to write the general form of behaviour in the university parking lot in an algorithmic language.

IF a given action
IS EQUIVALENT to an item included in the {set of prohibited activities}
THEN – the consequences from the {set of consequences}

An example of a specific notation in algorithmic form is as follows:

IF action = careless parking, partially occupying an adjacent space
IS EQUIVALENT to an item included in the {set of prohibited activities} – failure to comply with horizontal signs
THEN – consequences from the {set of consequences} – intervention of the university security services

An additional, theoretical rule can be formulated: if in the behaviour of a user there is no equivalent in the set of prohibited actions, then it can be assumed that this behaviour complies with the rules and is free from any consequences.

The presented behavioural parking rules can be extended to other rules of conduct in real space. It is necessary to compile a set of rules, standardise these rules and link them to a set of consequences. Then it is possible to write an algorithm for each specific case.

6. SUMMARY

This research paper addresses the problem of formal notation of legal rules applicable in the local geographical space. The space is divided into zones and then into smaller units – according to the hierarchical structure. Social groups that have relationships with zones and units of spatial objects are also formed into hierarchical sets. Individual groups have different rights to space – hierarchically – at the zone level.

Relationships to local space change over time, most frequently in a cyclical form. It can therefore be concluded that in this case the law is a function of time. Objects in spatial sets, at the lowest level of the hierarchy, are eliminated from the set in a procedure of random, temporary space occupation. Subsequent users must run the procedure of searching for the remaining free objects belonging to the set. This procedure was written in formal languages – in the form of a flowchart and a graph. The process of occupying space varies over time. Objects removed from the set (as they are occupied) may randomly return, so for effective searching this process is presented as repeatable – searching with restart. The repetition can be extended to the entire range of the hierarchy or the current zones can be searched repeatedly and then one moves on to the next zones (cascading search).

The third group of problems includes the behaviour of objects in the local road network and in elementary spatial fields. The network structure is represented by nodes, which are written as transition matrices.

In order to define the manner of behaving in local space, a list of rules was prepared and standardised. From a practical point of view, a form of prohibition is advantageous because it can be logically linked to a list (set) of consequences. After such assumptions, the general form of the algorithm of possible behaviours was formulated.

The presented issues demonstrate complex relationships to space and how space can be a factor integrating legal rules. This approach to law allows for space management using IT methods.

The notation of the structure of space and legal relationships related to time enable defining rules of conduct for diverse groups of local societies. This approach, different from equality of law for all and stability over time, occurs in local environments and raises the relationships of community, space and time to a higher level of generality.

It turns out that some seemingly obvious differences between space and time disappear on closer inspection. Time is similar to space in many respects, so much so that many philosophical problems concerning time have their counterparts in problems concerning space. Then we speak of a so-called temporal topology. This concept appears, for example, in the publication [11]. It seems that GIS technologies will develop towards additional dimensions. In the future, it will probably be necessary to introduce the time dimension into navigation tools, in the form of a solution to which users can be remotely connected, allowing detailed information to be obtained for authorised drivers. An example discussion of such solutions can be found in [12].

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