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Giuseppe Peano in Germany, His Connections with German Mathematicians, and the First English Translation of Gottlob Frege's Unpublished Letter to Adolph Mayer on Giuseppe Peano's Mathematical Logic

The article aims to answer whether Gottlob Frege's letter to Adolph Mayer, dated 8 July 1896, could help German mathematicians get acquainted with Giuseppe Peano's mathematical work, including his mathematical logic. It is the first publication of this letter in English. At the beginning, I present the main characters of this story. Next, I refer to the letters concerning Peano and his mathematical results. Thus, I show the background of Frege's letter to Mayer. In the last part, I collect information about Peano's contacts with German mathematicians – where he was quoted and by whom, who was quoted by Peano, and in which period of his life. I conclude that Peano was known in Germany before Frege wrote to Mayer in 1896. However, the letter could have helped publish five of Peano's articles in Germany, where Peano's mathematical logic was hardly known then. Undoubtedly, the letter promoted that knowledge.

Keywords: Gottlob Frege, Giuseppe Peano, Adolph Mayer, mathematical logic, scientific correspondence

Słowa kluczowe: Gottlob Frege, Giuseppe Peano, Adolph Mayer, logika matematyczna, korespondencja naukowa

> I have some difficulty in reading your symbols;
> but I shall get better at it, and if I still find difficulties,
> I shall take the liberty of writing to you.
> (Peano–Frege 30.01.1894)

> I believe I have perfected my conceptual notation somewhat in this work [*Grundgesetze der Arithmetik*]
> […] I regarded it in many respects as better than Peano's, even though it may appear less simple at first glance.
> (Frege–Jourdain, 23.09.1902)

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Introduction

Gottlob Frege exchanged letters for over fifteen years with two Italian mathematicians interested in his scientific activities: Giuseppe Peano (1859–1932) and Giovanni Vailati (1863–1909), who belonged to Peano's school.[1] Frege, Peano and Vailati studied mathematics and had limited education in philosophy. However, from today's point of view, both Frege and Vailati were also philosophers. In addition, Vailati's friends called him 'the Philosopher', and actually many of his papers are on philosophical topics, such as language. Peano was denied competence in the field of philosophy,[2] although he successfully participated in philosophical congresses in Paris in 1900, in Heidelberg in 1908, and in Bologna in 1911 with Vailati.[3]

The years from 1894 to 1903, when Frege and Peano exchanged letters, were a 'golden period' of their academic activities. It was the peak of Frege's logicism and time to develop a philosophy of language. At that time, Peano worked on his famous five editions of Formulario,[4] called by Erika Luciano and Clara Silvia Roero 'an encyclopedia of mathematics written in logical-symbolic language.'[5] During this period, in a letter to Adolph Mayer (1839–1908) from 8 June 1896, Frege was asking to publish his article devoted to Peano's logic in the journal "Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse" ("Proceedings of the Royal Saxon Society for the Sciences in Leipzig, Math-physics Class"). It finally happened.[6] Mayer was a member of the society, so he was allowed to present at a society meeting another author's paper and recommend it for publication.[7]

Based on my findings of altogether 48 letters and other documents, I will discuss the aspect of the first English publication of Frege's unknown letter to Mayer concerning an article on Peano's logic (Frege–Mayer, 8 June 1896) and shed light on the connections between Peano and German mathematical community in the time of Frege's letter to Mayer and after the appearance of Frege's article on Peano's logic. I refer to Peano's contacts or connections with (in alphabetic order): Georg Cantor, Richard Dedekind, Gottlob Frege, Hermann Grassmann, Felix Klein, Adolph Mayer, Moritz Pasch, Alfred Pringsheim, Adolph

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1   Frege's collaboration with Giovanni Vailati will be presented in a separate paper. On the rich characterization of Peano's school see E. Luciano, *Characterizing a Mathematical School. Shared Knowledge and Peano's Formulario*, "Revue d'Historie des Mathématiques" 2017, vol. 23, no 2, p. 183–231.

2   H. Kennedy, *Peano. Life and Work*, San Francisco 2002, p. 63.

3   E. Luciano, C.S. Roero, *From Turin to Göttingen. Dialogues and Correspondence (1879–1923)*, Piza, Roma 2012, p. 28; H. Kennedy, *Peano. Life and Work*, p. 118.

4   G. Peano, *Formulaire de mathématiques*, vol. 1, Turin 1895; idem, *Formulaire de Mathématiques*, *publié par la Revue de Mathématiques*, vol. 2, no 3, *Logique mathématique. Arithmétique. Limites. Nombres complexes. Vecteurs. Dérivées. Intégrales*, Torino 1899; idem, *Formulaire de Mathématiques*, vol. 3, Torino 1901; idem, *Formulaire mathématique. Édition de l'an 1902–03*, vol. 4, Torino 1903; idem, *Formulario Mathematico*, vol. 5, Torino 1908.

5   E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 71.

6   G. Frege, *Über die Begriffsschrift des Herrn Peano und meine eigene*, "Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse" 1896, vol. 48, p. 361–378. In this context it is worth adding that Klein rejected to publish another Frege's paper in "Mathematische Annalen." See [Klein–Frege, 14.08.1881], [in:] G. Frege, *Wissenschaftlicher Briefwechsel*, ed. by G. Gabriel, H. Hermes, F. Kambartel, Ch. Thiel, A. Veraart, Hamburg 1976, p. 134–135; also *Korrespondenz Felix Klein – Adolph Mayer. Auswahl aus den Jahren 1871–1907*, ed. by R. Tobies, D.E. Rowe, Leipzig 1990, p. 37. Frege's article was published after his death as G. Frege, *Booles Rechnende Logik und die Begriffsschrift [1880/1881]*, [in:] idem, *Nachgelassene Schriften und wissenschaftlicher Briefwechsel*, vol. 1, Nachgelassene Schriften. ed. by G. Gabriel, H. Hermes, F. Kambartel, F. Kaulbach, W. Rödding, Hamburg 1983, p. 9–52.

7   R. Tobies, a private letter of 5.11.2022.

Schepp, Ernst Schröder, and Hermann Amandus Schwarz. I am interested in the period from 1891 to 1903 when Frege stayed in epistolary contact with Peano.

I refer to the correspondence and papers of the people in question and the literature devoted to Frege's collaboration with Peano. My paper is divided into three parts. I start by briefly introducing four main scholars to whom I will refer: Gottlob Frege, Giuseppe Peano, Adolph Mayer, and Felix Klein. In the next chapter, I describe the preserved and lost correspondence of Frege, Klein, Mayer, Pasch, and Peano. It consisted of at least 48 items and illustrated the appearance of Peano's mathematical logic in Germany. Frege's letter to Mayer, previously unpublished in English, is placed in this chapter. In the last paragraph, I collect information on German mathematicians who collaborated with Peano, when that happened, and whether they quoted him and where.

The subjects of artificial languages, interlingua (invented by Peano) and pasigraphy (invented by a German mathematician Ernst Schröder) are beyond the scope of this article and require a separate study.

The protagonists

Gottlob Frege (1848–1925) was a German mathematician, logician and philosopher from the University of Jena. His system of symbols is called concept script (*Begriffsschrift*). Frege assumed a logical framework, i.e. a standpoint in which arithmetical (not geometrical) notions, definitions, and rules are said to have been reduced to pure logical ones.

Frege's correspondence with Peano started probably in 1891 or later, after Frege's publication of *Die Grundlagen der Arithmetik* (Breslau 1884) and rather before the first volume of *Grundgesetze der Arithmetik* (Jena 1893) had come out. At that time, Frege also wrote his famous papers on the philosophy of language: *Über Sinn und Bedeutung* (1892) and *Über Begriff und Gegenstand* (1892). Their correspondence stopped after Frege had published the second volume of *Grundgesetze der Arithmetik* (Jena 1903), which was sent to Peano.

Frege and Peano were deeply influenced by the Leibnizian project of universal language, and they both built logical systems; however, their aim of doing logic was only partly the same. Peano's notation is easier to understand for us. On the other hand, Frege considered his conceptual notation being better than Peano's in many respects.[8] It was Peano who wrote the first letter and, unfortunately, this document is lost.

Frege referred to Peano directly by name in a lecture, papers and a book:

1. In a lecture on Peano's logic given on 17 September 1895 during the 67th meeting of the Society of German Scientists and Physicians (Versammlung Deutscher Naturforscher und Ärzte), which took place in1895 (16–20 September) in Lübeck (Germany),[9] with David Hilbert among the audience.

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8   See the motto; also G. Besler, „*Podążamy tymi samymi lub podobnymi drogami myślowymi"*. *Tematyka korespondencji logicznej Gottloba Fregego z Giuseppe Peanem, Davidem Hilbertem i Bertrandem Russellem*, Katowice 2019, p. 50–71.

9   See G. Frege, *Über die Begriffsschrift des Herrn Peano und meine eigene*, p. 361; also idem, *Kleine Schriften*, ed. by I. Angelelli, Hildesheim 1990, p. 220.

2. In the following year, the same topic on Peano's logic was presented by Mayer on 6 July 1896 in the Royal Saxionian Academy of Science (Königlich Sächsischen Gesells-chaft der Wissenschaften zu Leipzig).[10]
3. In a published paper *Über die Begriffsschrift des Herrn Peano und meine eigene.*[11]
4. In an answer to Peano's review, called *Lettera del sig. G. Frege all'Editore.*[12]
5. In an unpublished paper *Begründung meiner strengeren Grundsätze des Definierens* (1897/1898).
6. In a book *Grundgesetze der Arithmetik*, vol. 2, Jena 1903, p. 70–71, 77.

Shortly after 1880, Giuseppe Peano (1858–1932) learned about Frege's logical system from a detailed review of Frege's first book,[13] written by a German mathematician Ernst Schröder.[14] Peano taught at the University of Turin from 1880 to 1932, from 1890 as a Full Professor,[15] dealing with mathematical logic. In 1891 he established the journal "Rivista di Matematica", later called "Revue de Mathématiques." Before starting the correspondence with Frege he had published among others: *Arithmetices principia, nova methodo exposita* (Roma, Florentia 1889), and the following papers during the correspondence: *Notations de logique mathématique* (Turin 1894), *Formulaire mathématique* (Torino 1895).

It should be noted that Peano was also in epistolary contact with at least four other German mathematicians: Felix Klein, from 1885; Hermann Amandus Schwarz, from 12 April 1890; Adolph Mayer, before 1896; Georg Cantor, from 6 April 1895 to 10 January1896.[16]

Peano's letters to Frege are kept in the Darmstaedter collection (Sammlung Darmstaedter) at the Staatsbibliothek Preussischer Kulturbesitz (Berlin). The letters Frege wrote to Peano have not yet been found. We have only two drafts (more precisely: preliminary and incomplete versions of the letters) found in Frege's legacy and a letter published in the journal "Rivista di Matematica." Frege's letters were written in German, whereas Peano's letters were in Italian or French.

As far as I have been able to establish, Peano met in person solely with two German mathematicians – Felix Klein and Ernst Schröder (see the following sections). Three German mathematicians, Gottlob Frege, Georg Cantor and Rudolf Mehmke (1857–1944) published in "Rivista di matematica."[17] Peano wrote reviews of books published by Gottlob Frege, Hermann Grassmann, Ernst Schröder and published them in his journal.[18]

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10 An announcement of this lecture appeared in "Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse" 1896, vol. 48, p. 361.
11 G. Frege, *Über die Begriffsschrift des Herrn Peano und meine eigene*, p. 366, 376; also idem, *Kleine Schriften*, p. 223–224, 231.
12 Idem, *Lettera del sig. G. Frege all'Editore*, "Revue de Mathématiques/Rivista di Matematica" 1896–1899, vol. 6, p. 53–59; also idem, *Kleine Schriften*, p. 234–239.
13 Idem, *Begriffsschrift, eine der arithmetischen nachgebildete Formalsprache des reinen Denkens*, Halle 1879.
14 E. Schröder, *Gottlob Frege, Begriffsschrift. Halle a. S., Verlag von Louis Nebert, 1879, 88 S* [book review], "Historisch-literarische Abteilung der Zeitschrift für Mathematik und Physik" 1880, vol. 25, p. 81–94.
15 See H.C. Kennedy, *Biographical Sketch of Giuseppe Peano*, [in:] *Selected Works of Giuseppe Peano*, transl. and ed. by H.C. Kennedy, London 1973, p. 3–4.
16 Details of the correspondence are presented in the following section.
17 G. Frege, *Lettera del sig. G. Frege all'Editore*, p. 53–59; also idem, *Kleine Schriften*, p. 234–239; G. Cantor, *Sopra una questione elementare della teoria degli aggregati*, "Rivista di matematica" 1892, vol. 2, p. 165–167; R. Mehmke, *Über eine allgemeine construction der Krümmungsmittelpunkte ebener Curven und eine neue Begründung der Fundamentalsätze der Flächentheorie*, "Rivista di matematica" 1892, vol. 2, p. 65–71; idem, *Über die Aenderung der Hauptkrummungen einer Flache bei einer beliebigen Berührungstransformation*, p. 159–161.
18 G. Peano, *E. Schröder, Vorlesungen über die Algebra der Logik* [book review], "Rivista di matematica" 1891, vol. 1, p. 164–70; idem, *H. Grassmann, Gesammelte mathematische und physicalische Werke, Bd. 1* [book review],

Based on Frege–Peano correspondence, we also knew what books and articles they exchanged and how they inspired each other.[19] It should be noted that Frege is not mentioned in Peano's *I principi di geometria logicamente espositi*,[20] where Peano regretted that no one dealt with logic in a similar way.[21] Nevertheless, shortly after, the name 'Frege' is mentioned in Peano's following works and a lecture:

1. In 1891, in *Principii di logica matematica*, "Rivista di Matematica."
2. In 1894, in *Notations de logique mathématique.*
3. In 1895, in a review of Frege's *Grundgesetze der Arithmeti*k, published in "Rivista di Matematica".
4. In 1896 – 1899, in *Risposta*, which was his commentary on Frege's letter published in "Rivista di Matematica".
5. In 1897, in *Studii di logica matematica* and two years later in the German translation: G. Peano, *Anhang I. Über mathematische Logik.*
6. In 1897, in *Formulaire de mathématiques*, vol. 2.
7. Peano referred to Frege in his lecture given during the first congress of mathematicians in Zürich in 1897 – the lecture was subsequently published.
8. For the last time in 1900 in *Formules de logique mathématique*, where Peano wrote: 'Mr. Frege reached in his own way, in 1893, an ideography by which he expressed propositions on the idea of numbers.'[22]

Let me quote from Peano's paper (1897), published in German in 1899 and translated into English in 1973:

> The works of Frege are independent of those of numerous authors of mathematical logic. […] I am not now able to say whether the ideography of Frege is or not complete, i.e. whether its symbolic propositions can be read independently of the accompanying text, Frege's formulas are, for me, much more difficult to read than those of other authors.[23]

Peano published three papers in Germany in "Mathematische Annalen" and one in another journal (all in French), five papers as five appendixes to Genocchi's book (in German), and a book on geometrical calculus (in German) in the years 1887–1899. The details follow in the next paragraphs.

"Rivista di matematica" 1894, vol. 4, p. 166–169; idem, *G. Frege, Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet* [book review], "Rivista di matematica" 1895, vol. 5, p. 122–128; also idem, *Opere scelte*, ed. by U. Cassina, vol. 2, *Logica Matematica*, Roma 1958, p. 189–195; English translation: Peano's Review of Frege's *Grundgesetze*, transl. by V. Dudman, "The Southern Journal of Philosophy" 1971, vol. 9, p. 25–38; G. Peano, *E. Schröder, Über Pasigraphie, ihren gegenwärtigen Stand und die pasigraphische Bewegung in Italien, Verh. d. erst. Int. Math. Kongresses in Zürich 1897* [book review], "Revue de Mathématiques/Rivista di matematica" 1896–1899, vol. 6, p. 95–101.

19 I partly presented the subject in a book devoted to the content of their correspondence: G. Besler, „*Podążamy tymi samymi lub podobnymi drogami myślowymi*", p. 44.

20 G. Peano, *I principi di geometria logicamente espositi*, Torino 1889.

21 See H. Kennedy, *Peano. Life and Work*, p. 98.

22 In Kennedy's translation; see H. Kennedy, *Peano. Life and Work*, p. 103.

23 *Selected Works of Giuseppe Peano*, p. 191. Italian original: G. Peano, *Studii di logica matematica*, Torino 1897, p. 6; German translation: idem, *Anhang I: Über mathematische Logik*, [in:] A. Genocchi: *Differentialrechnung und Grundzüge der Integralrechnung*, ed. by G. Peano, transl. by G. Bohlmann, A. Schepp, Leipzig 1899, p. 338; also idem, *Arbeiten zur Analysis und zur mathematischen Logik*, ed. by G. Asser, Leipzig 1990, p. 12.

Peano asked Frege to publish a paper devoted to his logic, and Frege wrote about it in a letter to Adolph Mayer (1843–1908). During the correspondence with Frege, Mayer was a professor of mathematics at the University of Leipzig. Mayer was also in correspondence with Peano because Frege wrote to Mayer: 'he [Peano] repeatedly forwarded me your kind regards';[24] however, this correspondence has been lost. Fortunately, Mayer's exchange of letters with Felix Klein survived and is also important to our study. Peano is mentioned in Mayer's three letters to Klein. They discussed some points connected with Peano's results in mathematics and the occurrence of his papers in Germany.

The preserved Frege–Mayer correspondence started on 8 June 1896, almost one year after Frege's lecture on Peano's logic in Lübeck. Mayer read the same paper (or improved it; there is no evidence) during the meeting of the Royal Saxon Society for the Sciences in Leipzig (Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig); it was subsequently published.[25]

Peano was also in epistolary contact with Felix Klein (1849–1925) who taught in Göttingen then. There are four Peano–Klein letters preserved from 1887 to 1894.[26] However, it was Klein who contacted Peano first and asked him to publish a paper in the prestigious German journal "Mathematische Annalen," which happened – as Peano wrote – long before 10 October 1887.[27] To be more precise:

> The first letter from Klein to Peano, at that time a young assistant to the chair of Analysis but already known in Germany thanks to the 1884 treatise with A. Genocchi, dates to May 1885, when Klein asked Segre to pass on one he had written to Peano.[…] Klein invite Peano to present the extension of his proof of the existence and uniqueness of the solution of the systems of ordinary differential equations in the "Mathematische Annalen."[28]

When Peano published his articles in Germany in "Mathematische Annalen," Klein was one of the journal's editors. The famous journal was founded in 1869 by Alfred Clebsch and Carl Neumann. It is worth mentioning that Klein encouraged Peano to publish it, and Mayer thanked him for it in a letter.[29] As Luciano, Roero and Tobies remarked:

> they [Peano and Klein – G.B.] sharply disagreed from the very beginning about their assessment of ideographic symbolism, and in a second moment, by different attitudes towards metamathematical questions and the psychological implications of logical-foundational themes.[30]

24 [Frege–Mayer, 8.06.1896], published in this article.
25 G. Frege, *Über die Begriffsschrift des Herrn Peano und meine eigene*.
26 Bibliographical details in the following section.
27 [Peano–Klein, 10.10.1887], [in:] M. Segre, *Le lettere di Giuseppe Peano a Felix Klein*, "Nuncius. Annali di Storia della Scienza" 1997, vol. 12, p. 117; H. Kennedy, *Peano. Life and Work*, p. 34.
28 E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 29. There are more details on Klein–Peano collaboration.
29 See [Mayer–Klein, 21.08.1887], [in:] *Korrespondenz Felix Klein – Adolph Mayer*, p. 163; R. Tobies, *Felix Klein. Visions for Mathematics, Applications, and Education*, transl. by V.A. Pakis, Cham 2021, p. 483; E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 34.
30 E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 34.

Klein had a skeptical opinion of Peano's logical-axiomatic studies […] and […] overcame this skepticism […] around the year 1908.[31]

Peano, in turn, encouraged Klein and Frege to publish in a journal established by himself in 1891, i.e in "Rivista di mathematica/Revue de Mathématiques."

As mentioned above, Peano and Klein met in person. It was on 11 August 1897, during the First International Congress of Mathematicians in Zürich, while they had their talks during the second general session.[32] They could have also met in Turin, which Klein visited several times.[33]

Klein also corresponded with other Italian mathematicians, such as Corrado Segre (1883–1923) and Gino Fano (1895–1899).[34]

Frege, Klein, Mayer, Pasch and Peano's correspondence. Preserved and lost documents

I have found information about 48 letters pertaining to Peano, encompassing both extant and lost documents written in the period in question. There are:

1. 26 letters from Peano to German mathematicians: Cantor, Frege, Klein, and Schwarz.
2. 15 letters from German mathematicians to Peano: Cantor, Frege, Klein, and Schwarz.
3. 7 letters exchanged between German mathematicians Frege, Mayer, Klein and Pasch, which mention Peano and his results in mathematics or mathematical logic.

I have put them in order on the timeline to show the place of Frege's letter to Mayer of 8 June 1896, published here for the first time in English. I have to add that there is evidence of Peano–Mayer correspondence, but I have not managed to establish whether it was preserved and stored anywhere.

1. Klein–Peano, May 1885*[35]
2. Peano–Klein/Mayer, March 1887
3. Mayer–Klein, 21 June 1887[36]
4. Mayer–Klein, 21 August 1887
5. Mayer–Klein [undated]
6. Klein–Peano, before 10 October 1887*
7. Peano–Klein, 10 October 1887[37]
8. Peano–Klein, 8 April 1888

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31 R. Tobies, *Felix Klein. Visions for Mathematics*, p. 483. Cf. E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 33–34.
32 H. Kennedy, *Peano. Life and Work*, p. 95.
33 On Klein's trips to Italy see R. Tobies, *Felix Klein. Visions for Mathematics*, p. 153–158. On Italian mathematicians who studied under Klein see ibidem, p. 199–201, 246–248.
34 E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 83–219.
35 See [Segre–Klein, 20.05.1885], [in:] E. Luciano, S. Roero, *From Turin to Göttingen*, p. 138, also p. 29. The * sign means that the document is lost; we know about its previous existence from the content of other letters.
36 Mayer's letters to Klein are published in *Korrespondenz Felix Klein – Adolph Mayer*, p. 161–163, 181.
37 Peano's letters to Klein are published in: M. Segre, *Le lettere di Giuseppe Peano a Felix Klein*, p. 117–122. The German translation of [Peano–Klein, 19.09.1894] can be found in: G. Peano, *Arbeiten zur Analysis und zur mathematischen Logik*, p. 126. The English translation of excerpts of [Peano–Klein, 25/29.08.1894] is published. See H. Kennedy, *Twelve Articles on Giuseppe Peano*, San Francisco 2002, p. 63–64; C.S. Roero, *The* Formulario *between Mathematics and History*, [in:] *Giuseppe Peano between Mathematics and Logic*, ed. by F. Skof, Milan, Dordrecht, Heidelberg, London, New York 2011, p. 85–86.

9. Schwarz–Peano, before 12 April 1890[38]

10. Peano–Schwarz, 12 April 1890

11. Peano–Schwarz, 27 April 1890

12. Peano–Frege, before the first saved letter*

13. Frege–Peano, undated, 1891 or later[39]

14. Pasch to Klein, 19 October 1891[40]

15. Peano–Schwarz, 4 April 1893

16. Peano–Frege, 30 January 1894

17. Peano – Frege, 10 February 1894

18. Peano – Klein, 29 August 1894

19. Peano – Klein, 19 September 1894

20. Cantor–Peano, 6 April 1895, beginning of the correspondence[41]

21. Peano–Frege, 4 October 1895

22. Cantor–Peano, end of the correspondence 10 January 1896

23. Peano–Frege, 5 April 1896

24. **Frege–Mayer, 8 June 1896, Jena, a letter[42]**

25. Mayer–Frege, 9 June 1896

26. Mayer–Frege, 6 July 1896

27. Peano–Frege, 3 October 1896

28. Peano – Frege, 14 October 1896

29. Frege–Peano, undated (draft version), between 1896 and 1903

30. Frege–Peano, 29 September 1896, published in "Rivista…", 1899

31. Peano–Frege, without date, published in "Rivista…", 1899

32. Peano–Frege, 7 January 1903.

Frege–Mayer correspondence started on 8 June 1896, one month after Frege's lecture on Peano's logic in Lübeck (Germany). The subject matter of the correspondence was the publication of Frege's article on Peano's mathematical logic. The transcription of Frege's original first letter to Mayer, dated 8 June 1896, has been published in the Polish edition of Frege's letters. What follows is the first English translation of the first letter, made by Philip Ebert:[43]

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38   Kennedy referred to four documents from the Schwarz–Peano correspondence. See H. Kennedy, *Peano. Life and Work*, p. 22, 50, 61.

39   The Frege–Peano correspondence is published and translated. See G. Frege, *Wissenschaftlicher Briefwechsel*, p. 176–198; idem, *Philosophical and Mathematical Correspondence*, ed. by G. Gabriel, H. Hermes, F. Kambartel, Ch. Thiel, A. Veraart, transl. by H. Kaal, Oxford 1980, p. 108–129; idem, *Alle origini della nuova logica. Epistolario scientifico con Hilbert Husserl Peano Russell Vailati e altri*, preface by G. Lolli, Torino 2020, p. 142–168; idem, *Korespondencja naukowa. Wydanie krytyczne*, ed. by G. Besler, J. Zwierzyńska, Katowice 2020, p. 291–322.

40   Transcribed German originals and their English translations by D. Schlimm can be found in: *The Correspondence between Moritz Pasch and Felix Klein*, "Historia Mathematica" 2013, vol. 40, p. 192–193, 199–200. The name 'Peano' does not appear in the remainder of Pasch–Klein correspondence.

41   'The correspondence in both directions has apparently been lost […] the "letter books" of Cantor […] contain drafts of 10 letters from Cantor to Peano during the period from 6 April 1895 to 10 January 1896 and give evidence of at least 8 letters and postcards from Peano.' H. Kennedy, *Peano. Life and Work*, p. 84.

42   The letter is stored in Bereich Sondersammlungen Univesitätsbibliothek Leipzig (Germany). German transcribed original and its Polish translation appeared in the Polish edition of Frege's letters. See G. Frege, *Korespondencja naukowa. Wydanie krytyczne*, p. 278–279.

43   I would like to thank Philip Ebert (University of Sterling, Scotland) for his translation.

Jena, 8 June 1896

Dear Colleague,

On numerous occasions, Mr Peano from Turin urged me to publish something on his *logique mathématique.* Since he himself reviewed my *Grundlagen der Arithmetik*[44] and sent me some of his essays, I also feel somewhat obliged to do so. When I then recently talked to my colleague Thomae about where I could publish an essay on this topic, he suggested that you may be inclined to submit it to the Königl. Sächs. Gesellschaft der Wissenschaften [Royal Saxon Society for the Sciences]. Given that he repeatedly forwarded me your kind regards, which I hereby gratefully return, I dare to trouble you with this request, and I enclose the essay of which I hope that it is suitable for the above-mentioned purpose. Furthermore, I take the liberty to enclose a printed article which also concerns logical issues that seem to me of fundamental importance for a scientific symbolic language.

In the hope that the enclosed material is not undesirable, I remain respectfully yours.

Sincerely yours, G. Frege[45]

Mayer had known Peano's academic activities at least since 1886. Two years later, in April 1888, Peano's first paper was published in Germany, in "Mathematische Annalen," and Mayer was one of the editors.[46] Owing to Frege's letter to Mayer and Frege–Peano correspondence, we know about Frege–Peano collaboration:

1. Peano encouraged Frege to publish on his mathematical logic in Germany. It was Carl Joannes Thomae (1840–1921) who suggested publishing it in the journal "Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse".

2. Frege felt obliged to do so, because Peano published a review of *Grundgesetze der Arithmetik* (1893).

3. Peano was in contact with Mayer before the middle of 1896, because there is a sentence in Frege's letter to Mayer saying: 'he [Peano] repeatedly forwarded me your kind regards.' I don't know whether Peano–Mayer correspondence has survived.

Mayer responded positively to Frege's request to publish his article on Peano's mathematical logic and read Frege's paper during the meeting of the Königlich Sächsische

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44   I suppose Frege made a mistake here, Peano reviewed *Grundgesetze der Arithmetik*. See G. Peano, *G. Frege, Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet* [book review], p. 122–128.

45   Jena, den 8. Juni 1886. Sehr geehrter Herr Kollege! Herr Peano in Turin mahnt mich immer wieder, etwas über seine logique mathématique zu veröffentlichen. Und da er selber meine Grundlagen der Arithmetik angezeigt und mir manche von seinen Aufsätzen zugeschickt hat, so fühle ich mich dazu auch einigermaßen verpflichtet. Als ich nun neulich mit dem Kollegen Thomae darüber sprach, wo ich wohl eine Abhandlung über diesen Gegenstand veröffentlichen könnte, meinte er, dass Sie vielleicht die Güte haben würden, sie der Königl. Sächs. Gesellschaft der Wissenschaften vorzulegen. Und da er mir wiederholt freundliche Grüsse von Ihnen bestellt hat, die ich hiermit bestens erwidere, so wage ich es, Sie mit dieser Bitte zu belästigen, und lege die Abhandlung bei, von der ich hoffe, dass Sie für den genannten Zweck geeignet sein wird. Ich erlaube mir noch einen gedruckten Aufsatz beizufügen, der gleichfalls logische Fragen behandelt, die mir für eine wissenschaftliche Symbolik von grundlegender Wichtigkeit zu sein scheinen. Indem ich hoffe, dass Ihnen diese Zusendung nicht unerwünscht sein wird, verbleibe ich mit Hochachtung. Ihr ergebenster, G. Frege.

46   G. Peano, *Intégration par séries des équations différentielles linéaires*, "Mathematische Annalen" 1888, vol. 32, p. 450–456.

Gesellschaft der Wissenschaften zu Leipzig, which took place on 6 July 1896. Afterwards, Frege's paper was published.[47]

The following section discusses the broader context of their collaboration.

Peano in Germany – general remarks

Peano presented himself as an inheritor of Gottfried Wilhelm Leibniz's (1646–1716) tradition; therefore, let me refer to two quotations on this topic. Kennedy and Roero have stated:

> Leibniz may or may not have been Peano's original inspiration—probably not—but more and more he became the inspiration for the project. Peano delighted in seeing his work as a continuation of that of Leibniz, and he would soon see it as a fulfillment of Leibniz' dream.[48]

> In 1896 Peano several times returned to the fulfilment of Leibniz's dream of producing 'une spécieuse générale ou une manière de langue où d'écriture universelle, où toutes les vérités de raison seraient réduites à une façon de calcul'. Like Leibniz, he held that it was one of the main problems to be faced and that it had as much value in science as the discovery of the telescope and the microscope. The echoes of Leibniz's remarks are almost identical to the German's original words, and are scattered throughout the writings of Peano and his collaborators.[49]

Luciano and Roero claimed that Peano 'did all he could to spread the results of his School on an international scene'.[50] Peano's connections with the living German mathematicians from 1891 to 1903 are the following.

It seems that Peano learned about the achievements of German mathematics from his academic teacher Angelo Genocchi (1817–1889), who was in touch with them.

In 1880 Schröder wrote a review of Frege's first book *Begriffsschrift*,[51] from which Peano most likely learned about Frege's new logic, as he reported to Frege. Peano graduated in July 1880, but we do not know when he read the review.

Hermann Amandus Schwarz (1843–1921) discovered an error in a handbook written by Joseph Alfred Serret (1819–1885)[52] and reported it to Genocchi in a letter of 7 January 1881. A year later, the same error was found by Peano when he was replacing Genocchi in teaching because of his illness. As early as 26 May 1882, Genocchi wrote about Peano's discovery to Schwarz. However, it was Peano who first published the discovery in *Sulla*

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47 G. Frege, *Über die Begriffsschrift des Herrn Peano und meine eigene*, "Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse" 1896, vol. 48, p. 361–378; Peano mentioned this paper in G. Peano, *Anhang I: Über mathematische Logik*, p. 338; see also idem, *Arbeiten zur Analysis und zur mathematischen Logik*, p. 12.

48 H. Kennedy, *Peano. Life and Work*, p. 65.

49 C.S. Roero, *The* Formulario *between Mathematics and History*, p. 89. See also P. Cantù, *The Right Order of Concepts: Graßmann, Peano, Gödel and the Inheritance of Leibniz's Universal Characteristic*, "Philosophia Scientiæ" 2014, vol. 18, no 1, p. 157–182.

50 E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 73.

51 G. Frege, *Begriffsschrift und andere Aufsätze*, ed. by I. Angelelli, Hildesheim, Zürich, New York, 1998 (1st ed. – *Begriffsschrift, eine der arithmetischen nachgebildete Formalsprache des reinen Denkens*, Halle 1879).

52 J.A. Serret, *Calcul intégral*, Paris, 1868.

definitione dell'area d'una superficie.[53] He mentioned Schwarz in his paper but did not add the time sequence of the discovery.[54] This time it was Schwarz who sent a letter to Peano (now lost), and Peano answered, referring to the topic in two letters, of 12 April 1890 and 27 April 1890.[55] He also discusses other matters. There was at least one more Peano's letter to Schwarz, of 4 April 1893, without a reference to the error.[56]

Peano's first letter to Klein was written between March 1887 and 24 August 1887; unfortunately, it is a lost document, as all Klein's letters to Peano. However, from the context of other letters we know that they were sent. In March 1887, Peano sent his papers to Mayer and Klein to let them know about his mathematical research. In a letter of July, Mayer asked Klein to answer Peano's letter and encouraged him to publish a paper in "Mathematische Annalen," even in Italian, because German mathematicians also published in German in Italian journals.[57]

In 1888 Richard Dedekind (1831–1916) published *Was sind und was sollen die Zahlen?* In 1889 one of the most important Peano's books, *Arithmetices principia, nova methodo exposita* (only 36 pages), appeared. His first statement of the famous postulates for the natural numbers is published. Peano referred here to Dedekind's work (p. V); nevertheless, the similarities and differences between the two pivotal works are still discussed.[58]

Another German mathematician, Hermann Grassmann (1809–1877), had a significant influence on young Peano. In 1888, a thirty-year-old Peano wrote a book including Grassmann's name in the title: *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann.*[59] From today's point of view, the work is important also due to a concept of the so-called definition by abstraction, investigated thoroughly in Peano's school. As Mancosu has observed, the book

> contains the first description by Peano of definition 'by abstraction'. The terminology is not there yet (one has to wait to 1894 for the first full explicit use of 'definizione per astrazione' in a review by Vailati) but all the elements are in place. It is important to point out that the title of Peano's work […] unequivocally shows Grassmann's influential role in shaping reflection on abstraction in the nineteenth century. […] Peano defines equality between two entities of a certain system, written a = b, to mean a relation between elements of the system that satisfies symmetry and transitivity.[60]

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53 G. Peano, *Sulla definizione di integrale*, "Atti della Accademia Nazionale dei Lincei, Rendiconti, Classe di scienze fisische, matematische e naturali" 1890, vol. 6, p. 54–57.

54 H. Kennedy, *Peano. Life and Work*, p. 21–22; see also Kennedy's comment in *Selected Works of Giuseppe Peano*, p. 137, 138.

55 H. Kennedy, *Peano. Life and Work*, p. 21–22.

56 Ibidem, p. 61.

57 [Mayer–Klein, 21.07.1887], [in:] *Korrespondenz Felix Klein – Adolph Mayer. Auswahl aus den Jahren 1871–1907*, p. 162.

58 See H. Kennedy, *Peano. Life and Work*, p. 41.

59 G. Peano, *Calcolo geometrico secondo l' Ausdehnugslehre di H. Grassmann, preceduto dale operazioni della logica deduttiva*, Torino 1888. More on Grassmann's legacy in P. Freguglia, *Geometric Calculus and Geometry Foundations in Peano*, [in:] *Giuseppe Peano. Between Mathematic and Logic*, ed. by F. Skof, Milan, Dordrecht, Heidelberg, London, New York 2011, p. 69–82. Peano also referred to Hermann Grassmann's younger brother, Robert (see I. Grattan-Guinness, *Giuseppe Peano. A Revolutionary in Symbolic Logic?*, [in:] *Giuseppe Peano Between Mathematics and Logic*, ed. F. Skof, Milan, Dordrecht, Heidelberg, London, New York 2011, p. 139).

60 P. Mancosu, *Definitions by abstraction in the Peano School*, [in:] *From Arithmetic to Metaphysics*, ed. by C. De Florio, A. Giordani, Berlin, Boston 2018, p. 262.

One more quotation referring to Grassmann's influence on Peano, written by Grattan-Guinness reads as follows:

> A rather unusual feature of the book was Peano's deployment of new algebraic techniques, especially vectors and determinants. One of his sources for the former was the calculus of extension of Hermann Grassmann […]; Peano made limited use of it, but he recognised its significance to the extent of quickly producing a short textbook […] on it. Grassmann had developed his theory in the 1840s, especially the book Ausdehnungslehre (1844) […] but it had gained attention from the 1860s onwards after a second edition of his book. Most of the interest was taken in German-speaking countries, but there was some also in Italy, and Peano saw it worthy of reaching a wider audience. […] In his book on Grassmann Peano followed the main symbols and properties of Grassmann's algebra.[61]

As Grattan-Guinness has stated, Peano followed Ernst Schröder's (1841–1902) calculus of-classes in 1888[62], but I omit this topic. In the same year, Peano's first short paper appeared in French in "Mathematische Annalen."[63]

Next, in 1889, in the book *I Principi di geometria logicamente espositi*[64] Peano referred to another German mathematician, Moritz Pasch (1843–1930). Peano was impressed with his book *Vorlesungen über Geometrie* (Leipzig 1882) discussing

> several axioms of Pasch's axiomatization of geometry – which are formulated in natural language – and contrasts them with his own axioms – which are symbolised […]. In this discussion he highlights the ambiguities involved in the expression of mathematical laws by means of natural language.[65]

Moritz Pasch wrote to Felix Klein in 1891. However, from his point of view

> with regard to external representation, I do not want to go so far as, for example, Peano (in *I principii di Geometria logicamente esposti*, Torino 1889), who criticised me a little because of this on pp. 32–33, 36.[66]

At age 26, Peano edited his teacher's (Angelo Genocchi's) book published in Italy in the autumn of 1884[67]; in the same year, Frege's *Die Grundlagen der Arithmetik* appeared. Six years later, in 1890, Peano published two papers in "Mathematische Annalen," both in

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61 I. Grattan-Guinness, *Giuseppe Peano. A Revolutionary in Symbolic Logic?*, p. 136–137.
62 Such a note was given by Peano in P.E.B. Jourdain's article on Peano's logic: Ph. Jourdain, *The Development of the Theories of Mathematical Logic and the Principles of Mathematics*, "Quarterly Journal of Pure and Applied Mathematics" 1912, vol. 43, p. 273; see also I. Grattan-Guinness, *Giuseppe Peano. A Revolutionary in Symbolic Logic?*, p. 138.
63 G. Peano, *Intégration par séries des équations différentielles linéaires*, "Mathematische Annalen" 1888, vol. 32, p. 450–456. See Mayer's letters to Klein referring to the publication. [Mayer–Klein 21.07.1887], in *Korrespondenz Felix Klein – Adolph Mayer*, p. 161–163; [Mayer–Klein, 21.08.1897], p. 163. Also E. Luciano, C.S. Roero, *From Turin to Göttingen*, p. 29.
64 G. Peano, *I principi di geometria logicamente espositi*, p. 32–33.
65 J. Bertran-San Millán, *Frege, Peano and the Interplay between Logic and Mathematics*, "Philosophia Scientiæ", 2021, vol. 25, no 1, p. 22.
66 *A.9. Pasch to Klein, October 19, 1891 [Klein 11, 184]*, [in:] D. Schlimm, *The Correspondence between Moritz Pasch and Felix Klein*, p. 200.
67 A. Genocchi, *Calcolo differenziale e principii di calcolo integrale*, ed. by G. Peano, Torino 1884.

French.[68] Among the editors of this issue were two German professors, Klein and Mayer, who were important for Peano's work in that country. It is worth adding that three years later, Gustaw Mie (1868–1957) devoted a paper to Peano's proof of the integrability of ordinary differential equation systems, also published in „Mathematische Annalen".[69]

In 1891 some next triumphs ensued. Peano published his book[70] for the first time in German, translated by Adolf Schepp (1837–1905), and the first volume of "Rivista di matematica", a journal founded by Peano and published in Turin until 1908, came out. There was a place for publishing the works of German mathematicians and evaluating their achievements. In the first issue, Peano published his review of the book by Ernst Schröder, *Vorlesungen über die Algebra der Logik*.[71]

Peano's correspondence with Gottlob Frege started after 1891; unfortunately, the first document is lost. From the contents of Frege's letter to Peano we know that Peano sent Frege a postcard and some of his papers, among them *Sul concetto di numero*[72] Peano's first preserved letter to Frege is from 1894. In the same year, Peano, in a letter to Klein dated 25/29 August, wrote about a mathematical logic which uses seven signs to express all possible relations between sentences and classes, including those that are difficult to express in everyday language.[73]

In 1892, Georg Cantor (1845–1918), yet another German mathematician, published his first paper in Peano's journal "Rivista di matematica", in Italian.[74] From Cantor's preserved letter book, we know that he stayed in epistolary contact with Peano at least from 6 April 1895 to 10 January 1896. Unfortunately, their correspondence has been lost. Nonetheless, there are drafts of 10 letters from Cantor to Peano in Cantor's letter book and evidence of at least eight letters and postcards from Peano to Cantor.[75] Another topic concerns Peano's knowledge of Cantor's set theory at this time. Grattan-Guinness has noted:

> He [Peano] was aware of its origins in Cantor's work on Fourier series in the early 1870s, for he cited the paper involved (and also the short book of 1872 on irrational numbers by Richard Dedekind (1831–1916)) in the first page of his notes 1884c of Genocchi's textbook. But the main development of Cantor's theory was much more recent, effected in around a dozen papers published between 1877 and 1885, and it is not clear how familiar was this material to Peano. The use

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68 G. Peano, *Démonstration de l'intégrabilité des équations différentielles ordinaires,* p. 182–228; idem, *Sur une courbe, qui remplit toute une aire plane,* "Mathematische Annalen" 1890, vol. 36, no 1, p. 157–160. See also [Mayer–Klein, undated], in *Korrespondenz Felix Klein–Adolph Mayer,* p. 181; E. Luciano, C.S. Roero, *From Turin to Göttingen,* p. 30.
69 G. Mie, *Beweis der Integribarkeit gewöhnlicher Differentialgleichungssystem nach Peano,* „Mathematische Annalen" 1893, vol. 43, p. 553–568.
70 G. Peano, *Die Grundzüge des Geometrischen Calculus,* ed. by A. Schepp, Leipzig 1891. It was a translation of the Italian edition. See idem, *Gli elementi di calcolo geometrico,* Torino 1891.
71 G. Peano, *E. Schröder, Vorlesungen über die Algebra der Logik* [book review], p. 164–170.
72 [Frege to Peano, undated], [in:] G. Frege, *Philosophical and Mathematical Correspondence,* p. 109; G. Peano, *Sul concetto di numero* "Rivista di matematica" 1891, vol. 1, p. 87–102, 256–267.
73 See: C.S. Roero: *The Formulario between Mathematics and History,* p. 85; M. Segre, *Le lettere di Giuseppe Peano a Felix Klein,* p. 119–120.
74 G. Cantor, *Sopra una questione elementare della teoria degli aggregatii,* "Rivista di matematica" 1892, vol. 2, p. 165–167.
75 H. Kennedy, *Peano. Life and Work,* p. 84.

made of it in his book 1887d on geometry suggests that he knew at least the basic properties of manipulating sets and of point set topology, especially in the chapter on 'geometrical magnitudes' where he attempted to extend the notion of integral, as indeed Cantor had tried to do before him. He also knew the short book of 1888 on the foundations of arithmetic by Richard Dedekind (1831–1916), for he cited it in his own account of arithmetic […]; and he could have picked up some elements of Cantor's theory there also.[76]

Later Peano devoted a paper to the Cantor-Bernstein theorem.[77]

Next, Peano's review of the next German work appeared in 1894 – this time it was H. Grassmann's *Gesammelte mathematische und physicalische Werke.*[78] Peano's one-page text was published in French in another German journal, "Monatshefte für Mathematik und Physik".[79]

After all the previously announced events, the next one on the timeline is Frege's letter to Mayer, published here for the first time. As we remember, Mayer's correspondence with Frege started on 8 June 1896, almost one year after Frege's lecture on Peano's logic in Lübeck. In the same year Frege published *Über die Begriffsschrift des Herrn Peano und meine eigene* (1896). He focused there on comparing his conceptual notation with Peano's. He complained about less rigor of proofs, less clarity, logical defects and ambiguity of notions. However, Peano's notation was found as more convenient for readers.

Peano quoted another German mathematician, Alfred Pringsheim (1850–1941).[80] Pringsheim also referred to Peano and his teacher Genocchi, e.g. in two articles published in *Encyklopädie der Mathematischen Wissenschaften.*[81] It is worth adding that Peano 'like many of his contemporaries, […] was fascinated by the *Encyklopädie der Mathematischen Wissenschaften,* which was being published in Leipzig because it provided "an excellent collection of results."'[82]

In 1899 Peano's five works appeared as five appendices to the German translation of Genocchi's book[83]. It was three years after Frege's letter to Mayer concerning the publication of the article on Peano's logic. Translators of Genocchi's and Peano's papers,

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76 I. Grattan-Guinness, *Giuseppe Peano. A Revolutionary in Symbolic Logic?*, p. 137.
77 G. Peano, *Super theorema de Cantor-Bernstein*, "Rivista di matematica" 1902–1906, vol. 8, p. 136–143.
78 Idem, *H. Grassmann, Gesammelte mathematische und physicalische Werke* [book review], p. 166– 169.
79 Idem, *Sur les systèmes linéaires*, "Monatshefte für Mathematik und Physik" 1894, vol. 5, p. 136.
80 For example idem, *Formulaire de matématiques*, p. 83, 105; idem, *Formulario mathematico*, p. 105, 225, 229, 304, 347.
81 A. Pringsheim, *Grundlagen der allgemeinen Funktionenlehre*, [in:] *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, vol. 2, ed. by H. Burkhardt, W. Wirtinger, R. Fricke, Leipzig 1899, p. 1–53; idem, *Irrationalzahlen und Konvergenz unendlicher Processe*, [in:] *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, vol 1, part 1, ed. by W.F. Meyer, Leipzig 1898, p. 47–146.
82 C.S. Roero, *The* Formulario *between Mathematics and History*, p. 85.
83 G. Peano, *Anhang I: Über mathematische Logik*, p. 336–352; see also idem, *Arbeiten zur Analysis und zur mathematischen Logi*k, p. 10–26; G. Peano, *Anhang II. Definitionene der Arithmetik*, [in:] A. Genocchi, *Differentialrechnung und Grundzüge der Integralrechnung*, p. 353–358; also in: G. Peano, *Arbeiten zur Analysis und zur mathematischen Logik*, pp. 27–32; G. Peano, *Anhang III. Über Taylor'sche Formel*, [in:] A. Genocchi, *Differentialrechnung und Grundzüge der Integralrechnung*, p. 359–365; G. Peano, *Anhang IV. Über die Definition des Integrals*, [in:] A. Genocchi, *Differentialrechnung und Grundzüge der Integralrechnung*, p. 366–370; G. Peano, *Anhang V. Die komplexen Zahlen*, [in:] A. Genocchi, *Differentialrechnung und Grundzüge der Integralrechnung*, p. 371–395.

G. Bohlmann and A. Schepp, wrote in the introduction that it was Adolph Mayer (and J. Knoblauch) who strived for the German translations.[84]

A year later, in 1900, there was a famous discussion between Peano and Schröder during the first International Congress of Philosophy in Paris, which influenced Bertrand Russell.[85]

Nothing more from Peano's papers was published in Germany. However, in 1990 a new edition of his five articles and the two from "Mathematische Annalen" appeared as a separate volume.[86]

It was the last event presented here in the time sequence of collaboration of Peano with German mathematicians.

There is no evidence of any collaboration between Peano and David Hilbert (1862–1943). However, they had a lot in common due to working on similar topics and establishing a new approach to mathematics, like seeking to guarantee consistency, axiomatisation, the belief that properties of objects could be studied by making a system of signs, and the view that numbers have the simplest structural properties of objects or an objection to the view that mathematical notions could be reduced to logical ones[87]. In a recently discovered Hilbert's paper from 1917 he 'quoted and praised Peano's contribution to the axiomatic method'.[88] Next, in 1928, during the International Congress of Mathematicians held in Bologna, 'Hilbert acknowledged Peano's contribution to the creation of the ideography.'[89]

Conclusions

Giuseppe Peano was known in Germany before his correspondence with Gottlob Frege, starting in 1891 or later; he stayed in contact with Felix Klein from 1885. In addition, he also maintained various types of academic contacts with the following German mathematicians: Georg Cantor, Richard Dedekind, Hermann Grassmann, Adolph Mayer, Moritz Pasch, Alfred Pringsheim, Adolph Schepp, Ernst Schröder, and Hermann Amandus Schwarz.

Peano's correspondence with Frege and Frege's with Mayer could have helped publish German translations of Peano's five articles in 1899, which appeared as five appendices to the German translation of Genocchi's book. Moreover, Frege was the first German mathematician to appreciate the importance of Peano's mathematical logic. I have argued that Frege's letter to Mayer, dated 8 June 1886, was crucial in this regard, as later Frege's article in which he referred to Peano's mathematical logic.

84 G. Bohlmann, A. Schepp, *Vorwort der Übersetzer*, [in:] A. Genocchi, *Differentialrechnung und Grundzüge der Integralrechnung*, p. V.
85 H. Kennedy, *Peano. Life and Work*, p. 121, 123.
86 G. Peano, *Arbeiten zur Ananlysis und zur mathematischen Logik*.
87 M. Segre, *Peano's Axioms in their Historical Context*, "Archive for History of Exact Sciences" 1994, vol. 48, no 3–4, p. 311.
88 Ibidem.
89 Ibidem.

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Giuseppe Peano w Niemczech, jego związki z niemieckimi matmatykami i pierwszy angielski przekład niewydanego listu Gottloba Fregego do Adolpha Mayera o logice matematycznej Giuseppego Peana

Celem artykułu jest odpowiedź na pytanie, czy list Gottloba Fregego do Adolpha Mayera, nadany 8 lipca 1896 r., mógl pomóc niemieckim matematykm zapoznać się z pracami matematycznymi Giuseppego Peana, w tym z jego logiką matematyczną. Jest to pierwsze wydanie tego listu w języku angielskim. Najpierw przedstawiam główne postaci tej historii. Następnie omawiam listy dotyczące Peana i jego wyników matematycznych. W ten sposób pokazuję tło listu Fregego do Mayera. W ostatniej części zbieram informacje o kontaktach Peana z niemieckimi matematykami – gdzie był cytowany i przez kogo, kogo cytował Peano i w którym okresie swojego życia. Kończę z wnioskiem, że Peano był znany w Niemczech zanim Frege napisał do Mayera w 1896 r. List mógł jednak pomóc w wydaniu pięciu artykułów Peana w Niemczech, gdzie jego logika matematyczna była wówczas niemal nieznana. Niewątpliwie list pomógł w szerzeniu tej wiedzy.