Structural Recognition of Curves Using a Neural-Aided Fuzzy-Statistic Method with Applications to Graphs of Heart-Rate Ratios

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1. Introduction

Pattern recognition is one of principle problems in computer science. Many issues such as controlling, making decisions or predictions are related to it. It also has the main position in robotics. Therefore, this branch of computer science has been developing for a long time both in theoretical and implementation aspects. In a lot of cases pattern recognition can be a difficult problem and consequently the only method commonly used to sort out this issue does not exist. Presently, a wide range of methods based on various elements of mathematics, for instance calculus of probability or approximation theory, is applied. However, a universal recognition method does not exists – a given one can be effective for a specific sort of tasks and can fail for others. This is the reason why new methods are created and the existing ones developed. For example, syntactic methods are supported with probabilistic mechanisms and methods, which are combination of different basic methods such as neural-fuzzy ones, are created or hybrid expert systems are built. This paper concerns recognition curves in relation to their structural features. The considered problem is situated in a group of problems where pattern representation is a sequence of primitives being elements of a context language. For this group of languages, admittedly, automata which analyze these languages exist but their complexity is non-polynomial and consequently their usefulness in practical applications is limited. Moreover, an algorithm of grammar inference does not exist, consequently a method of automatic creation of tables controlling parsers (conversion functions in automata) does not exist, which in a practical nontrivial application disqualifies these languages. So, for structural patterns whose representations belong to context languages syntactic

methods allowing to analyze them do not exist. Therefore, an application of nonsyntactic methods to structural features analyzing seems to be valuable. The aim of this paper is to propose a new methodology of curves recognition in relation to their structural features taking advantage of fuzzy methods statistically aided. The possibility of a neural implementation of a recognition system based on the proposed methodology is tested. In the second chapter of this paper, the methodology of a decision function construction in an axiomatic recognition of patterns is presented. In the third chapter the proposed methodology is applied to classification curves describing relative changes in the cardiac rhythm between different people with and without a cognitive load, respectively. The curves were obtained in the Department of Psychophysiology of the Jagiellonian University. The experiment is described in details in [14, 15, 27]. The fourth chapter contains the description of a neural network computing the value of membership functions for each class.

2. A method of membership function specification

In the presented approach recognized classes of objects are considered to be fuzzy sets. Thus a definition of membership functions should be introduced. An algorithm that classifies recognized objects uses the values of membership functions calculated for each class. Therefore the construction of membership functions and the algorithm calculating their values is a theoretical stage of the method. The proposed methodology allows to construct the membership functions in the following way:

Step 1. Division of a set of objects into classes.

Division of a set of objects into classes must be done if it is not given *a priori*. In case of the complete information about structural features being known the division can be done on the basis of the given knowledge. Otherwise it has to be done by an expert or automatically, for example, by a neural network trained using unsupervised methods. We can also take advantage of taxonomic methods - see [6].

Step 2. Selection of distinctive features.

If the information about structural features of classes is not given, then the following steps should be done.

- 1. Features C_i^k , typical of each class, (k indexes classes, whereas i is the number of a feature in the class), are selected by analyzing a learning set X. Provided that Y_i is a set of values of the *i*-th feature a function $c_i : X \longrightarrow Y_i$ is defined as a result of features selection. While selecting distinctive features and founding the mapping c_i the following problems should be taken into consideration:
 - (a) How does correlation between features affect a membership function,
 - (b) How do perturbations of patterns affect a membership function,

- (c) Specification of asymptotic properties of membership functions on the bordering line between two classes.
- 2. A hypothesis that a found feature is distinctive, i.e., values of functions c_i are correlated with particular classes, is stated.
- 3. The hypothesis is verified using a statistical test.
- 4. If the feature has turned out to be a distinctive one, then for each class, a normalized histogram of relative frequencies of the feature values is plotted.
- 5. Three last steps are repeated until a sufficient number of distinctive features is selected.
- 6. A mapping $s_i^k : Y_i \longrightarrow [0, 1]$ is postulated based on histogram analysis for each class and feature, indexed by k and i, respectively. The following rules should be taken into consideration.
 - The value of mapping s_i^k is equal to one in every interval in which the frequency has the maximal value these values of the feature are the most typical of the class.
 - If such an interval of the feature values that can be found only in one class exists, then the function s_i^k also takes value 1 in this interval because this interval is significantly distinctive.
 - In the remaining intervals, values of function s_i^k depend on the frequency. The bigger frequency in the interval the higher value is taken by the function s_i^k in this interval. The bigger difference between frequency for the same feature but different classes the bigger difference between values taken by s_i^k for particular classes.
- 7. The membership functions are defined as:

$$\chi_k = \sum_{i=1}^n (w_i \cdot s_i^k),\tag{1}$$

where n is the number of selected distinctive features, w_i is the weight of the *i*-th feature, $w_i \in [0, 1]$. The value of s_i^k gives information how the *i*-th feature affects the value of membership function for the k-th class whereas w_i gives information about the relative power of distinction considering the feature.

Step 3. A definition of a decision function.

For a given problem classification criteria are postulated. The simplest way is to assign the recognized patterns to the class for which the calculated value of the membership function is the highest. However, also a more sophisticated classification algorithm should be taken into account especially in a case when either all values are small or a distinction between them is too low.

Notices

- The presented methodology can be applied also when distinctive features are statistical. It means that a given feature is only correlated with a given class in statistical meaning. It can happen that some objects from this class do not have a considered feature whereas others from different classes do.
- On the whole, in classical recognition methods it is assumed that considered distinctive features are not correlated. However, it can happen that the correlation between features is not very high but it is significant, and resigning from one of them can be a disadvantage to the whole recognition process. In a postulated axiomatic method of constructing a membership function considered features can be correlated but then the influence of this correlation on the properties membership function should be taken into account.
- If considered features are not correlated, then the second point can be omitted. Weights are calculated mainly for statistical features.

In Fig. 1 the membership construction process is presented as a block diagram. It is more general than the afore described method considering only the case when complete information about features in each class is unknown a priori. The block diagram includes both instances, when full information is given and when features have to be selected. Particularly, points 4 and 5 in step 2 are dedicated to creating axioms for a membership function while point 6 postulates their algebraical formulae. Parameters calibration corresponds to weights fitting in formula (1).

3. Analysis of heart rates

3.1. Structural analysis of graphs of a cr-function

The method of pattern recognition taking into account structural features aided by fuzzy sets theory, was applied to the data obtained in the Department of Psychophysiology of the Jagiellonian University. A comparison was drawn based on relative changes in the cardiac rhythm between different people with and without cognitive load, respectively. The experiment is described in details in [14, 15, 27]. The implemention of a system that recognizes whether a given graph of the function was obtained with or without cognitive load was our aim. For every person under examination the measurement of a heart rate was done many times and the final value is the average of all of them. The examination was conducted on a group of 54 people with and without cognitive load, respectively. Thus, as a result we obtained a set of 108 curves – 54 graphs of the function with cognitive load make up the first recognized class 'count' and 54 ones without cognitive load make up the second class 'ignore'. The aim was to specify structural features differing these two given classes. Given 'a priori' classes of graphs of the function were treated as fuzzy



Fig. 1. A scheme of construction of membership functions in a fuzzy system

sets and for each of them suitable membership functions were constructed according to the methodology described in the previous section. The obtained results of the classification were the starting point for constructing a neural classifier, presented in the next section. Examples of the obtained curves are shown in Fig. 2.

3.2. Features specification and a definition of membership functions

The structure of function graphs was investigated in order to find such distinctions that could describe each class in the explicit way. In the considered case only statistical features could be found. It means that a distinguishing factor for both classes is a distribution of considered features in each of them. In order to check whether a found feature is distinctive a statistical method was used. A test, verifying the hypothesis that two samples have the same meaning, has been used ([12], pages 249–250, [20]). If the feature distributions in the classes are unknown, then for a sufficiently large sample, the distribution of statistics is asymptotically normal ([12], page 249, case C) which is implied by the Central Limit Theorem of Levy ([12], page 180). The set of 108 curves was divided into a learning set containing 80 curves (function graphs numbered from 15 to 54) and a testing one with 28 curves. In the applications, according to the Central Limit Theorem, a sample consisting of tens of elements is sufficiently large. Considering the learning set, the following distinctive features were chosen:

- c_1 a relative position of the first minimum computed by dividing a coordinate position by a support function measure;
- c_2 a quotient of the sum of function values for all 15 points and the absolute sum of these values;
- c_3 a relative half-width of the first minimum computed by dividing a half-width by a support function measure;
- c_4 a relative depth of a niche fixed by an inflexion point at the end of a graph of the function see Fig. 2.

The following form of the membership function was proposed:

$$\chi_k = \sum_{i=1}^4 w_i \cdot s_i^k, \qquad k \in \{ig, cn\},$$
(2)

where w_i are weights for a given feature, s_i^k are values of function s for the *i*-th feature in the k-th class. The values of s_i^k are defined on the basis of the features distribution in each class obtained using histograms of features – see Fig. 3.



Fig. 2. Graphs of heart rates with (a) and without (b) cognitive load

The following staircase functions were received. For the ignore set:

$$s_1^{ig} = \begin{cases} 1.0 & \text{for } c_1 \in [0.20, \ 0.80] \\ 0.8 & \text{for } c_1 \in [0.10, \ 0.20) \\ 0.3 & \text{for } c_1 \in [0.05, \ 0.10) \\ 0.0 & \text{in other cases,} \end{cases}$$
(3)

$$s_2^{ig} = \begin{cases} 1.0 & \text{for } c_2 \in [-1.0, \ 0.0] \\ 0.4 & \text{for } c_2 \in (0.0, \ 0.4] \\ 0.0 & \text{in other cases}, \end{cases}$$
(4)

$$s_{3}^{ig} = \begin{cases} 1.0 & \text{for } c_{3} \in [0.20, \ 0.80] \\ 0.7 & \text{for } c_{3} \in [0.16, \ 0.20) \\ 0.5 & \text{for } c_{3} \in [0.10, \ 0.16) \\ 0.0 & \text{in other cases,} \end{cases}$$
(5)

$$s_4^{ig} = \begin{cases} 1.0 & \text{for } c_4 \in [0.00, \ 0.03] \\ 0.5 & \text{for } c_4 \in (0.03, \ 0.07] \\ 0.3 & \text{for } c_4 \in (0.07, \ 0.10] \\ 0.0 & \text{in other cases.} \end{cases}$$
(6)

For the *count* set:

$$s_1^{cn} = \begin{cases} 1.0 & \text{for } c_1 \in [0.0, \ 0.1] \\ 0.9 & \text{for } c_1 \in (0.1, \ 0.2] \\ 0.0 & \text{in other cases,} \end{cases}$$
(7)

$$s_2^{cn} = \begin{cases} 1.0 & \text{for } c_2 \in [0.0, \ 1.0] \\ 0.4 & \text{for } c_2 \in (-0.4, \ 0.0] \\ 0.00 & \text{in other cases,} \end{cases}$$
(8)

$$s_3^{cn} = \begin{cases} 1.0 & \text{for } c_3 \in [0.00, \ 0.21] \\ 0.0 & \text{in other cases,} \end{cases}$$
(9)

$$s_4^{cn} = \begin{cases} 1.0 & \text{for } c_4 \in [0.10, \ 0.24] \\ 0.5 & \text{for } c_4 \in (0.04, \ 0.10] \\ 0.3 & \text{for } c_4 \in (0.01, \ 0.04] \\ 0.0 & \text{in other cases.} \end{cases}$$
(10)



Fig. 3. Histograms of the distribution of features in every class

$$w_1 = 0.235,$$

 $w_2 = 0.290,$
 $w_3 = 0.325,$
 $w_4 = 0.150.$

Thus membership functions have forms:

$$\chi_{ig} = 0.235 \cdot s_1^{ig} + 0.290 \cdot s_2^{ig} + 0.325 \cdot s_3^{ig} + 0.150 \cdot s_4^{ig}, \tag{11}$$

$$\chi_{cn} = 0.235 \cdot s_1^{cn} + 0.290 \cdot s_2^{cn} + 0.325 \cdot s_3^{cn} + 0.150 \cdot s_4^{cn}.$$
(12)

Assuming that a given graph of the function belonging to the 'ignore' set is classified correctly, provided that the value of χ_{ig} is bigger than the value of χ_{cn} the following results were obtained:

- for the learning set 5 objects were classified incorrectly in the 'ignore' set (i.e. 87.5% curves were recognized correctly) and 6 objects were classified incorrectly in the 'count' set (i.e. 85% correctly), thus for the whole set 86.25% of events were recognized correctly;
- for the testing set 4 objects were classified incorrectly in the 'ignore' set (i.e. 71.5% curves were recognized correctly) and 1 object was classified incorrectly in the 'count' set (i.e. 93.1% correctly), thus for the whole set 82.3% of events were recognized correctly.

4. A neural implementation of structural recognition of function graphs of a heart rate

The next stage of the research consisted in inspecting whether the calculated membership functions, by means of formulae (11) and (12) can be reproduced with AANs. This is a classical problem of neural networks, signal classification and pattern recognition. The application of ANNs allows to reduce indispensable time to calculate a membership function for each class for all objects. One has to bear in mind that in formula (2) the measurement of distinctive features is needed, which is troublesome and time-consuming. The application of ANNs allows to calculate these values only on the basis of curves representations. It is also vital that the ANN used in such a context is not a classifier but an approximator. Thus the classification is not performed by the network which only calculates values of the fuzzy sets membership functions describing each class. In the experiment a measurement was performed on a constant interval of time. The vector for which the *i*-th component is the value of *i*-th measurement of a given curve, was an input signal for the ANNs. The problem of recognition of curves is dealt with a multi-layer ANN which was trained with a back-propagation algorithm which is a kind of a gradient algorithm. For a learning set the membership functions were calculated by means of forms (11) and (12).

Various configurations of multi-layer AANs were tested 14 - 1 - 2, 14 - 2 - 2, 14 - 3 - 2, 14 - 4 - 2 and 14 - 5 - 2. The set of 108 curves was divided into learning sets and testing ones also in various ways. The proposed divisions were: 80 and 28 or 70 and 38 or 60 and 48. In all these cases both in learning sets and testing ones the number of curves with cognitive load (*count*) and without it (*ignore*) were the same. During the learning process changes of weights were carried out after each presentation of vectors from the learning set. The succession of these vectors within each epoch was random. The formula

$$SSE := 0.5[(y_{ig} - \chi_{ig})^2 + (y_{cn} - \chi_{cn})^2]$$
(13)

was used to estimate the correctness of an answer of the net for a given curve, where y_{ig} and y_{cn} are values of the membership function estimated by the net. Values χ_{ig} and χ_{cn} are calculated by means of formulae (11) and (12). The classification of a given curve corresponded to theoretical calculations if $(y_{ig} - y_{cn}) \cdot (\chi_{ig} - \chi_{cn}) > 0$. Generally, the nets well reproduced both values of membership functions and the classification itself. Depending on the configuration of the nets and division of the set of 108 curves to the learning sets and testing ones the measure of correctness was as follows:

- from 91.7% to 98.4% for the testing set,
- from 89.3% to 96.5% for the learning set,
- from 93.5% to 96.3% for the whole set.

The nets also reproduced in an efficient way the values of membership functions achieving the following values of mean errors:

- from 0.088 to 0.12 for the testing set,
- from 0.079 to 0.100 for the learning set,
- from 0.083 to 0.100 for the whole set.

In all these cases the error of learning was stable after 200 epoches.

The net with the configuration 14-1-2 the division into 70 curves in the learning set and 38 in the testing one achieved the best result. It incorrectly classified 4 objects in the whole set (2 in the learning and 2 in the testing one). So the net performed the task with 97.14% efficiency for the learning set and 94.74% for the testing one and 96.30% on the whole set. The net with the configuration 14-3-2and the division into 60 curves in the learning set and 48 in the testing one achieved the efficiency for the learning set 93.33%, for the testing one 95.88% and 94.5% in the whole set.

The average squared errors for the learning set, the testing one and the whole set for two afore mentioned nets were 0.095, 0.090, 0.093 and 0.090, 0.120, 0.099, respectively.

5. Concluding remarks

The application of the proposed methodology allowed to separate given classes in an efficient way and implement a neural classifier. All the found features turned out to be distinctive only in a statistical sense, which was the main problem. In spite of this the result of the conducted experiment was satisfactory -86% correctly recognized curves for the learning set and 82% for the testing one, which in comparison to other recognition systems applied in medical science, gaining from 68% to 98% correctness, is a good achievement. A classification performed by the neural net, reproduced in 98% for the learning set and in 96% for the testing one the classification carried out by means of analytic formulae. Furthermore, the neural net reproduced the membership function values with a high accuracy.

The proposed methodology is valid also when considered classes are not clearly separated, which means that some objects lie on the border line between them. A precise specification of membership functions, describing attachment to individual classes, will allow both to point out domains of complete attachment to classes and exact tracing of membership values in border regions.

It should be stressed that the presented methodology is complementary to the frequently used methodology based on fuzzy sets consisting in creating rules of fuzzy reasoning (see [22], Chapters 3 and 5, [7, 17, 29]) being useful when producing rules would be hard to obtain for instance in cases where dependence of the membership function on features is complicated.

The vital problem in the considered example was a fact that both classes of function graphs were difficult to be separated. The application of fuzzy methods with statistics elements allowed both to separate these classes in an efficient way and to implement a proper neural network learned using the descent gradient method. On a certain stage of construction of membership functions statistic methods were applied to:

- verify wether a given feature is distinctive,
- propose values of feature weights for membership functions.

In the described cases fundamental statistic methods (standard tests) were applied. However, in a given case, more sophisticated methods can be put into practice what under no circumstances influences the structure of the algorithm of membership function construction described in Chapter 2. The classes of patterns were given but it was only known which one was obtained with and which one without cognitive load. There were not specified any graph features determining belonging to individual pattern classes. Therefore, it was essential to find distinctive features and then work out an axiomatic method and define membership functions. The vital problem was that all features turned out to be statistical. Apart from this, the size of the sample used in the research was quite small. It was only 108 graphs. Despite these problems, the method turned out to be sufficient. In particular

• the application of statistic methods allows to verify that successive structural features are distinctive; such approach allows to separate four distinctive features what turned out to be enough to recognize patterns in a sufficient way,

- the classification carried out by the proposed analytic formula for the membership function was sufficient – more than 86% correctly recognized patterns for the learning set and about 82% for the testing one, what in comparison to other recognition systems implemented for medical data reaching from 68% to 98% correctness is good enough ([2, 23, 24, 25, 21, 26, 28]),
- the classification by the neural network was similar to the classification carried out by the analytical formula, received results were 98% for the learning one and 96% for the testing one,
- the neural network well reconstructed the values of the membership function computed by the analytical formula.

The described methodology also allows to analyze when classical methods can be insufficient. Namely, if for example two classes are not separated well, it means that a great number of objects lies near to the boundary between both of them, then classical methods treat these classes as the same one. Whereas, the precise description of the membership function for each class as membership functions for fuzzy sets, allows to show that there are objects belonging to a given class completely and there are objects belonging to that class partially. In this way additional information about objects is gained.

There does not exist a commonly used structural feature definition. One of the known says that structural features should be unchangeable for calibration. It is obvious that features used, in the considered example, are unchangeable on account of graphs calibration because in each case a numerator and a denominator are multiplied by a rate of calibration.

6. References

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