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Cross Entropy Clustering Approach to Iris Segmentation for Biometrics Purpose

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Abstract. This work presents the step by step tutorial for how to use cross entropy clustering for the iris segmentation. We present the detailed construction of a suitable Gaussian model which best fits for in the case of iris images, and this is the novelty of the proposal approach. The obtained results are promising, both pupil and iris are extracted properly and all the information necessary for human identification and verification can be extracted from the found parts of the iris.

Keywords: CEC, cross entropy clustering, iris recognition, biometrics.

1. Introduction

The usability of biometrics is increasingly demanded in the environment of security, user authentication and varieties of applications where human identification or verification is required. As a more comfortable alternative, biometrics features are replacing the classical methods of people recognition that are based on the documents we have or a password, a PIN code to remember. The most popular and usable biometric features that can be widely seen in the airports and human crowds, wherever there exists a large number of people, are the face and iris in addition to the fingerprints. In this paper the iris feature is considered for its usability rapid increase as an attractive feature with a very high variability. In his paper [1] Daugman computed the distribution of Hamming Distances for 9.1 million possible comparisons between different pairs of irises in the database. He counted and found out that the cumulative from 0 to 0.300 was 1 in 10 billion. This leads to the fact that even poor degree of match for two different iris images would still give an evidence of identity.

The small size of iris makes it difficult to process for classification. Although Daugman's mathematical model of integrodifferential operator [1] has so far been proved to be one of the best (if not the best at all) methods of iris recognition, still exist many other trials to work out a simple model for both iris and pupil image segmentation for feature extraction and hence code calculation.

In this paper a method is presented as a rather new contribution to the segmentation of iris and pupil after detecting the eyelids and removing the eyelashes. All that takes place in one algorithm based on cross entropy clustering [2]. One of the promising applications that has always shown difficulties in iris segmentation, the authors are currently working on, is the detection of iris in a sick eye, where the iris image is rather occluded [3–5].

2. Algorithm statements and its motivation

The main idea of the algorithm proposed in this paper lies on the natural interpretation of iris image – see Figure 1. Namely, we can interpret the illumination value of each pixel on the image as a third coordinate – beside the first two describing the location of the pixel in the image. Thus we can represent the image as a 3D surface (subset of $\mathbb{R}^3$), as in Figure 1. We used the following scale: 1.0 is the value for black color while 0.0 is the white color. Other colors varying from black at the strongest intensity to white at the weakest.

By careful study of Figure 1b one can distinguish the interesting regions of the image: the highest one which corresponds to the pupil (Figure 1c) and the region below and around the pupil – the iris region (Figure 1d). If we find them then the iris is localized.[1]

In the next part of the paper we present the algorithm for iris localization which will find those two regions basing on cross entropy clustering (CEC). The segmentation can be processed in the following steps:

[1] In this paper we will not investigate the issues of the eyelids and eyelashes, which are one of our goals in the future studies.

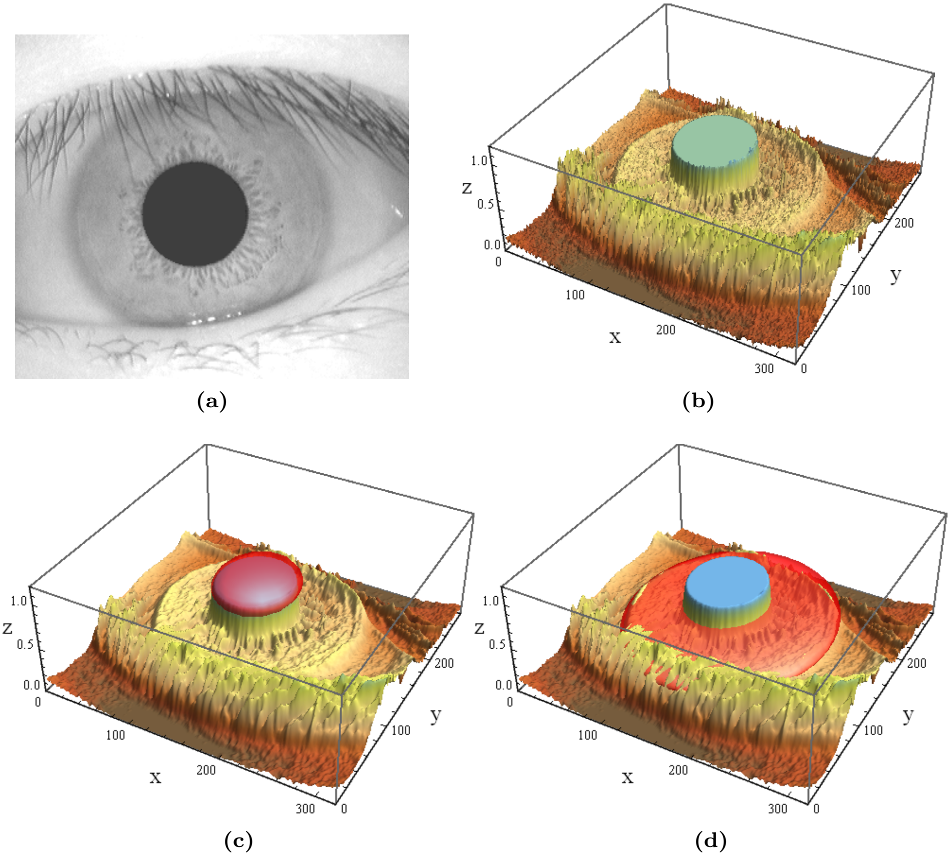


Figure 1. Iris image (a) interpreted as 3D image (b) with marked pupil (c) and iris (d) regions.

1. Gaussian correction;
2. regression correction;
3. CEC clustering;
4. result enhancement.

In the next subsections we give a brief description of each step with its intuition and basic theoretical background.

Before we proceed with the further part of the paper we would like to point out that the aim of the first two steps is to reduce the complexity of the data structure. Namely, our goal is to increase the number of white pixels on the image, thus the white pixels will not be precessed for example in the clustering step.

2.1. Gaussian correction

In Figure 1a we can notice that the same regions of the skin are white or have a color very close to white – we want to increase those regions. Moreover, we observe that the interesting information, i.e. iris, is placed around the pupil (centre of the pupil), which can be seen as the mean point of the image.

Furthermore, we can notice that the main, important for us, information is placed at the center of the image and is connected with the position of the pupil in the image. Thus we apply the operation which will bring to white color regions on image which are very far for the pupil. We will do this by constructing the optimal Gaussian distribution for investigated image. To do this we calculate the mean and the covariance matrix of our image. As a result we can build the Gaussian mask which we convolve with the image. The result of this step is presented in Figure 2a – for a comparison we also put the 3D interpretation of the current image (Figure 2b).

We can observe that we reach the goal – we increase the white part of the image (i.e. the lowest region in Figure 2b).

2.2. Regression correction

By the performing previous step we can bring the same abnormalities to image. Namely, the surface of the pupil can change, especially if its centre the pupil does not correspond to the mean of Gaussian distribution from the previous step. To fix such inconvenience we can calculate the optimal plane and subtract it from the image.

The construction of the optimal plane can be found by solving the multivariable linear regression, where pixel coordinates are the input variables while the pixel color intensity is the measured variable.The standard and detailed solution of multivariable linear regression can be found for example in [6].

In Figure 2c we can see the result of regression correction – for a comparison we also put the 3D interpretation of current image (Figure 2d). The image contains the explicitly selected areas corresponding to the pupil and iris.

2.3. Clustering with CEC

In this section we will give a brief description of the CEC and show how to cluster iris image with respect to ellipsoids with the shape best fitted to "red" ellipsoid marked in Figures 1c and 1d.

Let us start with a short introduction to CEC and for more detailed explanation we refer the reader to [2]. We shall recall that the standard Gaussian density in $\mathbb{R}^d$ is defined by

$$\mathcal{N}(\mathrm{m}, \Sigma)(x) := \frac{1}{(2\pi)^{d/2} \det(\Sigma)^{1/2}} \exp\left(-\frac{1}{2}\|x - \mathrm{m}\|^2_{\Sigma}\right),$$

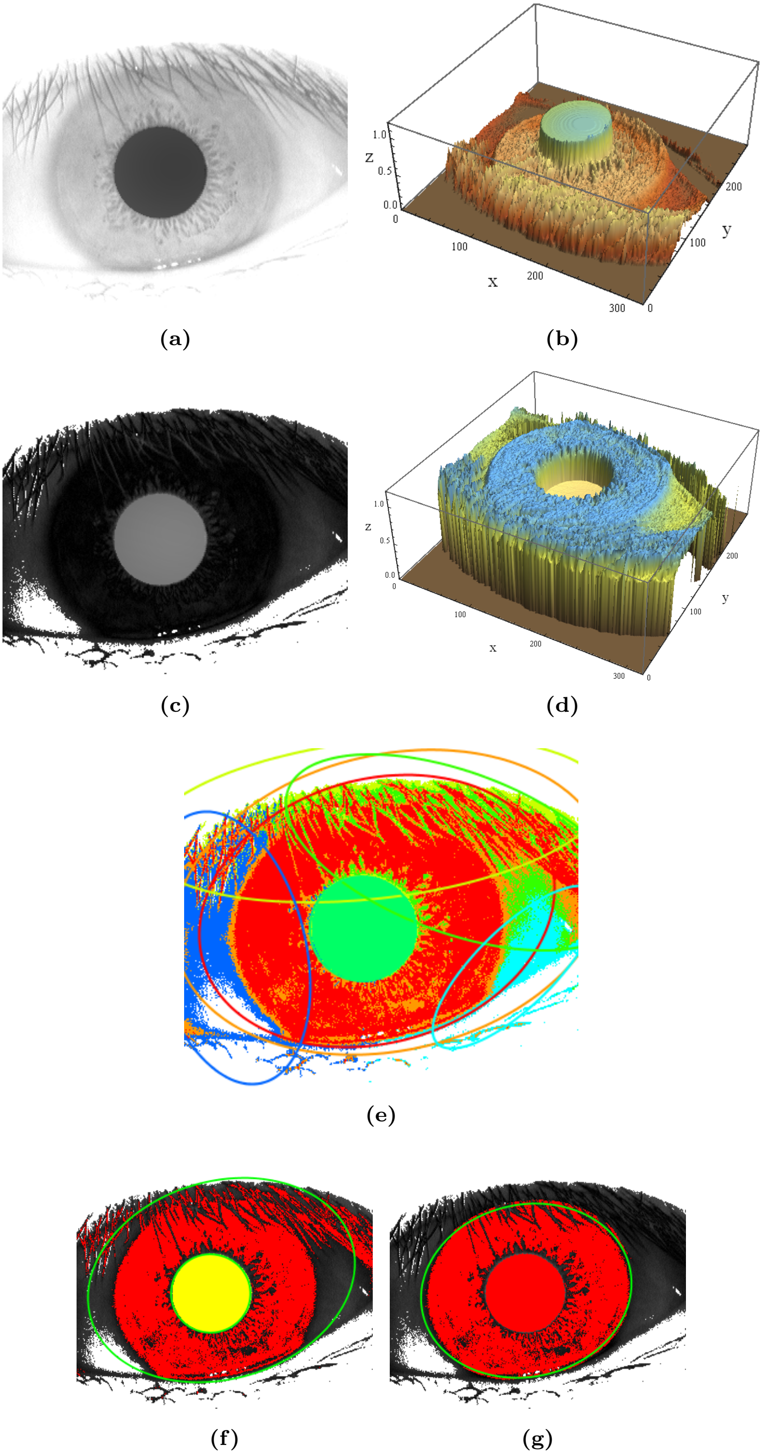


Figure 2. Iris image changes during the algorithm processing: (a) and (b) – after Gaussian correction; (c) and (d) – regression correction; (e) – CEC clustering result; (f) and (g) – after results optimization.

where m denotes the mean, Σ is the covariance matrix and $\|v\|_{\Sigma}^{2} := v^T \Sigma^{-1} v$ is the square of the Mahalanobis norm.

The general idea of CEC relies on finding the splitting of $X \subset \mathbb{R}^N$ into pairwise disjoint sets $X_1, \ldots, X_k$ such that the overall inner information cost of clusters is minimal. Consequently, to explain CEC, the cost minimizing function needs to be introduced. To do so recall that by the *cross entropy of a data set X with respect to density f* is given by

$$H^{\times}(X\|f) = -\frac{1}{|X|}\sum_{x\in X}\ln(f(x)).$$

In the case of splitting of $X \subset \mathbb{R}^N$ into $X_1, \ldots, X_k$ respectively to function $f_1, \ldots, f_k$ the cost function of CEC depends on memory needed for coding element of cluster X_i respectively to density function f_i and identification of the coding algorithm

$$\mathrm{CEC}(X_1, f_1; \ldots; X_k, f_k) := \sum_{i=1}^{k} p_i \cdot \big(-\ln(p_i) + H^{\times}(X_i\|f_i)\big)\,, \text{ where } p_i = \tfrac{|X_i|}{|X|}.$$

More precisely, the first component $-\ln(p_i)$ in the above formula corresponds to the memory needed for identify algorithm which is used for coding the element $x \in X_i$ and the second one, $H^{\times}(x\|f_i)$, is the mean code-length of coding X_i by the density f_i.

The CEC algorithm dived a dataset into clusters which are described by functions belonging to arbitrary fixed family of probability distributions. In such a case, we need the definition of cross entropy respect to the family of densities, which is given by

$$H^{\times}(X\|\mathcal{F}) = \inf_{f\in\mathcal{F}} H^{\times}(X\|f).$$

In the classical general version of CEC we use family $\mathcal{G}$ containing all Gaussian functions. Nevertheless, we can easily adapt the method for specialize class of Gaussian functions. For example we can use: $\mathcal{G}_{\Sigma}$ – Gaussian densities with fixed covariance Σ; $\mathcal{G}_{(\cdot I)}$ – spherical Gaussian densities; $\mathcal{G}_{\mathrm{diag}}$ – Gaussians with diagonal covariance, etc.

In this paper we need another family of Gaussian densities. Since the basic goal of the paper is to apply CEC to iris segmentation we introduce the density model in $\mathbb{R}^3$ (Figures 1c and 1d). More precisely, we use Gaussian densities with covariance matrix $\Sigma \in \mathcal{M}_3(\mathbb{R})$ ($\mathcal{M}_d(\mathbb{R})$ denotes the set of d-dimensional square matrices) which have the diagonal block matrix form

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix},$$

where $\Sigma_1 \in \mathcal{M}_2(\mathbb{R})$ and $\sigma_2 \in \mathbb{R}$.

For fixed $\varepsilon \in \mathbb{R}$ the family of disc like Gaussian distributions are denoted by

$$\mathcal{G}_{\varepsilon} = \left\{ N(\mathrm{m}, \Sigma)\colon \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \varepsilon \end{bmatrix},\ \text{where } \Sigma_1 \in \mathcal{M}_2(\mathbb{R}) \right\}.$$

Consequentially, for adaptation of CEC to the Gaussian family $\mathcal{G}_{\varepsilon}$ we need to introduce the formula for $H^{\times}(X\|\mathcal{G}_{\varepsilon})$, which in general case is not easy. Nevertheless, intuitively the covariance matrix which realized infinitum of cross entropy is given by

$$\Sigma_\varepsilon = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \varepsilon \end{bmatrix},$$

where the empirical covariance of dataset X is

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}.$$

Thus,

$$H^\times(X\|\mathcal{G}_\varepsilon) = \frac{N}{2}\ln(2\pi) + \frac{1}{2}\operatorname{tr}(\Sigma_\varepsilon^{-1}\Sigma_\mu) + \frac{1}{2}\ln\det(\Sigma_\varepsilon)$$

– compare with [2].

The value of ε, which is a parameter of family Σ_ε, was selected by empirical simulations, and was set to 10.

Applying our calculations to CEC we get the result presented in Figure 2e. The CEC was run with initial 20 clusters and end up with 7 clusters, the ε was set to 10.

The first look at the clustering, the results are promising, because the pupil and iris are in separate clusters – the green and red ones. The clusters, especially iris cluster, contain the additional (unwanted) data, which we will remove in next step.

2.4. Result enhancement

At the beginning of this step we have to face with one challenge – how to 'automatically' select clusters of the iris and pupil from the results of CEC (see Figure 2e). There are many strategies dependent on the situation and they give us a good solution[2], e.i., image quality, the iris position in the image, etc. In our case we decided to pick up the two clusters with smallest empirical color variance. The selected clusters are presented in Figure 2f. It is easy to notice that the clusters contains unwanted pixels besides pupil and iris regions. In this stage we reduce the cluster size.

To reduce the size of a cluster we recall the following theorem, which shows that ellipse is a ball in terms of Mahalanobis distance.

Theorem 1 ([7]) *Consider the uniform probability density on the ellipse $E \subset \mathbb{R}^2$ with covariance Σ_E and mean μ_E. Then*

$$E = \mathbb{B}_{\Sigma_E}(\mu_E, 2).$$

The above observation is crucial, namely, for a given set with covariance Σ and mean μ we can consider the ball $\mathbb{B}_\Sigma(\mu, 2)$ – the boundary of this shape is marked as a green ellipse on Figure 2f.[3] The above theorem indicates that if the set is ellipse

[2] Automatic selection of pupil and iris from CEC results is one of the goal of further work on this algorithm.

[3] Figure 2e also contains ellipses constructed in this way.

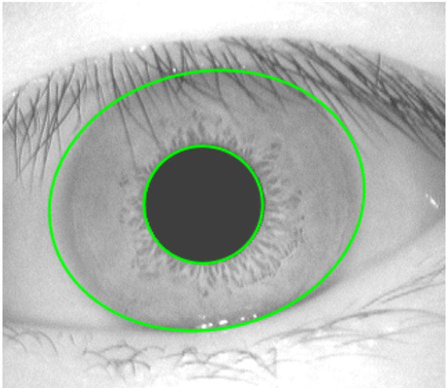


Figure 3. Results of the iris and pupil localization.

then these shapes – original set E and $\mathbb{B}_{\Sigma}(\mu, 2)$ – are similar. In the case of pupil (the yellow cluster in Figure 2f) we have such a situation. However for iris (the red cluster) it is not the case.

To fix this issue of iris cluster we will add points to iris from pupil cluster – we call the new cluster `Iris`. Then Ellipse Shrinking[4] is performed.

The Ellipse Shrinking algorithm finds iteratively the optimal ellipse describing the given set. We start with all point of the given set classified as members of optimal ellipse. Then we proceed with the following two steps:

1. compute the optimal ellipse for the current set, namely $\mathbb{B}_{\Sigma}(\mu, 2)$ (where Σ and μ are calculated for the current set);
2. from the optimal set delete points outside the optimal ball, namely, points which Mahalanobis distance from the mean of current set is greater than 2 (compare with Theorem 1).

We repeat the above two steps until no points are removed in the second step.

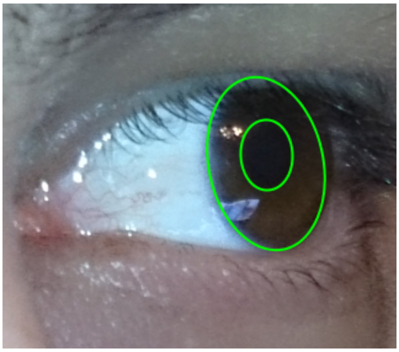
The result of Ellipse Shrinking algorithm for the iris cluster is presented in Figure 2g.

2.5. Iris segmentation results

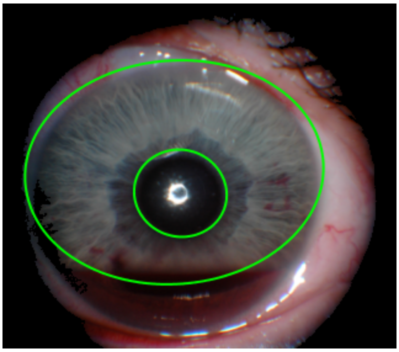
The results of the iris segmentation for the image processed in the previous subsections of Section 2 are presented in Figure 3.

The iris image presented in Figure 3 seams to have a good quality and can be treated as almost idea example. However, in general cases, the iris images can be affected by many factors that influence the shape, pattern or at least it may disturb the information collected from the iris, for example off-angle or tilted images (Figure 4a) or when the iris is damaged by a disease (Figure 4b).

[4] This algorithm is similar to Ellipse Growing [7].



(a)



(b)

Figure 4. Results of the iris and pupil localization for: (a) – tilted image; (b) – iris damaged by a disease.

3. Conclusions

A new method for iris segmentation is presented for human identification purpose. The algorithm is based on cross entropy clustering (CEC). The numerical experiments have demonstrated that the CEC algorithm works effectively in this cases. However it is necessary to develop a new model of Guassian densities which fits better in the case of iris segmentation.

Noticeable is that the new algorithm for iris segmentation can be applied for nonideal iris recognition, for example off-angle or tilted images (Figure 4a) or when the iris is damaged by a disease (Figure 4b).

In the future work the authors will concentrate on automatizing the step of optimization and extension of the preprocessing steps (Gaussian and regression image correction). The authors will also work on the development of human identification and verification procedures to propose a complete solution for the recognition by iris.

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4. References

[1] Daugman J., *How iris recognition works.* Circuits and Systems for Video Technology, IEEE Transactions on, 2004, 14(1), pp. 21–30.

[2] Tabor J., Spurek P., *Cross-entropy clustering.* Pattern Recognition, 2014, 47(9), pp. 3046–3059.

[3] Li P., Ma H., *Iris recognition in non-ideal imaging conditions.* Pattern Recognition Letters, 2012, 33(8), pp. 1012–1018.

[4] Ma L., Tan T., Wang Y., Zhang D., *Efficient iris recognition by characterizing key local variations.* Image Processing, IEEE Transactions on, 2004, 13(6), pp. 739–750.

[5] Misztal K., Saeed E., Tabor J., Saeed K., *Iris pattern recognition with a new mathematical model to its rotation detection.* In: *Biometrics and Kansei Engineering.* Springer, 2012, pp. 43–65.

[6] Freedman D., *Statistical models: theory and practice.* Cambridge University Press, Cambridge, United States of America, 2009.

[7] Misztal K., Tabor J., *Mahalanobis distance-based algorithm for ellipse growing in iris preprocessing.* In: *Computer Information Systems and Industrial Management.* vol. 8104. LNCS, Springer, London, 2013, pp. 158–167.