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DYNAMICS OF SNOW MELTING ON TENTS DURING POSSIBLY THREATENING PRECIPITATION

PRZEBIEG TOPNIENIA ŚNIEGU NA DACHACH HAL NAMIOTOWYCH PODCZAS POTENCJALNIE NIEBEZPIECZNYCH OPADÓW

A b s t r a c t

Lightweight tent structures are allowed under European codes to be designed for reduced snow load if the interior of a tent is appropriately heated. The dependence of the resultant snow load on snowfall rate is considered in this paper for such cases. A simple thermodynamic model of the heat flux is derived. Numerical simulation reveals the occurrence of a precipitation rate threshold value above which safety measure provided by heating becomes ineffective.

Keywords: snow load, tents, safety

S t r e s z c z e n i e

Zgodnie z zapisami Eurokodów lekkie hale namiotowe mogą być projektowane na zmniejszone obciążenie śniegiem, jeżeli wnętrze hali jest odpowiednio ogrzewane. W artykule na podstawie prostego modelu termodynamicznego rozważana jest zależność wartości zgromadzonego obciążenia dachu od intensywności opadu śniegu. Obliczenia numeryczne ujawniają istnienie pewnej progowej wartości intensywności opadu śniegu powyżej której ogrzewanie hal staje się nieefektywne jako środek bezpieczeństwa.

Słowa kluczowe: obciążenie śniegiem, namioty, bezpieczeństwo

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1. Introduction

Most regular roof structures are designed for a 50-year return period of snow loads. This strict regulation makes less sense in the case of light, temporary tent-like structures. Code EN 13782 [3] in section 6.4.3.2 allows applying only 0.2 a kNm^{-2} load (or 8 cm of snow cover) in its design, provided that the inside volume of the tent is heated by appropriate equipment 'in such a way, that the whole cladding has an outside temperature of more than +2°C' and that the heating begins prior to snowfall. This approach became quite common on the market, and the so-called 'not full-season' tent halls are being offered together with heating equipment and recommendations to clear away the snow in case of heavy precipitation. This last part remains problematic due to the fact that the very operation of snow clearance is in itself an additional load applied to the structure in situations when it could be close to the limit of its carrying capacity.

The melting of already deposited snow cover may require a significant amount of heat due to the relatively high latent heat of ice. However, in the above mentioned scenario, snowfall intensity would be a key factor shaping demand for heating power. According to ICAO [4], snowfall may be considered as 'light' if the precipitation rate does not exceed 1 kg $h^{-1}m^{-2}$. 'Moderate' snowfall conditions are those not exceeding 5 kg $h^{-1}m^{-2}$ (in some studies a 2.5 kg $h^{-1}m^{-2}$ is also given as a threshold value). Levels higher than those defined as 'moderate' snowfall precipitation are described as 'heavy'. These three categories of snowfall intensity are probably rooted in return period statistics similar to those applied to ground snow load, but unlike the latter, the spatial distribution of snowfall intensity is generally unknown.

2. Snow cover melting model

The problem of snow melt is well-covered in literature (for example [1, 2, 5]), and may be derived from the basic rules of thermodynamics. The heat flux generated by the provided equipment is distributed among heat conduction through the walls and ground, conduction through the roof (and the snow cover), increase of inside air enthalpy and increase of snow cover enthalpy which eventually causes melting. Estimation of these components in real applications may not be easy due to complicated phenomena such as convection, possible outside air infiltration, various inhomogeneities of the air, the roof structure and the snow itself. Therefore, the +2°C rule seems to be a reasonable condition at which large enough areas of the bottom layer of the snow cover melts and initiate liquid water flow or even snow slides (another two complicated phenomena).

A simple thermodynamic model is derived taking into account the heat balance in a light tent structure. The snow cover is treated as a one-dimensional domain with phase change, obeying the equation [7]:

$$\rho \frac{\partial h}{\partial t} = \nabla \left(\lambda \cdot \nabla T \right) \tag{1}$$

where: ρ − density, h − specific enthalpy, λ − specific thermal conductivity, T − temperature.

The temperature may be expressed as a function of enthalpy (13). Spatial derivative in a one-dimensional case may be replaced by a derivative over water content coordinate:

$$\nabla = \frac{\partial}{\partial y} = \rho \frac{\partial}{\partial w} \tag{2}$$

which further simplifies the governing equation to:

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial h}\left(\frac{\rho\lambda}{c}\right)\left(\frac{\partial h}{\partial w}\right)^2 + \left(\frac{\rho\lambda}{c}\right)\frac{\partial^2 h}{\partial w^2} \tag{3}$$

where y – vertical coordinate, w – water content coordinate ($w = 0$ for the bottom of the snow cover), c − specific heat of a substance (snow, water or mixture, depending on the value of the enthalpy).

The specific conductivity of snow may vary significantly [6] and even show anisotropic properties, but in approximation may be expressed as a function of density:

$$\lambda_s = 3\times 10^{-6}\rho^2 - 1.06\times 10^{-5}\rho + 0.024 \tag{4}$$

where both density and conductivity are expressed in standard SI units.

Total heat balance is given by (Fig. 1):

$$q_c + q_w + \partial_t H_c + \partial_t H_a = q_0 \tag{5}$$

It is assumed that no heat flux is present at equilibrium $T_i = T_e$ and beyond this point the heat flux is linear function of temperature difference (in other words: there is no persistent heat flux from the ground). Hence, the heat flux through walls and ground:

$$q_w = \Lambda_w \left(T_i - T_e\right) \tag{6}$$

rate of change of enthalpy of inside air:

$$\partial_t H_a = c_a \partial_t T_i \cdot V \tag{7}$$

total change of enthalpy of the snow cover:

$$\partial_t H_c = \partial_t \int_0^D h\,dy \tag{8}$$

where: H_c − total enthalpy of snow cover, H_a − total enthalpy of inside air, q_c − heat flux conducted by roof and snow cover, q_0 − heat source power, Λ_w − total conductivity of walls and ground, T_i − internal temperature, T_e − external temperature, c_a − specific heat of the air, V – inside volume of the tent, D – depth of the snow cover.

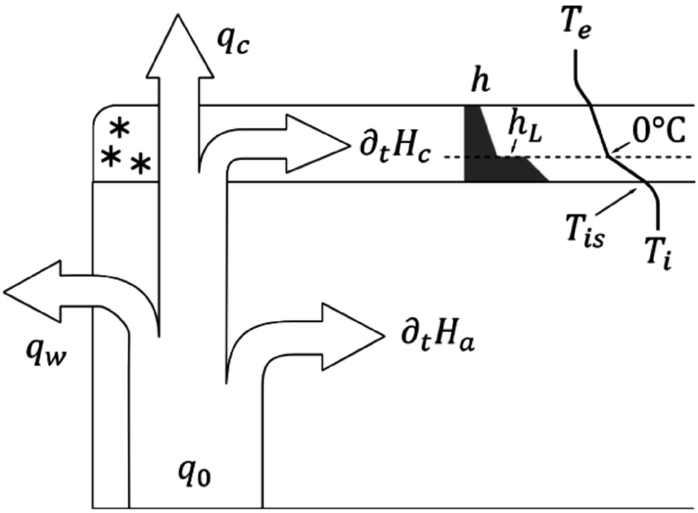

Fig. 1. Heat flow model in heated tent with snow cover

Air temperature inhomogeneity and cold air influx may be taken into account by appropriately modifying specific heat of the air to account for the loss of energy due to cold air mixing and gain in snow cover melting efficacy due to higher than average air temperature directly under the roof caused by convection.

The temperature at the top of the roof surface is expressed as:

$$T_{is} = T_i - \left(q_c + \partial_t H_c\right) \cdot \left(R_i + R_r\right) \tag{9}$$

where: R_i − thermal resistance of the heat interception, R_r − thermal resistance of the roof.

The thermal resistance of the roof in the case of tents covered with light fabric is negligible.

3. Calculation of snow melting during precipitation

The calculation involving phase change is susceptible to numerical instability due to stiffness of the governing equation in the transition region. To address this issue, known as the moving boundary problem, a dimensionless parameter (β) is used:

$$\beta = \frac{T_{is}}{w_t} \cdot \frac{w_t \kappa + R_e \rho_w \lambda_w}{\kappa T_{is} - T_e}, \quad \kappa = \frac{\rho_w \lambda_w}{\rho_s \lambda_s} \tag{10}$$

where: T_{is} − temperature on the internal surface, w_t − total water content in snow cover, $(\ )_w$ – water properties, $(\ )_s$ – snow properties.

The βw_t is a water content coordinate at which the phase transition occurs (shown as a "shelf" of h_L on Fig. 1). It is assumed that the curvature of the enthalpy function in snow and water layers is zero. This is a simplification, because the gain of the enthalpy in these regions can only occur by the second term of the RHS of (3). This approach may be called quasi-static – at each time point it is assumed that the enthalpy is at equilibrium and the change of system is accomplished only by a change of external conditions. In the case of this simulation, these external conditions are: the internal air temperature T_i, which changes due to the heating of the tent; the total water content of the snow layer w_t, which changes due to precipitation.

It is assumed that specific heat during melting is infinite, that is to say that the phase transition occurs at a constant temperature of 0°C, and the specific heats of the snow and water phases are constant.

With these assumptions, heat loss through the roof and snow layer may be expressed as:

$$q_c = \rho_w \lambda_w \frac{\kappa^\star T_{is} - T_e}{w_t \kappa + R_e \rho_w \lambda_w}, \quad \kappa^\star = \begin{cases} 1 \Leftarrow T_{is} \le 0 \\ \kappa \Leftarrow T_{is} > 0 \end{cases} \tag{11}$$

The snow enthalpy reference value is assumed to be 0 J kg^{-1} at the temperature of external air (T_e). This assumption simplifies the calculation of the heat budget during precipitation since falling snow does not add enthalpy to the system.

Total thermal energy contained in the snow layer may be expressed depending upon the location of phase change layer (β) as follows:

$$H_c = \begin{cases} \dfrac{h_e + h_i}{2} w_t & \Leftarrow \beta \le 0 \\ \dfrac{h_e - T_e c_s}{2} (1-\beta) w_t + \dfrac{h_L - T_e c_s + h_i}{2} \beta w_t & \Leftarrow \beta > 0 \end{cases} \tag{12}$$

where: h_e − specific enthalpy of the snow layer at external surface, h_i – specific enthalpy of the snow layer at internal surface, h_L – latent heat of ice melting, c_s – specific heat capacity of snow.

Temperature of the snow cover layer is a function of specific enthalpy:

$$T(h) = \begin{cases} \dfrac{h}{c_s} + T_e & \Leftarrow h < -c_s T_e \\ 0°\mathrm{C} & \Leftarrow -c_s T_e < h < h_L - c_s T_e \\ \dfrac{(h - h_L + c_s T_e)}{c_w} & \Leftarrow h > h_L - c_s T_e \end{cases} \tag{13}$$

where: c_w − specific heat capacity of water.

The problem in this form has two free variables: internal temperature of the roof surface T_{is}, and total water content of the snow layer w_t dependent on snow precipitation rate and time. Time derivatives of these variables may be calculated analytically.

4. Example

An example calculation is performed for a typical 20 m × 30 m × 8 m steel tent structure covered with a thin layer of PVC. The parameters of the simulation are gathered in Table 1.

Table 1

Simulation parameters

Simulation parameter			Snow	Water
Density	ρ	[kg m^{-3}]	200	1000
Conductivity · density	$\lambda\rho$	[kg m^{-2} s^{-3} K^{-1}]	≈ 28	580
Specific heat capacity	c	[J kg^{-1} K^{-1}]	2090	4190
Water/snow properties ratio (eq. 10)	κ	[1]	≈ 20.7	
Roof area		[m^2]	600	
Average tent height		[m]	8	
External temperature	T_e	[°C]	–4	
Latent heat of snow melting	h_L	[J kg^{-1}]	333 000	
Total heat conductivity of walls and ground	Λ_w	[W K^{-1}]	1500	
Thermal resistance of the inside surface of the roof	R_i	[K m^2 W^{-1}]	0.1	
Thermal resistance of the outside surface of the roof	R_e	[K m^2 W^{-1}]	0.04	

Initial condition of the simulation is $T_{is} = T_e$. It is a worst case scenario when the heating is being turned on only after precipitation starts. This condition violates the recommendation enclosed in EN 13782 [3] to heat the interior of the tent prior to the precipitation. The simulation of snow melting with initial temperature of the roof above 0°C becomes significantly more complex since in this case, snow-water mixture flow over sloped roofs must be taken into account. However, calculations show that the initial stage at which heating is mostly directed at increasing temperature of the internal air is relatively short compared to the time required to overcome the latent heat of precipitating snow. Moreover, this scenario is unfortunately quite probable in real-world situations.

Simulation is terminated when surface temperature exceeds +2°C which is (as assumed before) a time point when snow cover clearance occurs. The total amount of snow cover accumulated to that point is shown in Fig. 2 in relation to heat source power q_0. The simulation was resolved by the RK4 algorithm with a time step of 3 seconds. The time step was adjusted at 0°C crossing due to numerical instability.

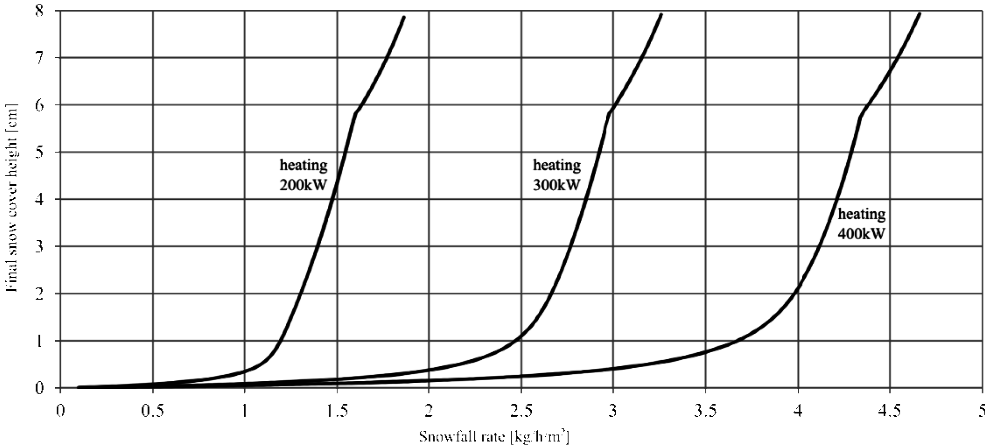

Fig. 2. Final (i.e. at T_{is} = ±2°C) snow cover height for different heating power

5. Conclusions

Although the snow melting phenomenon on a roof is a complex issue requiring many assumptions and simplifications, some general conclusions can be drawn regarding the reliability of safety measures obtained by heating of the interior of a tent. Despite the total heat power of heating devices, a threshold value of allowable snowfall intensity occurs in simulation. The final snow cover height (and roof load) is a step function for a relatively narrow range of snow precipitation rate, above which, weather conditions render the heating virtually ineffective as a means of preventing excessive snow loads.

Practical calculations aimed at assessing appropriate heating power must take into account many additional aspects of heat flux such as convection, outside air infiltration, snow permeability for melted snow, possibility of snow cover slide (roof fabric friction) etc. However, in any case, assumed snowfall intensity should remain as an important factor for such calculations.

The simulation shown in the paper shows that there is no safe level of heating power that can assure the safety of a large tent without relation to the expected snow-fall rate. Because the snow-fall rate is a random meteorological variable, the heating of a tent as a safety measure should be calculated with respect to the return period of this variable. An adequate value of this return period should be discussed among the engineering and scientific community.

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