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DISTINGUISHED GRADUATES IN MATHEMATICS OF JAGIELLONIAN UNIVERSITY IN THE INTERWAR PERIOD, PART I: 1918–1925

WYBITNI ABSOLWENCI MATEMATYKI NA UNIWERSYTECIE JAGIELLOŃSKIM W OKRESIE MIEDZYWOJENNYM. CZEŚĆ I: 1918–1925

Abstract

In this study, we present profiles of some distinguished graduates in mathematics of the Jagiellonian University from the years 1918-1925. We discuss their professional paths and scholarly achievements, instances of scientific collaboration, connections with other academic centers in Poland and worldwide, involvement in mathematical education and teacher training, as well as their later roles in Polish scientific and academic life. We also try to understand in what way they were shaped by their studies and how much of Kraków scientific traditions they continued. We find strong support for the claim that there was a distinct, diverse and deep mathematical stream in Kraków between the wars, rooted in classical disciplines such as differential equations and geometry, but also open to new trends in mathematics.

Keywords: history of mathematics in Poland, Jagiellonian University, Cracow

Streszczenie

W niniejszym artykule przedstawiamy sylwetki niektórych wybitnych absolwentów Uniwersytetu Jagiellońskiego w zakresie matematyki z lat 1918-1925. Omawiamy ich drogi zawodowe i osiagniecia naukowe, przykłady współpracy naukowej, związki z innymi ośrodkami akademickimi w Polsce i na świecie, zaangażowanie w nauczanie matematyki i kształcenie nauczycieli oraz ich późniejsze role w polskim życiu akademickim. Próbujemy także zrozumieć, w jaki sposób zostali oni ukształtowani przez swoje studia i na ile kontynuowali krakowskie tradycje naukowe. Znajdujemy mocne dowody na poparcie tezy, że w Krakowie międzywojennym istniał wyraźny, zróżnicowany i głęboki nurt matematyczny, zakorzeniony w dyscyplinach klasycznych, takich jak równania różniczkowe i geometria, ale również otwarty na nowe trendy w matematyce.

Słowa kluczowe: historia matematyki w Polsce, Uniwersytet Jagielloński, Kraków

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1. Introduction

Mathematical traditions at Jagiellonian University in Kraków (Cracow) are several centuries old. Founded in 1364, the university got its first chair for mathematics and astronomy in 1405. It functioned during the rule of the Jagiellonian dynasty and elected kings, as well as during the 123-year period of occupation by the Austro-Hungarian empire [9, 17, 29]. World War I brought some interruptions to its activity. Kraków was a fortress, important for the war operations. In anticipation of a Russian attack, emergency evacuation was ordered and one-third of the population left the city. A few university buildings, including its clinics, were requisitioned for the military needs. The university was closed for the winter semester of the academic year 1914/15. However, the offensive was thwarted before the Russians approached Kraków and soon the classes resumed. They continued until the bloodless liberation of Kraków on October 31, 1918, when Polish soldiers disarmed an Austrian garrison stationed in the City Hall Tower and the occupying army capitulated. To commemorate the event, the words "Finis Austriae" were entered in the book of doctoral promotions at the Jagiellonian University [13].

On November 11, 1918 (the same day the armistice was signed in Compiègne), the Regency Council handed in the military command to Józef Piłsudski. On November 14, it passed all remaining authority to him and dissolved itself. On November 16, Piłsudski notified the Entente Powers about the emergence of the independent Polish state.

The beginnings of the newly proclaimed Republic of Poland were turbulent. Even though the Great War was over, the state had to fight for its borders against the competing interests of Germans as well as Czechs, Slovaks, Lithuanians and Ukrainians, who also aspired to independence, and finally, in 1919–1921, to defend itself against the Bolshevik Russia. Many students interrupted their courses to volunteer for the military service. Internally, the emerging republic also faced many difficulties. It had to forge, among other things, a common legal, administrative and educational system, in place of no longer relevant disparate systems of the three occupying powers. The existing Polish-language academic institutions - that is, the Jagiellonian University, the Lwów (Lvov; Lviv) University and the Lwów Polytechnics - played a major role in these endeavors, educating the future forces to staff Polish administration, courts and schools at all levels, including the new or revived academic schools in Polish territories formerly under Russian or German occupation¹. Of course hardships of everyday life affected the educational process and the scientific research. At the opening of the academic year 1922/23, the vice-rector of the Jagiellonian University, Professor Stanisław Estreicher, a historian of law and a bibliographer, characterized the current conditions as follows:

"It cannot be denied (...) that the current relations, resulting from the tremendous shock of civilization which was the Great War, do not create by any means conditions that are favorable for the development of scientific institutions. The funding of our institutes and laboratories, which was not great

¹ Since 1871, Polish was the language of instruction in the institutions of higher education in the Polish province of Galicia of the Austro-Hungarian empire.

even before the war, in current conditions does not bear any proportion even to the most modest needs. Every year we lose more and more scientific contact with abroad, we are more and more isolated from intellectual trends, foreign books and journals reach our hands less and less frequently, with more and more difficulty. Even the most important scientific institutions lack the space or are unable to expand, and it is almost impossible to bring scientific staff from outside because of the difficulties in finding accommodations for them. The same difficulty affects the young people: our youth struggle not only with provisional difficulties and with dearness of scientific aids, but primarily with the difficulty of finding accommodations in Kraków. The number of youth does not increase at the rate at which it should, if one takes into account how big is the need of the society for intellectual forces, lacking in Poland in every way" [37] (translated by the second author).

A committee was formed to help the students. Institutions and more affluent private citizens answered the appeal and contributed cash and commodities. Despite all difficulties, young people were trying to pursue academic studies to satisfy their intellectual needs and prepare themselves for their roles in the public life of the new state. In the academic year 1921/22, the total number of students at the Jagiellonian University was 4580. Most of them, 2002, were enrolled in the faculty of philosophy (where mathematics also belonged). There were 976 women students, 763 of them in the faculty of philosophy [37].

In 1918 there were three professors of mathematics at the Jagiellonian University: Stanisław Zaremba, Kazimierz Żorawski and Jan Sleszyński. Lectures were also given by Antoni Hoborski, Alfred Rosenblatt and Włodzimierz Stożek, soon joined by Leon Chwistek, Witold Wilkosz and Franciszek Leja. The standard curriculum, established before 1918, comprised mathematical analysis (differential and integral calculus), analytic geometry, introduction to higher mathematics, differential equations, differential geometry, theory of analytic functions. Less frequently, lectures were given in number theory, power series, synthetic geometry, algebraic equations, higher algebra and projective geometry. Occasionally, there were also lectures in theory of elliptic functions, theory of transformations, theory of conformal mappings, spherical trigonometry, calculus of variations, analytic mechanics, algebraic geometry, elementary geometry, kinematics of continuous media, equations of physics, probability, theory of determinants, introduction to the methodology of mathematics, mathematical logic, integral equations. (see [29]) New subjects were added by Witold Wilkosz, who became an extraordinary professor in 1922: foundations of mathematics, set theory, theory of quadratic forms, theory of functions of a real variable, geometric topology, group theory. The course of studies could be concluded with either the teacher's examination or PhD examination. An additional subject was required at the examination (most commonly, physics was chosen).

A common view is that Kraków was lagging behind the mathematical centers of Lwów and Warsaw, neglecting developments in topology, set theory and functional analysis, in which the Polish school of mathematics achieved its most remarkable and lasting results. It has been claimed that the emphasis on classical mathematics, mostly on differential equations, caused the exodus from Kraków of younger mathematicians more interested in pursuing modern fields and eventually led to the dismantling of Kraków research group. Here are the impressions of the outstanding Russian mathematician Nicolai Lusin after his visit to Warsaw, expressed in his letter to Arnaud Denjoy, dated September 30, 1926:

"It seems to me that the mathematical life in Poland follows two completely different ways: one of them is inclined to the classical parts of mathematics, and the other to the theory of sets (functions). These ways exclude one another in Poland, being the irreconcilable enemies, and now a fierce struggle is going on between them" (cited after [11]; translated by S. S. Demidov).

"The classical side is currently represented only by the ancient (over 500-year old) Kraków University and Kraków Academy. Among Polish mathematicians the most stalwart supporter of this way is Professor Zaremba. Other supporters of this tendency stick close to Mr. Zaremba. (...) Thus currently only Kraków is a stronghold of classical mathematics. However, the Polish mathematicians whom I saw in Warsaw claim univocally that Mr. Zaremba's cause is doomed to failure, and this is why numerous colleagues and students are leaving him. Thus Mr. Zaremba's students, Dr. Kaczmarz and Dr. Nikliborc, are leaving Kraków for Lwów; Mr. Banach and Mr. Stożek already did so. A student of Mr. Zaremba, Mr. Leja, left for Warsaw. Mr. Nikodym also intends to leave Kraków and only material difficulties do not allow him to relocate to a different city. One can therefore speak about dissolution of the group of Kraków mathematicians" (cited after [17]; translated by the second author).

The truth is more complex than this. We already mentioned new subjects introduced (mainly by Wilkosz) to the students' curriculum which reflected the latest mathematical developments. These subjects also made their way into research of Kraków mathematicians, e.g. Ważewski did some significant work in topology and Gołab in geometry of metric spaces (see the subsequent sections devoted to individual mathematicians for more detail). Let us now address the relocations of Kraków mathematicians. In the first years of the Republic of Poland, it was necessary to provide Polish-speaking staff to the revived or newly created institutions of higher education. Lwów and Kraków mathematicians took the opportunity to engage in building academic and scientific life where there was none. Kazimierz Żorawski, who worked mainly in differential geometry and in the academic year 1917/18 served as the rector of the Jagiellonian University, moved to Warsaw in 1919, to become a professor first at the Warsaw Polytechnic, then at the University of Warsaw (both reopened in 1915) and to serve as an official in the Ministry for Religious Denominations and Public Enlightenment. He remained in Warsaw until his death in 1953. Franciszek Leja, an assistant in Kraków, whose works concerned differential equations, functions of a complex variable, potential theory and approximation theory², obtained a chair of mathematics at the Warsaw Polytechnics in 1923. However, he returned to Kraków to become an ordinary professor

² Leja was also first to define explicitly a topological group. For more information on him, see e.g. the article by M. Kosek in this volume.

of mathematics in 1936. A few mathematicians who were based in Lwów before World War I - Wacław Sierpiński, Zygmunt Janiszewski and Stefan Mazurkiewicz - decided to take positions at the University of Warsaw. Their departures (as well as the death of Józef Puzyna in 1919) left the Lwów mathematics understaffed. Some of the gaps were filled by Hugo Steinhaus, Włodzimierz Stożek, Stefan Kaczmarz, Władysław Nikliborc and Stefan Banach. Without formal training in mathematics and university diploma, Banach had no chance for a standard academic employment. The invitation from Lwów to work as Antoni Łomnicki's assistant (issued at the initiative of Steinhaus) came as a unique opportunity. Steinhaus himself did not have an academic appointment for a few years following his PhD, even though he got his habilitation in Lwów in 1917. After World War I he worked as a mathematical expert for a gas company and considered himself a "private scholar" before joining the Lwów University. Stożek became a professor of mathematics at the Lwów Polytechnics, but soon ceased to do research and devoted himself to writing textbooks. Kaczmarz and Nikliborc went on to get their PhD degrees from Lwów and to build their mathematical careers, ended by their premature deaths (Kaczmarz fell in the September campaign of 1939 and Nikliborc was driven to suicide by Communist security forces in 1948). While Kaczmarz's research concerned real analysis, orthogonal series and numerical methods, Nikliborc continued to work on potential theory, differential equations and mechanics. Otto Nikodym held on to his (secure and relatively well-paid) job as a high-school teacher, but ultimately left Kraków to pursue an academic career, getting his PhD (in 1924) and habilitation (in 1927) at the University of Warsaw. One can thus say that the relocations were motivated not so much by individuals' wishes to pursue particular mathematical interests, but rather by available career opportunities³. While Zaremba remained faithful to classical mathematical disciplines, he did not prevent his colleagues from pursuing topics in real analysis, set theory, topology and other modern developments, or from teaching them to students. His main neglect was in not taking care to create positions for junior mathematicians. Here is an account by Andrzej Turowicz, a student in the years 1922–26:

"When I came for my first year of studies at the university, there were no assistants. The so-called "proseminarium" [a pre-seminar], which preceded the recitations, was led by docent Leja. I was in this pre-seminar of his. He was a high school professor and had contract classes [at the university]. Only when I entered the second year, Wilkosz took care to have two deputy assistants nominated, that is Leśniak and Mrs. Wilkosz ([who was] a mathematician). Then also Turski made it. Even later, there was Krystyn Zaremba".

"I taught high school for 10 years. At that time, there was one adjunkt and one deputy assistant at the university. That was it. There was no chance of getting

³ There were also mathematicians who got their PhD in Kraków and were employed by other centers, e.g., Stefan Kempisty (PhD 1919), later a professor at the Stefan Batory University in Wilno, working mainly in real analysis, and Witold Pogorzelski (PhD 1919), later a professor of Warsaw Polytechnics, working mainly in differential and integral equations.

into university [as junior faculty]. It was Zaremba's fault. He did not take care to acquire an assistant" [61].

In the initial years, however, the Jagiellonian University and the Academy of Mining (created in 1919) were able to accommodate the mathematical talents of Tadeusz Ważewski and Stanisław Gołąb. They became world-class researchers and later created their own scientific schools. There were a few other distinguished graduates of mathematics in the period 1918–1926 (before the unification of university system and introduction of the master's degree for all disciplines) who later made an impact on Polish intellectual and academic life. Below we present their profiles.

2. Profiles



2.1. Tadeusz Ważewski (1896–1972)

Born in the village of Wygnanka, in the Austro- Hungarian province of Galicia (later in Tarnopol Voivodeship in the II Republic of Poland, then in USSR, now Ukraine), he passed his *matura* in the 1st Gymnasium in Tarnów. In the years 1914–1920 he studied in the philosophical faculty of the Jagiellonian University He started studying physics, then, influenced by S. Zaremba, switched to mathematics. Military service interrupted his studies. When Poland regained independence in November 1918, he served on a citizen patrol in Kraków. In the years 1920–1921 he taught in St. Anne State Gymnasium in Kraków as a substitute teacher. In the years 1921–1923 he studied at the University of Paris, where in 1924 he obtained PhD degree on the basis of the

thesis "Sur les courbes de Jordan ne renfermant aucune courbe simple fermeé de Jordan" concerning dendrites, i.e., locally closed continua which do not contain simple closed curves (later published as [65]). On the doctoral committee there were Émile Borel, Arnaud Denjoy and Paul Montel [12]. Paul Émile Appell is also signed on Ważewski's diploma, as the rector of the university of Paris at that time. Ważewski returned to Kraków, where he got his habilitation in 1927 for a thesis concerning rectifiable continua [66]. He became a deputy professor in 1926 and an extraordinary professor of the Jagiellonian University in 1933 [1, 2]⁴.

Ważewski was arrested along with other professors in the Sonderaktion Krakau on November 6, 1939, imprisoned in the Sachsenhausen concentration camp and released

⁴ The Council of the Philosophical Faculty of Jagiellonian University, of which mathematics was part, proposed Ważewski as a candidate for extraordinary professorship already in 1929. However, in a poll among mathematicians from other centers, Ważewski's name was not mentioned. Instead, other names appeared: of Władysław Nikliborc, Otto Nikodym, Antoni Zygmund, Bronisław Knaster, Alfred Rosenblatt, Alfred Tarski, Juliusz Schauder, Witold Hurewicz and Aleksander Rajchman. The ministry concluded that Ważewski's candidacy was inferior and rejected the proposal [17].

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in February 1940. He stayed in Kraków until the end of the Nazi occupation, officially lecturing at the Men School of Commerce while also engaging in clandestine teaching and conducting an illegal mathematical seminar with many participants who presented their results. In 1945 he became an ordinary professor and took active efforts to rebuild academic life in Kraków, not only at the Jagiellonian University, but also at the State Pedagogical College and Academy of Commerce as well as at the State Mathematical Institute (later Mathematical Institute of the Polish Academy of Sciences). He lectured, published papers, edited journals (e.g. *Annales Polonici Mathematici*), headed academic units (a university chair and a division of the Mathematical Institute), presided the Polish Mathematical Society (1959-61) and, perhaps most importantly, supervised PhD students, many of whom later became distinguished mathematicians. He gave short communications at the International Congresses of Mathematicians, in Oslo (1936) and Stockholm (1962), and a plenary address in Amsterdam (1954) [35].

Initially Ważewski was interested in point-set topology and obtained important results in this area. In his PhD thesis he constructed a universal dendrite, i.e., a dendrite containing a homeomorphic image of any other dendrite. This set, known as Ważewski's dendrite, still attracts interest of mathematicians, either in its own right [5] or as a tool in the study of Berkovich analytic spaces, lying at the intersection of algebraic geometry, number theory and topology [34]. In 1923 Ważewski proved a theorem on properties of the hyperspace of a locally connected metric continuum [64]. This result was obtained independently and published amost simultaneously by Leopold Vietoris [63] and later generalized by Menachem Wojdysławski [77] and other mathematicians [8]. In later years he occasionally returned to purely topological topics, e.g. in [73]. However, differential equations became his main interest, the area of his highest achievements and a springboard for new directions of research. He started working on them in 1930s and published over 25 papers concerning them before World War II. Two of these works he wrote jointly with Stanisław Krystyn Zaremba [74, 75], and in at least one he addressed questions inspired by the work of Adam Bielecki [67]. He also worked on differential inequalities. In 1947 Ważewski published first version of his fundamental result, later called Ważewski's retract principle [69] a creative application of topology to differential equations. The result has many variants now [58]. Roughly speaking, it says that certain properties of the solutions of a differential equation (or a system of equations) on the boundary of a given domain imply that some solutions to the equation must stay in the domain. The case of systems of equations requires significant use of the topological notion of retract. Versions for dynamical systems were also developed and led to the theory of so-called Conley index [54], which is a tool for analyzing topological structure of invariant sets of smooth maps or smooth flows. In 1948 [70], investigating the domain of existence of an implicit function, Ważewski introduced and studied a linear differential equation, later called Ważewski equation. The existence of polynomial solutions to this equation was later shown [44] to be equivalent to the famous Keller Jacobian Conjecture (which is still open in the complex case). In 1960s Ważewski wrote a series of papers in the optimal control theory, in which he created a new direction in the area by using the notion of a differential inclusion, introduced by S. K. Zaremba and A. Marchaud [31, 48].

Ważewski had a great pedagogical talent. He was ranked very highly by students as an instructor. He published lecture notes [71] and in his research he sometimes took up topics of educational value. For example, he gave a unified proof of all cases of the de l'Hôspital rule in calculus ([72]; later he also published a version for Banach spaces). He also had a strong sense of mission. During the occupation he entered twice the Kraków ghetto to talk to an amateur mathematician named Rappaport (first name unknown). The meetings were arranged by Tadeusz Pankiewicz [49, 51], a Pole who was allowed by the Nazis to run a pharmacy inside the ghetto until its end. Ważewski risked his own life, as he was a former concentration camp prisoner and the entrance was illegal, but he had discussions with Rappaport, who gave an approximate solution to the angle trisection problem. Rappaport did not survive the war; his result was published by Ważewski in 1945 [68]. Another memorable instance of Ważewski's doing what he considered the right thing was the following: his former student Andrzej Turowicz became a Benedictine monk and a priest (Fr. Bernard) after the war. It became hard for Turowicz to participate actively in the scientific life, as the communist authorities kept limiting the influence of the Catholic church in the public life. But thanks to Ważewski's firm stand, he was employed in the Mathematical Institute of the Polish Academy of Sciences since 1961, had his habilitation approved in 1963 and became an extraordinary professor in 1969. (For more information about Turowicz, see

[14] and the references therein).



2.2. Władysław Nikliborc (1899–1948)

Born in Wadowice, he finished high school there and passed his *matura* examination in 1916. His university studies were interrupted by military service. Released from the military in December 1920, he studied mathematics at the Jagiellonian University, graduating in 1922. He attended lectures in mathematics (by Stanisław Zaremba, Kazimierz Żorawski, Jan Sleszyński, Antoni Hoborski, Alfred Rosenblatt, Włodzimierz Stożek, Witold Wilkosz), physics (by Władysław Natanson, Czesław Białobrzeski, Konstanty Zakrzewski, Stanisław Loria) and astronomy (by Tadeusz Banachiewicz). In October 1922 he became an assistant in the Chair of Mathematics headed by Antoni Łomnicki at the Faculty

of Mechanics of the Lwów Polytechnics. He also taught mathematics at a private Ursuline gymnasium for women.

In 1924 Nikliborc obtained his PhD degree at the Faculty of Philosophy of the Lwów University, passing exams in mathematics, astronomy and philosophy and presenting a thesis "On applications of the fundamental theorem of Cauchy on the existence of solutions of ordinary differential equations to boundary value problems for the equation y'' = f(x, y, y')". His habilitation at Lwów University took place in 1927 and was based on a two-part paper "Sur les fonctions hyperharmoniques", published in Comptes Rendus of the Paris Academy of Sciences in 1925 and 1926 ([45, 46]). In this paper Nikliborc considered the Dirichlet problem in a polydisk for a function which is the real part

of a holomorphic function of two complex variables. Nowadays such functions are called pluriharmonic. The study of these functions was initiated in 1899 by Henri Poincaré and continued by Luigi Amoroso ([3]), who in 1912 also considered a Dirichlet problem (in a somewhat more general domain), but treated it as a problem in four real variables. It was Nikliborc who made subsequent advances in the theory of pluriharmonic functions. The referee of his habilitation, Hugo Steinhaus, praised not only his ingenuous methods but also his thorough knowledge of differential equations. In 1931 Nikliborc got habilitation in theoretical mechanics at the Lwów Polytechnics, on the basis of a paper about rotating fluid. He became interested in fluid mechanics and celestial mechanics and worked on these subjects. From 1937 to 1939 he was an extraordinary professor at the Warsaw Polytechnics. He wrote academic and high school textbooks (some of them jointly with Steinhaus or Włodzimierz Stożek). He spent the years of World War II in Lwów. During the Soviet occupation (1939-1941) he was a professor of mathematics in Lwów, and during the German occupation (1942–1944) he lectured at the Staatliche Technische Fachkurse, a school for vocational training created as partial replacement of the Polytechnics (which, like all Polish institutions of higher education, was closed by the Nazis). He took care of ailing Stefan Banach in his last days. In 1945 he went to Kraków, to take the Chair of Mathematics in the Faculty of Engineering at the Academy of Mining and Metalurgy and to give lectures at the Jagiellonian University, but soon he moved to Warsaw (Andrzej Turowicz was appointed to lecture in his stead, [61]). He became an ordinary professor there: first at the Polytechnics, then at the University. He committed a suicide in 1948 in Warsaw after being arrested and interrogated by the communist security forces [15]. His advanced textbook in differential equations was published posthumously in 1951 (see also [59]).

Nikliborc mastered differential equations as a student in Kraków. Rumor had it (as related later by Fr. Turowicz, [61]) that he and Stefan Kaczmarz, an earlier Jagiellonian student, moved to Lwów for their PhD in order to avoid taking further examinations with Zaremba. Because of lack of materials, we cannot confirm or deny personal factors playing a role, but both graduates were certainly well prepared for scholarly work in mathematics. While Kaczmarz's interests were closer to those of the Lwów mathematical school, Nikliborc continued working on classical analysis, differential equations and generalization of harmonic functions. However, he wrote a joint paper with Kaczmarz [36], and entered several problems in the Scottish Book, one as a "Theorem" (# 128, 129, 149 and 150 [39]). His later interests in mechanics were influenced mainly by Leon Lichtenstein, whom he visited in Leipzig in 1930–1931. Solid background in physics and astronomy acquired in Kraków helped Nikliborc in his work on equilibrium figures in hydrodynamics and on the three-body problem.

2.3. Stanisław Bilski (1893–1934(?))

Stanisław Bilski was born in Zgierz. He passed his *matura* examination in 1911 in Łódź Merchants' School of Commerce. In the years 1914–1918 he worked as a teacher in Zgierz. He started his studies in 1918 in Warsaw, then continued studying mathematics and philosophy in Kraków. In the school year 1925/26 he taught mathematics in the state gymnasium in Rybnik. In 1926 he obtained PhD in philosophy at the Jagiellonian University

on the basis of required exams and the thesis "A priori knowledge in Bertrand Russell's epistemology", supervised by Władysław Heinrich. He translated into Polish some writings of the dialectical materialistic philosopher Joseph Dietzgen and poetic works by Schiller, Goethe and Heine.

Since 1916, Bilski was active in the Polish Socialist Party-Left. He established first communist structures in Zgierz and organized worker demonstrations. In 1929, to avoid arrest by Polish authorities, he escaped to the USSR under the name of Stefan Biernacki. He supervised the Polish section of the Central Publishing House of the Nations of USSR in Moscow, then he lectured on the history of Polish labor movement in Kiev. He was arrested at the beginning of the Great Purge in 1934, sentenced as an alleged Polish spy and executed [76].



2.4. Jan Józef Leśniak (1901–1980)

Born in Ropczyce, he attended a gymnasium in Jasło. He passed his *matura* with distinction in 1919, then he studied mathematics at the Jagiellonian University. In 1922 he became a scientific aide of the Mathematical Seminar, then he was made an assistant. In 1928 he passed the exam for high school teaching licence and took a position in H. Sienkiewicz gymnasium in Kraków. At the same time he gave contract lectures on issues of elementary mathematics at the Jagiellonian University and – since 1930 – lectures on didactics of mathematics at the Pedagogical Institute of Studies. In 1939 he was arrested by Gestapo and taken to the concentration camps in Wiśnicz and Auschwitz. Released in 1940, he returned to Kraków, where he

taught mathematics in the School for Commerce and Industry. After the war he reassumed his position in a gymnasium and continued his lectures at the Jagiellonian University. In 1947 he obtained there his PhD degree (under the supervision of Franciszek Leja) on the basis of the thesis "Methods of solving equations". In the same year he started working at the Pedagogical College (WSP) in Kraków. In 1951 he submitted a habilitation thesis "Educational values of instruction in mathematics and their fulfilment in high school" and became an extraodinary professor. In 1963 he was promoted to the level of a professor of mathematics at the Faculty of Mathematics, Physics and Chemistry at the Pedagogical College. In 1961, in the paper [43], he proposed a new formal approach to indefinite integrals. His output counts over 20 items, including several books on elementary mathematics, theoretical arithmetics and functions of one variable.

2.5. Stanisław Gołąb (1902-1980)

Stanisław Gołąb was born on July 26, 1902, in Travnik (Bosnia), where his father, Walenty, was a judge; his mother, Jadwiga neé Skibińska, was a teacher. In 1910 the family moved to Kraków. Gołąb attended the V Gymnasium, where in 1920 he passed his maturity examination. One of his gymnasium teachers was Antoni Hoborski, later a professor of mathematics in the Academy of Mining, who also conducted lectures at the Jagiellonian University. In the years

1920–1924 Gołąb studied mathematics at the Jagiellonian University. In 1923 he completed the Pedagogical Study at the Jagiellonian University and in 1926 he passed examinations in mathematics and physics for candidates for high school teachers.

In 1923 he started working as a deputy assistant to Hoborski, and after his graduation in 1924 was promoted to an assistant in the Chair of Mathematics at the Academy of Mining. Influenced by Hoborski (who wished to establish a scientific school in geometry in Kraków), he chose geometry as his area of interest and started research in it. In the years 1925–1932 he published 14 papers.



In 1928, Gołąb, with Hoborski's support, obtained a scholarship from the Division of Science of the Ministry of Religious Denominations and Public Education for studying abroad. He went to Delft (the Netherlands) to work with J. A. Schouten. They wrote a paper together [55]; Gołąb also started working on his PhD thesis there. In 1929–1930 he got another scholarship, from the Fund for National Culture. He spent 3 months again in Delft, finishing his PhD thesis and writing the second part of his paper with Schouten [56]. Then he went to Rome, where he learned absolute calculus from E. Bompiani and relativistic mechanics from T. Levi-Civita. The last three weeks he spent in Prague, in private discussions with L. Berwald and V. Hlavatý. His trip resulted in new publications [32]. In 1932, Gołąb took part in the International Congress of Mathematicians in Zürich, giving two short talks there [22, 23, 35].

Since 1930, Gołąb conducted contract lectures at the Jagiellonian University. In 1931 he obtained the PhD degree at the Jagiellonian University on the basis of the thesis *O uogólnio-nej geometrii rzutowej*, which was presented in Polish (and published as [19]) and was related to his joint work with Schouten. His supervisor was Stanisław Zaremba; the referee was Witold Wilkosz. In 1932 he obtained habilitation at the Jagiellonian University on the basis of the paper *Zagadnienia metryczne geometrii Minkowskiego*, published in 1932 in the Proceedings of the Academy of Mining [21]. The topic of his habilitation lecture was *Metryka kątowa w ogólnych przestrzeniach*, chosen by the committee out of three topics proposed by the candidate (which he developed in his congress talks and several other publications, including a joint paper with Adam Bielecki, [6]). The examination questions concerned, among other things, Minkowski geometry, convex functions (Wilkosz), differential equations (Zaremba) and general metric spaces (Hoborski), and Gołąb's answers were evaluated as very good. The ministry extended Gołąb's licence to lecture to the Faculty of Mining at the Academy of Mining, and he received the title of professor in 1939.

On November 6, 1939, Gołąb was imprisoned along with other Kraków professors and taken to the concentration camps, first in Sachsenhausen, then in Dachau. He tried to take care of his teacher Hoborski, but ultimately witnessed his death. Released⁵ in December 1940

⁵ According to [57], Wilhelm Blaschke claimed to intervene personally on Gołąb's behalf. In private communication with the second author, Zofia Gołąb -Meyer, Staniaław Gołąb's daughter, could not confirm whether this was the case. On the basis of documents in her possession, she brought to our

together with a group of junior scholars, he worked in forestry administration, taking part in clandestine teaching since 1943. After the liberation he continued working at the Academy of Mining, becoming an ordinary professor there in 1948. When the State Mathematical Institute (later the Mathematical Institute of the Polish Academy of Sciences) was created, he became the head of the Division of Geometry there (he headed the division until his retirement). In the years 1950–55 he was a contract professor at the Pedagogical College in Kraków, and in 1954 he transferred from the Academy of Mining and Metalurgy to the Jagiellonian University (however, until 1962 he remained as a half-time employee at the Academy). He was the head of the Chair of Geometry and the dean of the Faculty of Mathematics, Physics and Chemistry of the Jagiellonian University. Because of his worthy behavior as the dean (supporting the rights of protesting students and faculty) during the March Events of 1968 he was demoted from the position of the head of the Laboratory of Geometry by the communist authorities. He retired in 1972 and died in Kraków on April 30, 1980 (see [16, 18, 41, 50]).

Golab's scientific output comprises over 250 publications, including two monographs: Tensor Calculus ([28]) and Functional Equations in the Theory of Geometric Objects (joint with J. Aczél; [30]). His main research interest was geometry, in particular clasical differential geometry, tensor calculus (under the influence of Schouten), spaces with linear or projective connection, Riemann, Minkowski and Finsler spaces, general metric spaces and other areas. He supervised 28 PhD degrees during his career. While working on the theory of geometric objects, in 1939 Gołąb [27] gave the first exact definition of a pseudogroup of transformations [33, 41]. He also promoted the methods of functional equations in geometry, starting with the so-called translation equation, for which he found a solution of class C^{1} [25] But his interest in functional equations dates earlier. In 1937 [24] he published a note in which he gave conditions characterizing the transformation $z \rightarrow \overline{z}$ of complex numbers as the only solution to the multiplicative equation f(uv) = f(u)f(v) (in the same journal issue, different conditions were given by A. Turowicz, [62]). He went on to obtain other results in functional equations and supervise PhD candidates in this field, and is considered as the founding father of the Polish school of functional equations ([42)]. In metric geometry, he proved e.g. that, under certain mild assumptions, approximation of a point x of a Minkowski space by points in a given closed subset C is unique if the distance function to C is differentiable at x ([26]; later on, many variants and refinements of this result were obtained by various authors, cf. [78]) Another memorable result by Gołab in this field concerns the unit disk in the plane with a norm. It says that the perimeter of the unit disk can take any value between 6 and 8, and the extremal values are taken if and only if the disk is a regular hexagon or a parallelogram [4, 21]. Golab also worked on problems in applied mathematics, mainly in mining and geodesy [7], in close collaboration with engineers. He was also interested in mathematics education and history of mathematics. He edited the monograph "Studies in the history of the chairs in mathematics, physics and chemistry of the Jagiellonian University", published in celebration of the University's 600th anniversary [29]. He lectured

attention the intervention of Hasso Härlen, one of Gołąb's coauthors. This intervention is described in detail in [52].

with lucidity and liked working with students. In turn, they were fond of him as an educator and treated him as an authority.

2.6. Zofia Krygowska (1904-1988)

Anna Zofia Krygowska (neé Czarkowska) was born in Lwów. She grew up in Zakopane, where she finished gymnasium. In the years 1923–1927 she studied mathematics at the Jagiellonian University. Then she passed the teacher's licence examination and taught in elementary and high schools. In 1931 she obtained the master's degree from the Jagiellonian University. She became interested in issues concerning the process of teaching mathematics, in particular curriculum development and bringing new trends in mathematics into school-level education. She engaged in the activities of the Methodological Center for Mathematics, participated in many conferences and discussions and published articles on the published articles on the



subject of teaching mathematics [10]. In 1937 she published a textbook *Mathematics for the 1st grade of high school*.

During the German occupation Krygowska was involved in clandestine teaching. Officially she worked as an accountant in a lumber company, while also travelling on behalf of the underground educational authorities to organize, teach and coordinate illegal classes (especially in the Podhale region), risking her life. In 1945 she resumed her work as a high school teacher. In the years 1948–1951 she headed the Methodological Center. In 1950 she obtained the PhD degree at the Jagiellonian University on the basis of the thesis "On the limits of rigor in the teaching of elementary geometry" prepared under the direction of Tadeusz Ważewski. In this thesis she developed and studied an original system of axioms of geometry and proved its equivalence to Hilbert's system [60]. She also took a full-time position at the Pedagogical College in Kraków. She began active efforts to modernize the education of teachers and to make didactics of mathematics a research discipline in its own right, while continuing her involvement in improving teaching of mathematics – especially geometry – in schools. In the years 67–71 she wrote or co-wrote five textbooks in geometry, which were subsequently adopted for high school use in Poland. She also published many articles and books on the issues of teaching of mathematics and its methodology.

In 1958 Krygowska became a professor in the newly created Unit of Methods of Teaching of Mathematics in the Pedagogical College in Kraków. She initiated post-graduate doctoral programs in didactics of mathematics. She supervised 22 PhD degrees and 4 habilitations in this field. She was a member of many national and international committees related to teaching mathematics, most importantly of the Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), founded in 1950. She served as a president, then as a honorary president, of this organization. She gave an address at the International Congress of Mathematicians in Nice in 1970. She was also instrumental in creating a series of lectures for teachers which were broadcast by Polish state television in the years 1968–1977. She supported launching a journal devoted to didactics of mathematics by the Polish

Mathematical Society and in 1982 the publication of "Dydaktyka Matematyki" started. She was an editor of this journal until her death [47].

Krygowska became acquainted with modern developments in set theory, topology, mathematical logic and other subjects as a student in Kraków. She gave credit to Witold Wilkosz, who taught these subjects, for initiating her interest in them. The new mathematical notions as well as emphasis on rigor made their way to her school textbooks and were consistent with the trends (started in 1930s) of reformulating mathematics in a modern way, advocated by the Bourbaki group. Krygowska also acknowledged the role of psychology in teaching. She was familiar with the work of Jean Piaget (whom she met at an international conference in public education in Geneva in 1956) and followed some of his ideas in her paper "Methodological and psychological basis for the activity-based method of teaching mathematics" [38].

Since her youth, Krygowska had a passion for mountaineering. She made a few first routes in the Tatra mountains, either in all-women teams or with her husband Władysław Krygowski, a lawyer [53].

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