

BEATA NIEZGODA-ŻELASKO*, JERZY ŻELASKO*

FREE CONVECTION ON THE OUTER SURFACE OF VERTICAL LONGITUDINALLY FINNED TUBES

KONWEKCJA SWOBODNA NA ZEWNĘTRZNEJ POWIERZCHNI RUR OŻEBROWANYCH WZDŁUŻNIE

Abstract

The paper presents methods of determination and the results of experimental studies of heat transfer coefficients on the outer surface of longitudinally finned tubes under free convection conditions.

Keywords: *free convection*, *vertical finned tubes*, *heat transfer coefficients*, *measuring methods*

Streszczenie

Artykuł przedstawia metody wyznaczania współczynników przejmowania ciepła w warunkach konwekcji swobodnej na zewnętrznej powierzchni rur ożebrowanych wzdłużnie.

Słowa kluczowe: *konwekcja swobodna, pionowe rury ożebrowane, współczynnik przejmowania ciepła, metody pomiarowe*

^{*} Ph.D. D.Sc. Eng. Beata Niezgoda-Żelasko, Ph.D. Eng. Jerzy Żelasko, Faculty of Mechanical Engineering, Cracow University of Technology.

Nomenclature

1. Introduction

This paper analyses the heat transfer process on the external surface of a vertical bi- -metallic tube with eight longitudinal aluminum fins situated central-symmetrically along the perimeter of the tube, with an inner core in the form of a copper pipe (Fig. 1a). Such tubes are frequently used in heat pump evaporators which are not fitted with fans. As evaporators operate under normal external conditions and the tubes are freely arranged, the tubes of the evaporator are not grouped into banks, but are located loosely. Null and low velocities of external air flowing from variable directions require the surface of the heat exchanger to be considerably enhanced – this is achieved thanks to fins being the correct length and them having a 'wavy' shape as well as their axisymmetric arrangement [1].

In the case of longitudinally finned tubes, it is necessary to differentiate between tubes fitted with single ribs or membrane tubes, and tubes where ribs are situated centrally and symmetrically along the circumference of the entire tube. For both types, various reference works propose equations for transverse air flow around a bank of tubes [2‒4].

The influence of a single longitudinal straight fin located on the outer surface of a horizontal cylinder on the natural convection heat transfer process was analysed in [5]. Tolpadi and Kuehn [5] found a slight influence of the fin on the averaged value of the heat transfer coefficient with respect to a smooth tube. The results of their studies indicated differences in the measured local and mean heat transfer coefficients for fins other than vertical fins.

Problems relating to the numerical analysis of heat transfer processes under free convection on the surface of longitudinal fins are often examined for flow inside longitudinallyfinned tubes [6, 7]. Paper [8] presents a numerical analysis of the impact of the thickness, height and number of fins in the free convection process on the distribution of velocities, temperatures and heat flux outside a horizontal cylindrical surface with longitudinal fins. The authors note that the maximum heat flux released by the finned surface can be achieved for fins thinner than 0.01 m (within the considered range of between $0.01-0.05$) and for heights not greater than 0.2 m. The maximum heat fluxes released by the finned surface were recorded for between 6 and 12 fins.

Publication [9] includes a numerical analysis of the influence of free convection on the increase of flow resistances and the heat transfer process occurring during laminar flow in a vertical ring pipe with longitudinal fins located on the outer tube. The authors analysed the influence of the number of fins and the relationship between the diameters of the channel's tubes on an increase of flow resistances and heat transfer coefficients. The authors pointed critical values of the Rayleigh number, at which there was clear influence of free convection on an increase of heat transfer coefficients. Among other things, it was indicated that the fewer fins there are, the closer the relationship between the outer tube diameter and the inner tube diameter; and the shorter the fin, the lower the value of the critical Rayleigh number and the greater the influence of free convection on the increase of heat transfer coefficients in the forced convection process. The critical Rayleigh numbers identified in [9] fitted within the range of 20 to $10⁴$.

2. Description of the measurement stand and methods

The experimental studies were conducted for a system including two 2 m-tall vertical tubes, whose axes were set apart at a distance of $S_q = 0.2306$ [m] (Fig. 1b). Measurements for airflow around the tubes were performed after the tubes were fed with ice water or R407C and R507 refrigerants. When air was cooled using the cooling fluids, the tubes made part of an evaporator system of a cooling device (Fig. 1b). Two shell heat exchangers fitted with electric heaters were an additional component of the evaporator of the cooling system. Eighteen NiCr-NiAl thermocouples were located on the external surface of the finned tubes in order to enable the measurement of temperature at the external surface of the pipe and at the tips of the fins (Fig. 1a). All the thermocouples were calibrated using a Quartz-35

calibration furnace (with a calibration accuracy of ± 0.15 [K]). The thermocouples were located on one of the tubes, at two levels: 0.2 and 1.6 m.

Fig. 1. a) Geometrical parameters of the investigated finned tube T_{woi} , T_{f} – thermocouple installation space, T_{ai} – air temperature measurement; b) Layout of the measurement stand: $\ddot{1}$ – condensing units; 2 – investigated tubes; 3 – electric heaters

When the tubes were fed with ice water, the heat flux transferred from the air to the water was calculated using the heat balance of the exchanger on the water side (Formula 1).

$$
\dot{Q} = \dot{V}_w \rho_w c_{pw} (T_{wout} - T_{win}) \tag{1}
$$

The water temperatures at the outlet and at the inlet of the exchanger were measured using PT100(7013) Hart Scientific resistance thermometers. In order to ensure high accuracy of the measurement of the temperature difference between the wall and the refrigerant, the sensors were calibrated over the full expected temperature range. The calibration was performed for the channel located in a cooling chamber with a controlled temperature level. Following the installation of all the sensors (thermocouples and resistance sensors) at the measurement stand, calibration was performed using precision sensors (Pt100(7013)). During calibration, air temperature in the cooling chamber was maintained at a level corresponding to the temperature of the refrigerant. A precise (tube wall and fluid) temperature difference measurement system created in this way has an accuracy comparable to the precision of Pt100(7013) sensors. The accuracy of a classical system used in such measurement is much lower. The volume flux of water was measured using a Hoffer turbine flow meter. The measurement values were supplemented with the measurement of the temperature of external air using PT100(7013) Hart Scientific resistance thermometers, and air humidity measurements (Hygrotest 80).

When the investigated tubes were fed with the cooling fluid, the heat flux transferred from the air was calculated using the heat balance equation on the side of the boiling refrigerant:

$$
\dot{Q} = \dot{m}_R (h_{\text{Rout}} - h_{\text{Rin}}) - (\dot{Q}_{\text{HEAT1}} + \dot{Q}_{\text{HEAT2}})
$$
\n(2)

In this case, the mass flux of the refrigerant was measured using a Mass 6000 Coriolis mass flow meter installed on the liquid line of the cooling circuit. The enthalpy of the refrigerant at the inlet and outlet from the evaporator of the cooling system were determined using RefproP.exe software, following earlier measurement of the temperatures (PT100 Hart Scientific resistance thermometers) and pressures (Vegabar17 pressure transmitter) of the refrigerant.

2.1. Determination of the mean values of heat transfer coefficient

The heat transfer coefficients of the air were obtained using two methods. The first method involved the calculation of the mean experimental values of the overall heat transfer coefficient $U_{\mu\nu}$ (3) and the mean heat transfer coefficients α_{μ} for the cooling medium (4–5):

$$
U_{Aw} = \frac{\dot{Q}}{A_w \cdot \Delta T_m},\tag{3}
$$

$$
\overline{\alpha}_R = \frac{\dot{Q}}{A_w \cdot (\overline{T}_{walli} - \overline{T}_R)},\tag{4}
$$

$$
\overline{T}_{walli} = \overline{T}_{wallo} - \frac{\dot{Q}}{A_w} \frac{d_i}{2} \left[\frac{1}{k_{\text{Al}}} \ln \left(\frac{d_{o1}}{d_o} \right) + \frac{1}{k_{cu}} \ln \left(\frac{d_o}{d_i} \right) \right],\tag{5}
$$

The mean temperature of the outer wall \bar{T}_{wall} was calculated as the arithmetical mean of the temperature of the outer wall along the circumference of the investigated tubes. Using formulas (2‒5) and solving equation (6) concerning the heat transfer coefficient (finned surface) for α _a for each measurement point, the heat transfer coefficient for air was calculated:

$$
U_{Aw} = \frac{1}{\frac{1}{\alpha_R} + \frac{d_i}{2} \left[\frac{1}{k_{Al}} \ln \left(\frac{d_{o1}}{d_o} \right) + \frac{1}{k_{cu}} \ln \left(\frac{d_o}{d_i} \right) \right] + \frac{A_w}{\alpha_a \cdot \text{RCJ} \cdot (\varepsilon_{fin} \cdot A_{fin} + A_t)}},
$$
(6)

In the case under consideration, the fin is treated as a prismatic straight fin with a height resulting from its heat transfer surface area. Hence, according to [10], fin efficiency is calculated using Schmidt's equation . The height of the fin $h_{\epsilon n}$ corresponds to the substitute height of a straight fin with a length of *L* and the actual fin surface area of:

$$
h_{\rm fin} = \frac{A_{\rm fin}}{n_{\rm fin} \cdot L} \tag{7}
$$

The openness of the RCJ process [11] applied to formulas (6) makes it possible to take into consideration the impact of condensation of water vapour from the air on the increase in the heat flux transferred by air to ice water. The RCJ coefficient determines the ratio of the heat flux related to the change in air enthalpy (change in temperature and degree of air humidity) and the visible heat flux related to the change in air temperature.

The openness of the RCJ process [11, 12] applied to formulas (8) and (10) makes it possible to take into consideration the impact of the condensation of water vapour from the air on the increase in the heat flux transferred by air to ice water. If the temperature of the cooler (\bar{T}_{wall}) is above the dew point, then the cooling process takes place without any change in air humidity, X_a = const, and the condition line coefficient of the transformation tends to infinity $\eta = \frac{\Delta h}{\Delta V} \rightarrow \frac{\Delta h}{\Delta X}$ → $-\infty$ [13]). Otherwise, humidity is condensed on the heat transfer surface. The air cooling process on the surface of the cooler may be treated as the mixing of two air streams: an air stream with the parameters, temperature and relative humidity corresponding to those of the cooler's surface ($T_a = T_{\text{wallo}}$, $\phi_a = 100\%$); an air stream with inlet parameters. The total heat flux transferred by the air to the heat transfer surface is the sum of the visible heat flux \dot{Q}_v – relating to a change in air temperature and the sensible heat flux \dot{Q}_s relating to the condensation of water vapour. The balance of the heat transferred by the air to the surface of the exchanger may be expressed by the following equation:

$$
\dot{Q} = \dot{Q}_{\nu} + \dot{Q}_{s} = (\varepsilon_{\text{fin}} \cdot A_{\text{fin}} + A_{t}) (\overline{T}_{a} - \overline{T}_{\text{wall}}) \alpha_{a} + (\varepsilon_{\text{fin}} \cdot A_{\text{fin}} + A_{t}) (\overline{X}_{a} - \overline{X}_{\text{wall}}) \beta_{a} h_{g}
$$
(8)

Equation (8) may be transformed to form (10) using Lewis's psychometric relationship for humid air (Chilton–Colburn heat and mass exchange analogy) describing the relationship between the heat transfer coefficient α_a and the mass transfer coefficient β_a (9) [14]:

$$
\frac{\alpha_a}{\beta_a} = c_{pa} \mathcal{L} e^{2/3} \approx c_{pa} \tag{9}
$$

$$
\dot{Q} = (\varepsilon_{fin} \cdot A_{fin} + A_t)(\overline{T}_a - \overline{T}_{\text{wallo}}) \alpha_a \left[1 + \frac{(\overline{X}_a - \overline{X}_{\text{wall}})_a \beta_a h_g}{(\overline{T}_a - \overline{T}_{\text{wallo}}) \alpha_a} \right]
$$
\n
$$
= (\varepsilon_{fin} \cdot A_{fin} + A_t)(\overline{T}_a - \overline{T}_{\text{wallo}}) \alpha_a \left[1 + \frac{(\overline{X}_a - \overline{X}_{\text{wall}})_b}{(\overline{T}_a - \overline{T}_{\text{wallo}}) c_{pa}} \right]
$$
\n(10)

For humid air, it is assumed that Le^{2/3} \approx 1 [14]. The expression in the rectangular brackets in equation (10) describes the RCJ coefficient used in equation (6)*:*

$$
RCJ = 1 + \frac{h_g}{c_{pa}} \frac{(\overline{X}_a - \overline{X}_{wall})}{(\overline{T}_a - \overline{T}_{wall})},
$$
\n(11)

where \bar{X}_{wall} is the degree of humidity of saturated air at temperature \bar{T}_{wall}

$$
\overline{X}_{wall} = \frac{0.622 \cdot p_w(\overline{T}_{wallo})}{10^5 - p_w(\overline{T}_{wallo})}.
$$
\n(12)

Equation (10) is ultimately reduced to the following form:

$$
\dot{Q} = (\varepsilon_{fin} \cdot A_{fin} + A_t) \cdot (\overline{T}_a - \overline{T}_{\text{wallo}}) \cdot \alpha_a \cdot \text{RCJ}
$$
\n(13)

The second method for determining the mean heat transfer coefficient of dry air α _{*x*} arises from the use of formula (13). Using formulas $(11-12)$, formula (13) was solved for α*a* at every measurement point. Both of the presented methods used to determine the mean heat transfer

2.2. Determination of local values of heat transfer coefficients

Local values of heat transfer coefficients were determined on the basis of temperature measurements at the base and tips of the fins. The following function is the solution of the energy equation for a straight prismatic fin:

$$
\Delta T(x) = T(x) - T_a = F_1 \cdot e^{m \cdot x} + F_2 \cdot e^{-m \cdot x}.
$$
 (14)

Constants F_1 and F_2 are calculated on the basis of boundary conditions, which take on the following form in the case under consideration: $\Delta T(x=0) = T_{\text{wallo}} - T_a$, $\Delta T(x=h_{\text{fin}}) = T_f - T_a$. Boundary conditions and relationship (14) enable the determination of integration constants F_1 and F_2 :

$$
F_1 = \frac{(T_a - T_{\text{wallo}}) \cdot e^{-m \cdot h_{\text{fin}}}}{2 \cdot \sinh(m \cdot h_{\text{fin}})},
$$
\n(15)

$$
F_2 = -\frac{(T_a - T_{\text{wallo}}) \cdot e^{m \cdot h_{\text{fin}}} + (T_f - T_a)}{2 \cdot \sinh(m \cdot h_{\text{fin}})},
$$
\n(16)

Local values of heat transfer coefficients for air were determined on the basis of the condition of equality of between heat flux absorbed by the fin (Q_{fin}) and the external surface of the tube without fins (Q_o) to the heat flux from the air to the cooling fluid:

$$
\dot{\mathcal{Q}}_{n_{\text{fin}}} = \dot{\mathcal{Q}}_{\text{fin}} + \dot{\mathcal{Q}}_{o} = -k_{\text{Al}} \cdot A_{\text{fin1}} \cdot m \cdot (F_1 - F_2) + \text{RCJ} \cdot \alpha_{\text{aloc}} \cdot A_{o1} \cdot (T_{\text{wallo}} - T_o). \tag{17}
$$

Formulas $(14-17)$ make it possible to calculate the local values of heat transfer coefficients α_{aloc} . The heat flux corresponding to the heat transfer surface area $A_{\text{fn1}} + A_{\text{d1}}$ (for a single fin) was determined using the heat balance of the investigated exchanger (eq. 2). In this case, the change of the heat flux at the tube circumference with respect to its mean value was taken into consideration according to the following relationship:

$$
\left(\frac{\dot{Q}}{n_{\text{fin}}}\right)_i = \frac{\dot{Q}}{n_{\text{fin}}}\cdot\frac{\Delta T_i}{\Delta T}.
$$
\n(18)

The goal of studying the local heat transfer coefficients was to perform a qualitative analysis of the distribution of heat transfer coefficients along the perimeter and heights of the investigated tubes. In addition, the research was used to compare directly measured mean heat transfer coefficients α_a with the mean heat transfer coefficients determined on the basis of the distribution of local values of $\bar{\alpha}_{loc}$.

3. Results of experimental studies

Experimental investigations of heat transfer coefficients on the outer surface of longitudinally finned tubes were conducted within the following range of parameters: air temperature $12 \le T_a \le 25$ [°C]; velocity of water flowing in the tubes $0.06 \le W_w \le 1.5$ [m/s]; density of the mass flux of coolants $90 \leq \dot{G}_R \leq 260$ [kg/(m²·s)]; water temperature T_R = 7 [°C]; evaporation temperature $-25 \le T_R \le -5$ [°C]; heat capacity $200 \le \dot{Q} \le 3700$ [W]. The measurement relative accuracy of values which were not directly measured are: $\delta(Q) = 2.5 - 8\%$; $\delta(\Delta T) = 0.01 - 0.24\%$; $\delta(k_{Aw}) = 2.7 - 10\%$; $\delta(\alpha_R) = 1.7 - 3.8\%$; $\delta(\alpha_a) = 8.6 - 14\%$.

Measurements of the distribution of temperatures at the base of the fin, along its perimeter and height have been shown in Figures 2a and 2c. Relationships $(14-18)$ and the obtained

Fig. 2. a, c) Distribution of temperatures on the outer heat exchange surface; b, d) Local values of heat transfer coefficients

measurements have been used to determine the local values of heat transfer coefficients along the perimeter of the tube (Fig. 2b and 2d).

Slightly higher values of heat transfer coefficients for the air have been recorded in the bottom part of the investigated tubes. In general, greater temperature differences $T_a - T_{wall}$ correspond to higher values of coefficients α_{abc} . In the upper part of the investigated tube, water temperature was higher than at the tube inlet. Under free convection, the buoyancy force is proportional to the difference in air density caused by differences in temperatures at the wall and at a large distance from the wall. A greater buoyancy force corresponds to higher airflow velocities and therefore, more intensive mixing of the fluid and an increased

Fig. 3. a) Relationship $\alpha_a (T_a - T_{\text{wallo}})$; b) Relationship Nu(Ra)

heat transfer coefficient. It also needs to be noted that the values of heat transfer coefficients obtained with the use of the two measurement methods (point 2.1) did not differ by more than 1.5%). In addition, the figure also shows the mean values of the heat transfer coefficients $({\bar \alpha}_{loc})$, obtained on the basis of the local values of heat transfer coefficients $({\alpha}_{abc})$. The obtained results indicate that the adopted measurement methods are correct and make it possible for the results to be described using a single dimensionless relationship.

Figure 3b shows the relationship between the Nusselt number and the Rayleigh number, determined both for cooling with ice water and coolants. The data included in Figure 3b provided the basis for determining the following dimensionless relationship:

$$
\text{Nu} = B_{1} \cdot (\text{Ra} / 10000)^{\left(B_{2} \frac{10000}{\text{Ra}} + B_{3}\right)} \cdot \left(\frac{\pi \cdot d_{o1}}{2 \cdot n_{\text{fin}} \cdot h_{\text{fin}} + (\pi \cdot d_{o1} - n_{\text{fin}} \cdot s_{1})}\right)^{0.15},\tag{19}
$$

$$
\text{Ra} = \text{Gr} \cdot \text{Pr} = \frac{\beta \cdot g \cdot d_{o1}^3 \cdot \Delta T \cdot \rho_a^2}{\mu_a^2} \cdot \frac{\mu_a \cdot c_{pa}}{k_a},\tag{20}
$$

where the characteristic dimension in the definition of Nu and Ra numbers is the outer diameter of the tube. The final element of equation (19) describes the influence of the heat transfer surface enhancement on the value of the heat transfer coefficient [10]. The obtained values of coefficients *A*, *B* and C in equation (19) are: $B_1 = 5.55 \cdot 10^{-5}$, $B_2 = 21.24$; $B_3 = 3.151$. Figure 3b) shows a comparison of measured and calculated values of Nu numbers.

4. Conclusions

The paper presents methods and the results of experimental research on heat transfer coefficients for airflow around vertical finned tubes with centrally-symmetrical longitudinal wavy fins. The research was carried out under free convection conditions. The mean values of heat transfer coefficients obtained during the study amounted to $2-8.5$ [W/(m²K)].

Studies of local values of heat transfer coefficients have confirmed the increasingly stronger relationship between those values and the difference between the temperature of the wall and of the air. The mean values of heat transfer coefficients obtained on the basis of the distribution of local values α_{abs} corresponded to mean values calculated from the balance equation and measurements of heat transfer coefficients.

As a result of the experimental studies, dimensionless relationships were determined to calculate the heat transfer coefficients for the cases under consideration. For free convection, for 85% of the measurement points, the differences between the measured heat transfer coefficients and those calculated on the basis of the dimensionless relationship were lower than 20%.

The results of the experimental research discussed in the paper, as well as the proposed dimensionless relationships, can be of great practical importance in designing air coolers and heaters working with natural airflow. The results presented in the paper will be especially useful in the design of heat pumps with evaporators without a fan.

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