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ANALYSIS OF THE INFLUENCE OF WOOD FRAMED DOME JOINT RIGIDITY ON INNER FORCES DISTRIBUTION

ANALIZA WPŁYWU SZTYWNOŚCI WĘZŁÓW DREWNIANEJ KOPUŁY PRĘTOWEJ NA ROZKŁAD SIŁ WEWNĘTRZNYCH

A b s t r a c t

The aim of this paper is to show the unfavourable influence of simplifications we adopt in modelling real structures numerically. The simplest way to model joints is to treat them as either rigid or flexible. However, there are structures whose stability strongly depends on joints' stiffness, an example of which is the structure of a dome whose joints have been designed as semi-rigid. It has been shown that even minor underestimation of the real rigidity of supports or joints has a negative impact on the static inner forces distribution.

Keywords: rigidity, joint, wood structures, forces distribution

S t r e s z c z e n i e

Głównym celem artykułu było wskazanie niekorzystnego wpływu uproszczeń, które przyjmujemy podczas numerycznego modelowania konstrukcji. Najprostszym znanym sposobem modelowania połączeń jest przyjęcie ich jako całkowicie sztywne bądź całkowicie podatne. Istnieją jednak konstrukcje, których stateczność silnie zależy od sztywności węzłów. Doskonałym przykładem okazuje się prezentowana poniżej konstrukcja kopuły, której węzły przyjęto jako częściowo podatne. Ukazano, że nawet małe niedoszacowanie rzeczywistej sztywności połączeń negatywnie pływa na pracę statyczną konstrukcji.

Słowa kluczowe: sztywność, węzły, konstrukcje drewniane, dystrybucja sił

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Notation

u – linear displacement [m]
ϕ – angular displacement [rad, –]
C, k, K – joints rigidity [kNm, kN/m]
E – Young's modulus [GPa]

1. Numerical model of the object

In order to verify if the influence of joints' rigidity is significant on the inner forces, the numerical model of the dome was created in RFEM 5 [9] on the basis of available material data following the currently valid norms. The dome was modelled as a system supported on columns, which represents the actual structure. The model consists of 1145 bars. The surface load is applied to the structure using the cell option, so that the program will transfer the surface load onto the bar immediately below the surface according to the generally accepted principles. There are 1035 wood bars and 110 steel bars. The steel columns are hinged based on rigid supports. It is an approximation of real case structures, since the real structure rests on a steel structure [2]. Wood and steel were modelled as linear elastic materials. Global view of the modelled structure is shown in Fig. 1.

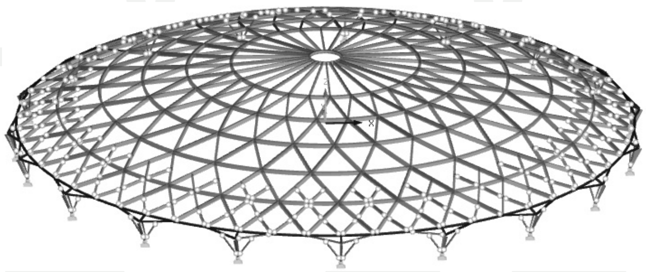

Fig. 1. Structure view form RFEM 5 program [9]

Plywood and glass roof was modelled and introduced as a constant load to the global model. There were two variants of the global model load varying in uniform or non-uniform snow distribution, adopting the basic load area per a dome quadrant, followed by connecting the quadrants with one another. The overall wind load operation was treated as suction.

The global structure was modelled as working linear, the stiffness matrix was calculated in the classical manner, followed by calculation of force results (without iteration). Initially the connections between the main bars were assumed as fully rigid and the secondary bars, with lower stiffness, as hinged joints operating in all directions. The minimal axial forces in Ultimate Limit State for this case are presented on Fig. 2. Next, an analysis of the joints stiffness of the structure was performed.

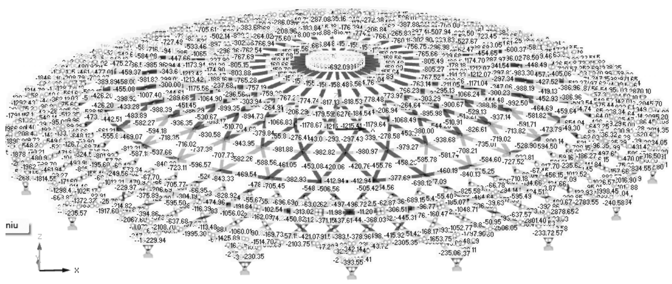

Fig. 2. Figure of minimal axial forces in Ultimate Limit State

2. Joints susceptibility designation

An analysis of the flexibility of the steel connector was carried in the most strained six-branched joint. The main goal of single joint analysis was the assumption of steel connectors stiffness. After that, calculated stiffness was put into the global model of the dome.

The single joint was modelled as panels of a constant thickness. The material of the joint was steel S 420. In order to check what the relationship between elements dimensions and joint rigidity is, six cases were taken under consideration:

Case 1 – the rotation stiffness for 20 mm thick sheets and pipe external diameter of 140 mm (without wood-bolt joint influence),

Case 2 – the rotation stiffness for 40 mm thick sheets and pipe external diameter of 170 mm (without wood-bolt joint influence),

Case 3 – the rotation stiffness for 40 mm thick sheets and pipe external diameter of 250 mm (without wood-bolt joint influence),

Case 4 – the joint rotational stiffness covering the stiffness of wood-bolt joint for the sheets dimensions from case 1,

Case 5 – the joint rotational and longitudinal stiffness covering the stiffness of wood-bolt joint for from case 1,

Case 6 – the joint rotational and longitudinal stiffness covering the stiffness of wood-bolt joint for from case 3.

In case of steel sheets, the joints panels were modelled as surfaces divided into tetragonal finite elements of 8 × 8 cm. The joint's central part was a pipe with wall thickness of 50 mm, divided into tetragonal finite elements of ca. 7 × 7 cm. The elements were made by program RFEM 5 [9] itself and, as a result, there were 432 finite elements for the connector shown on Fig. 3. The joint was loaded with forces derived from the global analysis to check the distribution of stresses. Loads were applied as point loads and point moments applied in the gravity centre of bolts configuration (Fig. 3). The values of these loads were taken from the dome model (from places where members were jointed) as axial and shear inner forces and moment in *XY* plane of sheets.

Detailed calculation of stresses in a single connector was carried out for case 2 in two steps. First, the forces were taken from the full rigid model of the dome, then the calculation of joint stiffness was made. After this, reduced stiffness of joints was applied to the global dome model, calculated once more, and than the inner forces were applied to the joint model once more.

Due to the fact that the connectors' metal components are situated between the rigid wood elements insensitive to rotation, they were supported on the entire plane, on the support of a given stiffness along the surface local axis z. The support stiffness was assumed as mean value of elasticity modulus perpendicular to the grains for timber [6, 7]. The angles of supporting timber surfaces are about 60 degrees, so this assumption was taken as a safety precaution.

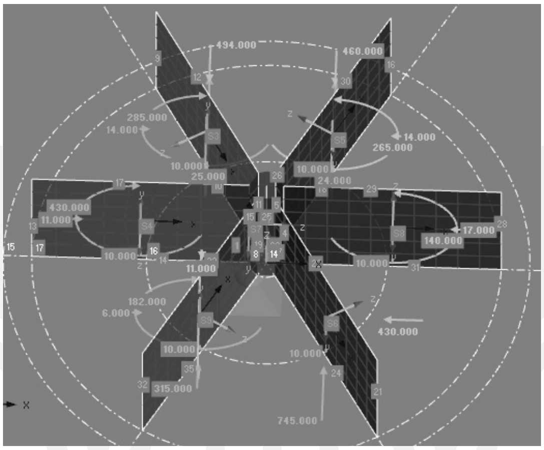

Fig. 3. Sheet joint's model loaded with maximum forces (axial forces, shear forces and moment in *XY* plane of the surface) for case 2

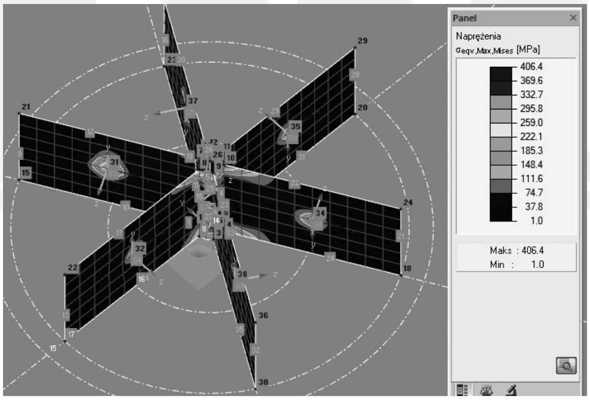

Fig. 4. Von Misses stresses in the most strained joint (case 2)

In order to maintain the stability of the model, a hinge support in one node was assumed, with the support receiving the entire vertical displacement.

As was mentioned before, the global model of the dome structure was modelled as linear. This assumption must have been made due to the size of the model. However, a single joint

(Fig. 3) was modelled as made of elastic-plastic material with a horizontal plasticity line. For a linear model, the stresses were too big for any available steel, so the plastic properties of steel were taken under consideration. The results of the calculation are shown as Von Misses Stresses in the Fig. 4.

In order to test the influence of joints rigidity on the structure's work, an analysis of the rigidity of a typical steel joint was carried out. This analysis was done separately for the deformation of the joint steel, as the most flexible part, and the deflection of the bolts, which are anchored in timber elements, due to the complexity of the calculations the following the PN-EN 1995-1-1 algorithm was determined [6, 8, 10]. The deformation of wood elements was not included due to their high flexural rigidity. The analysis was conducted by applying unit forces to the gravity centre of the bolts, with the displacements subsequently measured. It helps to give the final joints rotational and longitudinal stiffness.

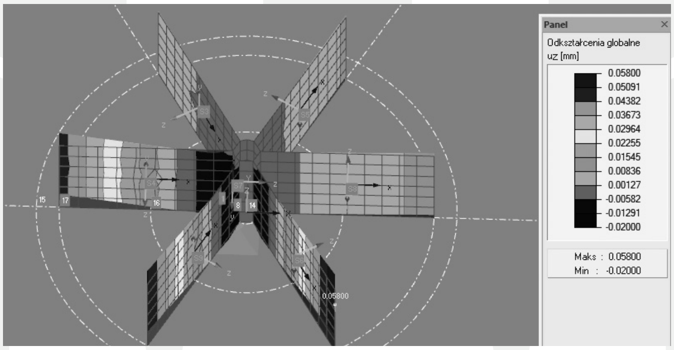

Fig. 5. Global element strain from unit moment

In a vector $\boldsymbol{u}^{T} = [0.001 \quad 0.014 \quad 0.017 \quad 0.020 \quad 0.004 \quad 0.003]\, m$, displacements of points corresponding to gravity centres of a group of bolt connectors (Fig. 5), which were used to determine the average stiffness of the connector are shown.

In order to simplify calculations for joint rigidity, the mean value of the displacement was calculated as an arithmetical mean. The next step was the calculation of a mean angular displacement of a single joint as mean displacement divided by the distance between gravity centres of bolts configuration and gravity centres of the joint. Rotational stiffness ($C_{\phi.\gamma}$) was calculated as unit moment value divided by mean value of the angular stiffness.

Equations (1) (6) show the way of stiffness determination for the bolt connections – the wood element after PN-EN 1995-1-1 [4, 6, 8, 10].

Due to PN-EN 1995-1-1 [6] slip modulus on Ultimate Limit State (K_{ser}) was calculated according to the formula 1 and the value for single bolt was K_{ser}= 19.4 MN/m

$$K_{ser} = \frac{2}{3} \times \rho_m^{1,5} \times \frac{d}{23} \tag{1}$$

Angular stiffness of a single joint depending on the bolts connection with timber in Ultimate Limit State was calculated as a geometrical sum given in formula 2 [3, 6]. In this formula, the long-term effects of loading and moisture content are taken under consideration by dividing it by $1+\psi_{2.1}\times k_{def}$. It equals 26.9 MNm.

$$k_{u_1} = \frac{2K_{ser}}{1+\psi_{2.1}\times k_{def}} \times \sum_{i=1}^{8} 4\left[x_{i.1}{}^2 + x_{i.2}{}^2 \right] \tag{2}$$

$$C_{\phi.\gamma.co} = \left(\frac{1}{k_{u_1}} + \frac{1}{C_{\phi.y}} \right)^{-1}$$

The final stiffness of the joints was calculated according to formula 3 [3], and for case 4, it equals: $C_{\phi.\gamma.co}$ = 18.2 MNm, for case 5, it equals: $C_{\phi.\gamma.co}$ = 20.7 MNm and for case 6, it equals: $C_{\phi.\gamma.co}$ = 26.2 MNm.

Angular stiffness in Serviceability Limit State was calculated according to formula 4, similar to the angular stiffness for Ultimate Limit State.

$$k_{ser_1} = \frac{2K_{ser}}{1+k_{def}} \times \sum_{i=1}^{8} 4\left[x_{i,1}{}^2 + x_{i,2}{}^2 \right] \tag{4}$$

Moreover, joints longitudinal stiffness and transversal stiffness were calculated for each case for a group of bolts in each wood-bolt connection, according to formulas 5 and 6. Longitudinal stiffness equals 134.1 MN/m and transversal stiffness equals 234.1 MN/m

$$K_{H_1} = \frac{n_{sz1}\times 2K_{ser}}{1+\psi_{2.1}\times k_{def}} \tag{5}$$

$$K_{V_1} = \frac{n_{sw1}\times 2K_{ser}}{1+\psi_{2.1}\times k_{def}}$$

3. Results based on static analysis of the dome structure

The resulting values of joints rigidity for six cases were introduced into the calculation model of the dome, and next re-calculated.

Cases analysed in the RFEM [9] program:

Case 0 – initial assumption of rigid joints,
Case 1 – the rotation stiffness of the joint steel sheet $C_{\varphi y1}$ = 52.6 MNm,
Case 2 – the rotation stiffness of the joint steel sheet $C_{\varphi y2}$ = 79.6 MNm,
Case 3 – the rotation stiffness of the joint steel sheet $C_{\varphi y3}$ = 445.7 MNm,
Case 4 – the joint rotational stiffness covering the stiffness of wood-bolt joint for $C_{\varphi y1}$ = 18.2 MNm,
Case 5 – the joint rotational and longitudinal stiffness covering the stiffness of wood-bolt joint for $C_{\varphi y1}$ = 18.2 MNm,
Case 6 – the joint rotational and longitudinal stiffness covering the stiffness of wood-bolt joint for $C_{\varphi y1}$ = 26.2 MNm.

The obtained results for all the above cases referred to the case 0.

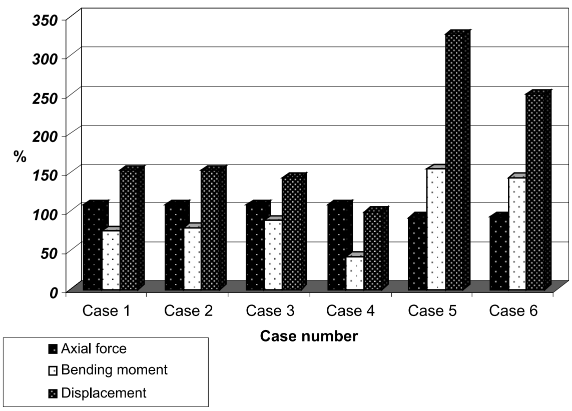

Fig. 6. Relationship between axial forces, bending moments and displacement for each case related to the case 0

As follows from the Fig. 6, the flexibility of joints does affect the forces distribution in both the bars and the joints. Whilst the rotational flexibility of joints reduced the joints moments, it increased the axial and shear forces. This phenomenon is favourable, since in moments transfer, the joints' work is non-symmetrical (only some joints are the most strained). On the other hand, assuming elastic strains of the connectors, on the transfer of axial and shear forces all joints are loaded uniformly. In wood-steel connections, the work in the connections is practically non-linear, the most strained joints are the extreme ones, even when purely axial forces are transferred.

What was initially considered was only the work of a steel joint in the form of a welded sheets pack which affected the forces redistribution only slightly (cases 1–3). After additionally taking into account the work of bolt connectors in the timber, there was another significant reduction in the rigidity of the joint (cases 4–6). What should be highlighted is the fact that the inclusion of longitudinal and transverse flexibility completely disturbed the distribution of forces in the structure (cases 5–6). It is generally believed that the global work of an element is affected mainly by flexural rigidity. In this case, also the reduction of compressive/tensile rigidity significantly affected the distribution of forces in the structure, which considerably changed the work quality (the reduction of the axial force in favour of the bending moment).

The impact of bolt connections in wood shear plate, adopted in the analysis, corresponds to the case 3, i.e. by analysing work most similar to rigid connections. When all the connections were assumed to be hinged, there was no convergence of calculations, unfortunately, and the displacement result was 13 m. From the above analysis, it can be concluded that the distribution of forces in the elements does not change significantly with the suggested variation of the rotational stiffness of the structure nodes analysed in this paper. Moreover, the moments M_z, transferred purely by the pressure of the wood members to each other decrease significantly when stiffness is reduced in this direction. Therefore, it can be stated that with some reserve for beam members carrying capacity, part of the moment will shift further away from the joint, which allows stresses to spread better in this direction.

The conspicuous change in the results of Serviceability Limit State is the only change that results directly from the reduction of the global stiffness of the structure. The design of a dome-shaped structure fortunately allows a large reserve in terms of displacements, which permits a partial loss of stiffness.

4. Conclusions

The analyses indicate that, as long as there are certain safety factors safeguarded in bar members design, assuming initially the joints as fully rigid, for more precise results it is possible to obtain less strained work of the joints. This phenomenon is favourable particularly in wood members, as the deformations in highly strained steel-wood connections are considerable and spoil the appearance of the structure. The aspect which was not analysed but may have a significant influence on the dome with a very small rise, is an equilibrium analysis which is shown in [1]. Domes with a small rise may be very sensitive to potential imperfections for a perfect structure. In the next stages of this analysis it should be verified.

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