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HORIZONTAL GROUND HEAT EXCHANGERS – ANALYTICAL SOLUTIONS AND NUMERICAL SIMULATIONS

GRUNTOWE WYMIENNIKI CIEPŁA – ROZWIĄZANIA ANALITYCZNE I SYMULACJE NUMERYCZNE

A b s t r a c t

This paper concerns the basics of modelling ground heat exchangers. Several specific cases were described, for which computational dependencies and calculation results were presented. Models based on analytical solutions were considered as well as models that use computer applications based on numerical calculations.

Keywords: transient heat conduction, numerical simulations, ground heat exchangers

S t r e s z c z e n i e

Praca dotyczy podstaw modelowania gruntowych wymienników ciepła. Opisano kilka szczególnych przypadków, dla których przedstawiono zależności obliczeniowe oraz wyniki obliczeń. Rozważono zarówno modele oparte na rozwiązaniach analitycznych jak również modele wykorzystujące aplikacje komputerowe bazujące na obliczeniach numerycznych.

Słowa kluczowe: nieustalone przewodzenie ciepła, symulacje numeryczne, gruntowe wymienniki ciepła

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Nomenclature:

a – thermal diffusivity coefficient, m^2/s
B – oscillation amplitude around the temperature T_b, K
k – heat conduction coefficient, W/(mK)
$\dot{Q}$ – rate of heat transfer, W
s – thickness of the plate, m
t – time, days
t_{max} – time since the beginning of the calendar year to the day in which the temperature is the highest, days
T – temperature of the ground, °C
T_a – average daily ambient air temperature, °C
T_b – average annual ambient air temperature, °C
T_i – initial temperature of the ground, °C
T_0 – temperature of the body, °C
x – position coordinate in the ground (vertical), m
y – position coordinate in the ground (horizontal), m
ω – frequency, 1/day

1. Introduction

Internal energy of the ground is used for heat pumps. When heat is received from the ground, usually the so-called closed systems are used, which utilise the diaphragm heat exchangers located in the ground. A liquid, low-temperature heat-carrying agent is circulating in the system.

Often, in modelling, ready applications are used. ANSYS Transient Thermal is frequently used for digital simulations.

Wu et al. [1] investigated the thermal performance of slinky heat exchangers for ground source heat pump systems for the UK climate. The authors presented the results of experimental measurements as well as of a numerical simulation using a three-dimensional CFD code ANSYS Fluent. This application was also used by Benazza et al. [2] to study the influence of the thermal conductivities and geometrical parameters on the heat exchanger efficiency. Condego et al. [3] performed calculations using Fluent for simulations of different configurations of a horizontal ground heat exchanger, in order to evaluate the characteristics of these systems in the most common layouts and under different working conditions. A 3D numerical model was also developed by Chong et al. [4]. The authors presented the thermal performance results for various heat exchanger configurations by comparing the heat transfer rate as well as the amount of pipe material needed.

This paper presents some simple cases concerning heat conduction in the ground. They have a practical significance for the modelling and simulation of ground heat exchangers coupled with heat pumps. The following have been considered:

- Heat conduction in the ground with no heat exchangers installed,
- Heat conduction in the ground with horizontal tubes arranged in parallel, assuming a homogeneous initial ground temperature,
- Steady heat conduction between horizontal pipes arranged in parallel on the ground surface.

None of the mentioned cases correspond precisely to the operating conditions of ground heat exchangers, since in reality, heat is extracted from the ground, the ground temperature varies with the position, and the process of heat transfer is transient. However, when constructing mathematical models, it is advantageous to be able to verify the obtained results for special cases. This work is dedicated to such extreme cases.

2. Analysis of climatic data

Parallel to the heat transfer between the ground and the working fluid in an exchanger, the heat is transported between the surface of the ground and the environment. It has a particular significance in the modelling of horizontal exchangers located at a small depth. In order to take this heat flux into consideration, the course of the ambient temperature as a function of time should be known. When examining the average daily temperature, this course can be described using trigonometric functions [5]. In order to elaborate the climatic data to form a mathematical relationship, the following model was applied:

$$T_a = T_b + B \cdot \cos\left[\omega\left(t - t_0\right)\right] \tag{1}$$

The frequency ω is:

$$\omega = \frac{2\pi}{t_c} \tag{2}$$

where t_c is the number of days in a year (≈365). Thus, $\omega = 0.0172$ day^{-1}. Relationship (1) was reduced to linear form:

$$T_a = T_b + b_1 X_1 + b_2 X_2 \tag{3}$$

where:

$$b_1 = B \cdot \cos\left(\omega t_0\right) \tag{4}$$

$$b_2 = B \cdot \sin\left(\omega t_0\right) \tag{5}$$

$$X_1 = \cos\left(\omega t\right) \tag{6}$$

$$X_2 = \sin\left(\omega t\right) \tag{7}$$

For calculations, climatic data for Cracow, shown in [6], were used. The coefficients were calculated using the method of least squares: $T_b = 8.5$°C, $b_1 = 10.17$ K, $b_2 = -2.755$ K. Next, when solving the system of equations (4)–(5), one can evaluate: $B = -10.4$ K, $t_0 = 15.4 \approx 15$ days. Therefore, the relationship, which describes temporal changes of ambient air temperature, takes the form:

$$T_a = 8.5 - 10.4 \cdot \cos\left[\omega\left(t - 15\right)\right] \tag{8}$$

Value of t_0 concerns the day, when the annual average temperature is the lowest and equals: 8.5 − 10.4 = −1.9°C. After a period of half a year ($t_c/2 \approx 183$ days), air temperature reaches a maximum of is 8.5 + 10.4 = 18.9°C. Thus, the relationship equivalent to (8) is:

$$T_a = 8.5 + 10.4 \cdot \cos\left[\omega\left(t - 198\right)\right] \tag{8a}$$

The course of this function is shown in Figure 1, including the climatic data used for calculations.

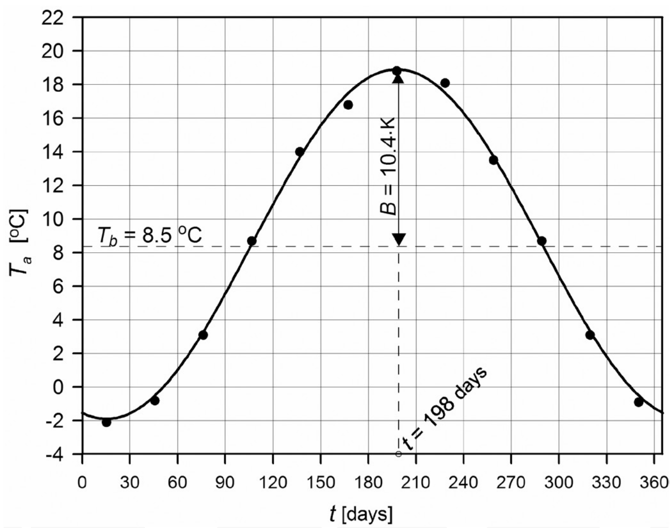

Fig. 1. Approximation of climatic data

3. Heat transfer in the ground taking into account an interaction with the ground surface

In order to determine the temperature distribution in the ground under natural conditions, the model of semi-infinite body is generally used. The analytical solution for both cases is known when the thermal resistance occurs only in the ground, as well as when the resistances occur in the ground and during heat transfer between the ground and the environment [7, 8].

Due to the fact that ambient temperature changes cyclically, the atmosphere interaction on the ground is limited. As a result of atmosphere interaction, the ground periodically heats up or cools down. A distant limited range of the climatic conditions' influence on the ground enables the use of an analytical solution for an infinite plate. However, the condition for the applicability of this solution is that the plate thickness exceeds the thickness of the ground zone in which temperature changes occur, caused by climatic conditions.

Below, an analytic solution for heat conduction for an infinite plate is presented, of which one surface (x = 0) is at a constant temperature and the second one (x = s) is subjected to cyclic temperature changes. The solution concerns a uniform plate temperature at the beginning of the process. In the presented system, during each time interval, the temperature profiles change until they reach a cyclic steady state. The adequate differential equation and conditions take the form:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \tag{9}$$

The initial condition:

$$t = 0 \qquad T = T_i \tag{10}$$

The boundary conditions:

$$x = 0 \qquad T = T_b = T_i \tag{11}$$

$$x = s \qquad T = T_b + B \cdot \sin\left[\omega\left(t - t_{\max}\right)\right] \tag{12}$$

The solution is [7]:

$$T = T_b + AB \cdot \sin\left[\omega t + \varepsilon + \phi\right] + \\ + 2\pi \sum_{i=1}^{\infty} \frac{i(-1)^i \cdot \left(i^2 \mu^2 \cdot \sin\varepsilon - 2R^2 \cdot \cos\varepsilon\right)}{i^4 \pi^4 + 4R^4} \sin\left(i\pi \frac{x}{s}\right) \cdot \exp\left(-i^2 \pi^2 \frac{at}{s^2}\right) \tag{13}$$

where:

$$A = \sqrt{\frac{\cosh\left(2R\,x/s\right) - \cos\left(2R\,x/s\right)}{\cosh\left(2R\right) - \cos\left(2R\right)}} \tag{14}$$

$$\varphi = A \cdot \tan \frac{p_1 - p_2}{1 + p_1 p_2} \quad \mathrm{mod} \quad \pi \tag{15}$$

$$p_1 = \frac{\tan\left(R\,x/s\right)}{\tanh\left(R\,x/s\right)} \tag{16a}$$

$$p_2 = \frac{\tan R}{\tanh R} \tag{16b}$$

$$R = \frac{s}{L} \tag{17}$$

$$\varepsilon = -t_{\max}\,\omega \tag{18}$$

The sum of the first two terms of the solution (13) refers to the cyclic steady state; in this state, the third term (containing the sum of the infinite series) resets.

For calculations, the following data were used: $T_b = 8.5°C$, $B = 10.4$ K, $t_{max} = 198$ days, $a = 0.384\cdot10^{-6}$ m²/s .The amount L with linear dimension calculated from formula:

$$L = \sqrt{\frac{2a}{\omega}} \tag{19}$$

equals $L = 1.96$ m. The thickness of the ground layer, beyond which there are no longer any temperature changes, was assumed as 7-times greater than L ($R = 7$). Thus, $s = 7\cdot1.96 = 13.75$ m.

In Fig. 2, the ground temperature profiles under natural conditions in a 3-months period (15 Jan – the lowest ambient temperature, 15 Jul – the highest ambient temperature) were presented. The value of $x = 0$ corresponds to the bottom surface of the plate at a constant temperature $T = T_b$ and the value of $x = 1$ refers to the upper surface of the plate, the same as the ground surface. The cyclic steady state was reached after approx. 5 years of process' duration (on the assumption that the initial temperature was uniform). In this state, the temperature course near $x = 0$ is position independent and the lines are practically vertical.

In Fig. 3, the temporal courses of temperature at several ground depths were presented. The value of $t = 1$ year corresponds to the value of dimensionless time $at/s^2 = 0.0641$. For a depth of 13.75 m (and higher), the temperature does not change with time. For a depth of 8 m, there are slight temperature changes, but they are significantly different in the subsequent years of the process. And for a depth of 2 m, the temperature changes are significant within 1 year, but the differences between successive years are insignificant.

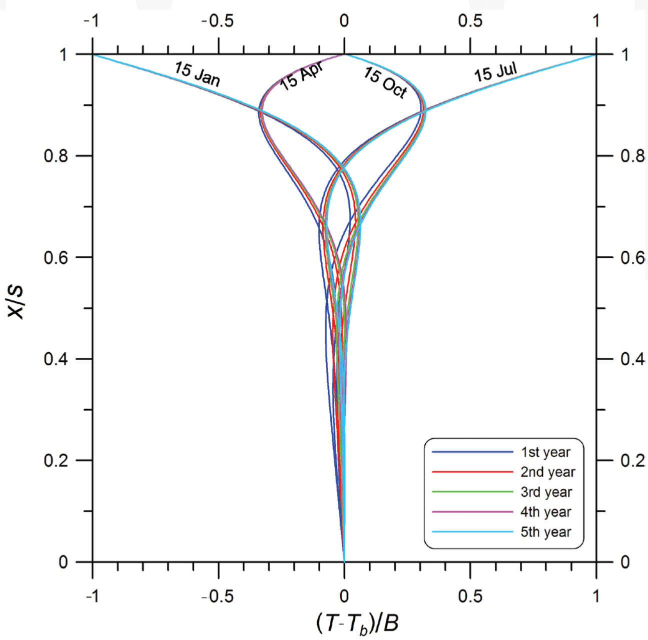

Fig. 2. Temperature profiles in the ground under natural conditions

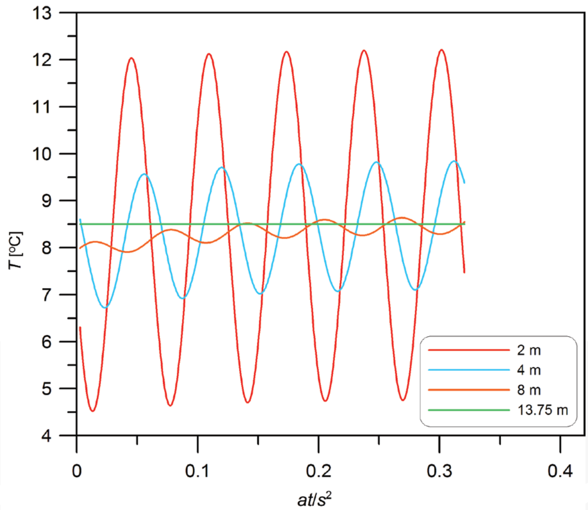

Fig. 3. The temporal courses of temperature of the ground

The analysis has the following practical significance. At numerical calculations relating to heat conduction in the ground, assuming a uniform initial temperature of the ground, the simulations should be carried out for a time value corresponding to at least a few cycles because a numerically determined temperature profile after one cycle (of the year) may differ significantly from the profile reached in the cyclic steady state.

4. Simulation of heat conduction in the ground heated by a horizontal pipe system – using ANSYS application

With slight changes in the temperature of the working fluid between the inlet and the outlet in a horizontal ground heat exchanger, heating or cooling of the ground can be treated as a two-dimensional problem. The flow through 7 horizontal tubes that are coupled in parallel and are located in the ground has been considered. The problem can be described by a two-dimensional equation of transient heat conduction.

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{20}$$

The initial condition follows from the assumption of a uniform ground temperature at the beginning of the process, whereas the boundary condition follows from the constant (with time and space) surface temperature value of all of the tubes. For the calculation, the following data was used: $a = 0.384 \cdot 10^{-6}$ m²/s, $T_0 = 5$°C, $T_i = 15$°C, the dimensions of the analysed ground fragment: 1.5×0.6×0.4 m. Furthermore, it was assumed that the outer diameter of the tubes is equal to 38 mm and that the distance between the axes of the pipes is equal to 150 mm. The simulation was carried out using the ANSYS application (number of nodes: 381270). The calculation results were presented in Figs. 4a, b, c.

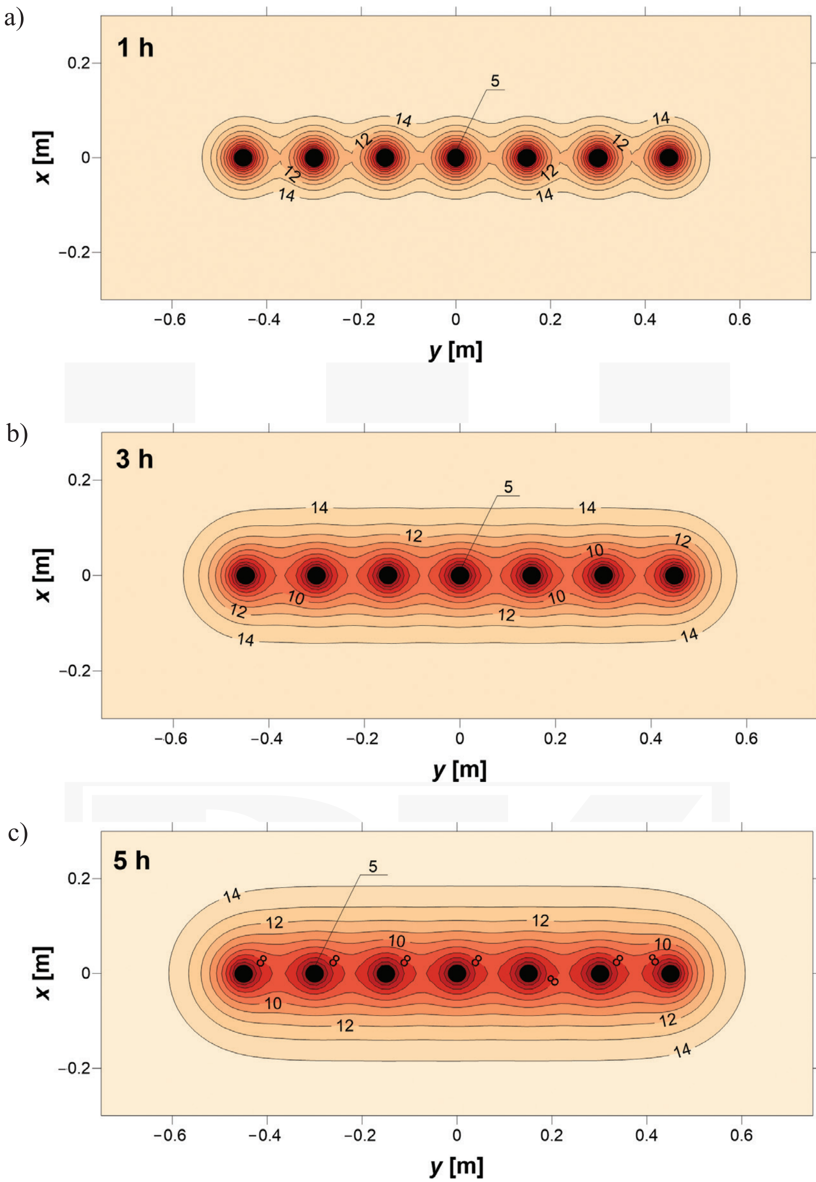

Fig. 4a, b, c. Heat conduction in the ground – digital simulation

The figures are related to the cross section of the pipe system. Lines of a constant temperature of the ground expressed by numbers (in °C) were shown. On the axes, suitable location coordinates are in the ground. Tubes are at a sufficient distance from the ground surface, providing the absence of interactions with external environmental conditions. As one can see, at the beginning, each pipe cools down the ground individually and independently of the other tubes. However, after some time, the temperature fronts come together and more lines of the constant temperature have a more rectilinear shape and are parallel to the course of the y axis. Thus, the system behaves similarly to the process of ground cooling by an infinite plate. The similarity is even larger when there are more tubes installed, and when they are more densely arranged side by side.

5. Steady heat conduction

When a body with a constant (surface) temperature T_0 (different with T_i) is located in the ground at a uniform temperature T_i, the heat transfer occurs between the body surface and the ground. When $T_0 < T_i$, the ground is cooling. Generally, the process has a transient character. However, when there is an object in the ground that can receive the heat, after some time, the heat transfer might become steady. This object can be, for example, the surface of the ground. When the heat flux from the ground to the body surface equals the flux from the environment to the ground surface, the heat transfer is steady. Depending on the configuration of the surfaces involved in heat transfer, there are relationships from which one can calculate the heat flux.

The rate of heat transfer related to single tube is calculated from the formula [9]:

$$\dot{Q} = Sk\left(T_i - T_0\right) \tag{21}$$

The case of horizontal pipes arranged in parallel in the ground, at a distance from the ground surface z, was considered. Coefficient S, in this case, can be computed from the formula (calculated on a single tube):

$$S = \frac{2\pi L}{\ln\left(\frac{2w}{\pi d}\cdot\sinh\frac{2\pi z}{w}\right)} \tag{22}$$

When there is only one tube in the ground, the coefficient S is as follows:

$$S = \frac{2\pi L}{\ln\frac{4z}{d}} \tag{23}$$

Relationships are valid for $z > 1.5d$.

In Fig. 5, a graphic interpretation of the mentioned dependencies was presented. For the calculation, it was assumed: $d = 0.038$ m, $k = 0.9$ W/(m·K) and $T_i - T_0 = 10$ K. As can be seen, the heat flux related to a single tube (refers to its length) strongly decreases with the depth z, where pipes are arranged. However, the opposite is the influence of the spacing between pipes: the longer the distances w, the greater the heat flux. For only one tube located in the ground, an appropriate curve ($w \to \infty$) is situated the highest.

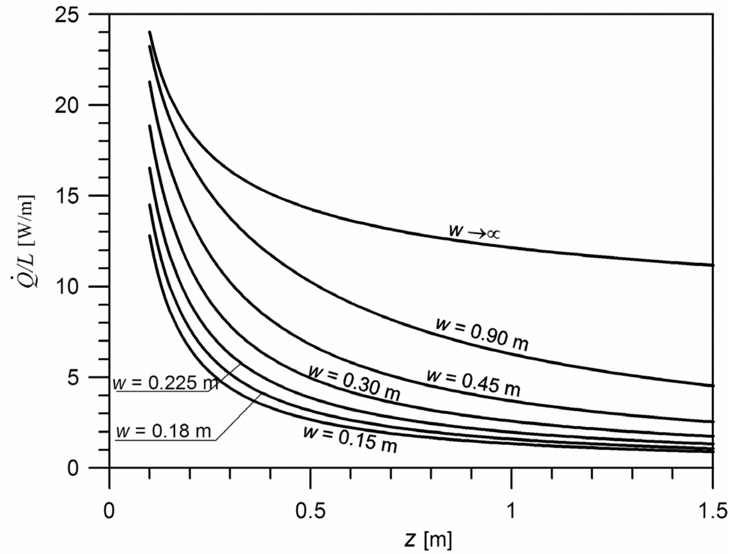

Fig. 5. Steady conduction in the ground

6. Conclusions

- With an application of the trigonometric functions used in the model in order to correlate to the climatic temperature data, a very good fitting was obtained.
- In numerical simulations related to heat conduction in the ground, taking into consideration the interaction with the surface, it should be taken into account that due to the existence of the cyclic steady state, it is necessary to carry out the calculations for at least a few annual cycles. For the calculation, one can use relationships for an infinite plate as long as its thickness exceeds the depth of the ground, where there are changes in temperature.
- With a heat conduction simulation of a series of parallel ground heat exchanger tubes, it has been found that the generated constant temperature lines become close to straight lines after a certain period of time. This gives the basis for the simplification of the modelling process of horizontal ground heat exchangers. Reference simulations were performed by using ANSYS codes.
- Calculation relationships for steady heat conduction in the ground can be used as a particular case of heat transfer in the ground, which is helpful for testing models concerning transient heat conduction.

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