

## The Precise Odometry Navigation for the Group of Robots

TOMASZ BURATOWSKI<sup>1</sup>, BORYS DĄBROWSKI<sup>2</sup>, TADEUSZ UHL<sup>1</sup>,  
MAREK BANASZKIEWICZ<sup>2</sup>

<sup>1</sup> AGH University of Science and Technology, Al. A. Mickiewicza 30, 30-059 Kraków  
email: *tburatow@agh.edu.pl*

<sup>2</sup> Space Research Center of the Polish Academy of Science,  
Bartycka 18A, 00-716 Warszawa

### 1. Introduction

The idea of the project is to trace the robot position and orientation and precise it by the implementation of the advanced algorithms based on triangulation and trilateration methods. Precise navigation is a difficult task since the odometry sensors used in this context are subject to systematic and random errors. The former ones result from inaccuracies in the mechanical construction of the vehicles, e.g. the wheel diameters can be unequal or the nominal direction of the wheels is different from real. The random errors arise due to interaction of wheels with the uneven terrain: slippage, faults, etc. [1]. Once one is able to determine the systematic errors and correct them, the problem is then made easier to attack but not yet solved. The random errors will accumulate as the robot progresses along its trajectory and the apparent position derived from odometry will be less and less accurate. The only way out, when a single robot is concerned, is to reference its estimated position to points with well-known coordinates. The situations seem to be different if many robots move together. Then the measurements of the relative position and orientation of the group should, in principle effectively reduce the random errors. Several authors analysed those problems in the past. The first issue, elimination of systematic errors, other words odometry calibration during the motion, has been approached by different methods. In the work by Borenstein and Feng [2], a calibration technique called UMBmark test has been developed to calibrate for systematic errors of a mobile robot with a differential drive. The Kalman filter approach employs robot equations of

motion and assumes some constant and initially unknown systematic errors as well as Gaussian random errors and solves for them by using kinematics and/or geometric measurements. This method requires dynamic motion equations to estimate desired parameters of equations. This method is interesting but requires strong computational unit for calculations. Larsen et al. [4] have introduced an augmented Kalman filter (AKF) which simultaneously estimates the robot configuration and the parameters characterising the systematic odometry error. This filter uses encoder readings as inputs and vision measurements as observations. These authors considered mobile robots with a differential drive system. Additionally optimal fusion sensor methods related with mobile robots in work were presented [3]. Martinelli [5, 6] adopted and extended AKF for a synchronous drive. This method seems to be very efficient in the process of precise navigation [7] because it is simultaneous, as an input requires encoder readings. The only problem is that it requires a vision system as an observer that is difficult to adapt in autonomous robots that require a vision system to find an object in the unknown terrain, and it is a computational problem. Another method of reducing both systematic and random errors is to couple the motion of vehicles by using a mechanical link [8]. Also Borenstein in work [9] is trying to reduce the influence of the navigation error in order to correct dead-reckoning errors. In that way one obtains, in fact, a single robot with two independently controlled wheel drives. Each vehicle uses the other ones as the reference. This method requires linking of robots, which means robots have additional tasks connected with precise navigation. This method has limitations especially in the unknown environment, e.g. where robots are responsible for fast object detection and localization. For group of vehicles, one approach to reduction of the navigation error is to use a mover-observer strategy, where one group of robots is stationary and observes the movement of other robots in the group [12]. In such a solution the measurements are used to correct the odometry error accumulated by the moving vehicles. In some cases odometry is abandoned and the position is estimated by means of observations by the stationary robots only [15]. While this method has proven successful, it slows down the speed of the entire group. In strategies where we are trying to optimize robots work in the group this method is not sufficient, because we are losing abilities to fast explorations of unknown. Another approach is to allow all the robots to move simultaneously. As an example of this strategy, Madhavan et al. [16] also uses an Extended Kalman Filter (EKF) to demonstrate cooperative localization of two rovers with a laser-rangefinder. In this case, one rover has absolute localization capabilities by means of a differential GPS. Thus, the position uncertainty remains limited, and the second rover can benefit from the accurate positioning information. This method is generally used in robots for outdoor solutions and requires special access to the technology based on radio communication with ground stations as landmarks. In this paper we attempt to solve the problem of cooperative positioning without the use of an absolute localization, determining the systematic odometry errors of each member of the moving group [19] as an

observer using ultrasonic and infrared sensors. To reach this goal we adopt an augmented Kalman filter introduced by Larsen [3, 4] to a group of robots that move simultaneously and use each other as beacons [22]. We also present an algorithm, which allows us to auto-calibrate the members of the group of robots without use of external reference points and observations. This may be advantageous over using natural landmarks or labelling a terrain before. For example, artificial beacons can be more easily employed in the unknown terrain than natural landmarks. Also, it is not necessary to prepare a terrain for navigation.

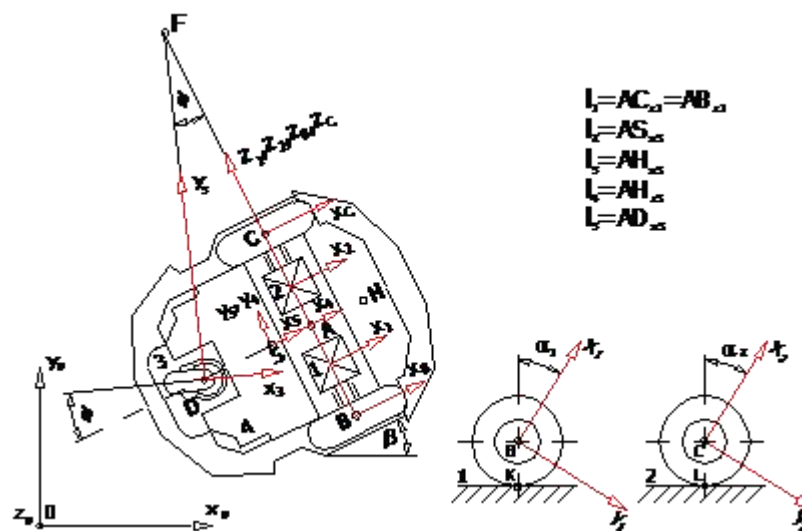


Fig. 1. Parameters for a 2-wheeled mobile robot

## 2. The model of computation formulation

On the basis of former works of authors [20, 22] a differential drive mode for the mobile robot model has been assumed. In this mode each wheel is controlled by its own engine and the turning is achieved by applying different rotation speed to the wheels. The characteristic points of the kinematics structure has been presented in Fig. 1.

Another example described and analyzed in the literature are computational models for mobile robots based on a synchronous drive mode [4, 5]. In this mode a single engine drives both wheels on the common axis and the turning of the vehicle is controlled by an independent small wheel.

### 2.1. Motion model

The position and orientation of the robot in a two dimensional environment, i.e. for a flat surface, can be described by three parameters:

$$X_o = [x \ y \ \theta]^T, \quad (1)$$

where  $x$  and  $y$  are Cartesian coordinates of the robot and  $\theta$  is its orientation expressed as an azimuth of the robot's direction in the  $(x, y)$  coordinate system. Since we want also to estimate the parameters responsible for systematic errors in robot motion, we add them to the description of the robot's state. For a differential drive we have:

$$X = [x \ y \ \theta \ \delta_R \ \delta_L \ \delta_d]^T, \quad (2)$$

where  $\delta_R$ ,  $\delta_L$  correspond to systematic errors in the distance covered by the right and the left wheel, respectively, while  $\delta_d$  is the error of the wheel axis length with respect to its nominal value. Hereafter we assume that all parameters responsible for systematic errors are time-independent.

We will model the odometry navigation as a discrete process, in which the current position  $X_{i+1}$  is calculated as the sum of the previous one  $X_i$  and the translation vector of the last step. The latter is determined from the encoder readings  $U_i$  and is subject to the Gaussian random error  $v_i$ .

$$X_{i+1} = f(X_i, U_i, v_i) \rightarrow X_{i+1} = X_i + \Phi_i \cdot (\Omega_i \cdot U_i + v_i). \quad (3)$$

The  $\Phi_i$  and  $\Omega_i$  matrices describe relationships between encoders readings, the random error and the translation of a robot. For a differential drive we obtained:

$$\Phi_i = \begin{bmatrix} \frac{\cos \theta_i}{2} & \frac{\sin \theta_i}{2} & \frac{1}{d \cdot \delta_{di}} & 0 & 0 & 0 \\ \frac{\cos \theta_i}{2} & \frac{\sin \theta_i}{2} & -\frac{1}{d \cdot \delta_{di}} & 0 & 0 & 0 \end{bmatrix}^T, \quad \Omega_i = \begin{bmatrix} \delta_{Ri} & 0 \\ 0 & \delta_{Li} \end{bmatrix}. \quad (4)$$

For a differential drive encoder readings are presented in the form:

$$U_i = [\delta p_i^{eR} \ \delta p_i^{eL}]^T, \quad (5)$$

where  $\delta p_i^{eR}$  and  $\delta p_i^{eL}$  correspond to translation of the right and left wheel in the  $i$ -th step.

For a synchronous drive, we can describe a vector of random errors  $v_i$  with the normal distribution and zero mean value as:

$$v_i = [v_i^\rho \ v_i^\theta]^T, \quad \begin{cases} v_i^\rho \propto N(0, \sigma_\rho) \\ v_i^\theta \propto N(0, \sigma_\theta) \end{cases} \quad \begin{cases} \sigma_\rho^2 = K_\rho \cdot \bar{\rho} \\ \sigma_\theta^2 = K_\theta \cdot \bar{\rho} \end{cases}, \quad (6)$$

where  $\bar{\rho}$  is a distance covered at one step, and the parameters  $K_\rho$  and  $K_\theta$  describe nonsystematic components of the odometry error. These parameters are related to properties of a terrain, in which robots move. The covariance matrix for a differential drive is

$$v_i = \begin{bmatrix} v_i^R & v_i^L \end{bmatrix}^T, \begin{cases} v_i^R \propto N(0, \sigma_R) \\ v_i^L \propto N(0, \sigma_L) \end{cases} \begin{cases} \sigma_R^2 = K_w \cdot |\delta\rho_i^{eR}| \\ \sigma_L^2 = K_w \cdot |\delta\rho_i^{eL}| \end{cases} \quad (7)$$

where  $|\delta\rho_i^{eR}|$  and  $|\delta\rho_i^{eL}|$  are absolute values of translation of the right and left wheel, and the parameter  $K_w$  characterizes the random translation error for each wheel. The covariance matrix for a differential drive reads:

$$Q = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_L^2 \end{bmatrix} = K_w \begin{bmatrix} |\delta\rho_i^{eR}| & 0 \\ 0 & |\delta\rho_i^{eL}| \end{bmatrix}. \quad (8)$$

## 2.2. Multi-robot motion model

When modelling a group of  $n$  robots, the individual motion models have to be combined to an overall motion model. A motion model for the entire system state,  $X_i = [X_{1i}^T \dots X_{ni}^T]^T$ , is a block-diagonal superposition of individual robot matrices:

$$\begin{aligned} \Phi_i &= \text{diag}\{\Phi_{1i}, \dots, \Phi_{ni}\}, \\ \Omega_i &= \text{diag}\{\Omega_{1i}, \dots, \Omega_{ni}\}, \end{aligned}$$

We can describe encoder readings and the vector of the random error for a multi-robot motion model as:

$$U_i = [U_{1i}^T \dots U_{ni}^T]^T, v_i = [v_{1i}^T \dots v_{ni}^T]^T. \quad (9)$$

## 2.3. Observation model

In the assumed model it is possible to observe distances only or distances and bearing angles between themselves. For a single pair of robots, observations are given by formulas:

$$\begin{aligned} d_{oi} &= \sqrt{(x_i - x_o)^2 + (y_i - y_o)^2} + \omega_o^d, \\ \varphi_{oi} &= a \tan\left(\frac{y_i - y_o}{x_i - x_o}\right) - \theta_o + \omega_o^\varphi, \end{aligned} \quad (10)$$

where:

$d_{ot}$  – the distance between the observer and the target,

$\varphi_{ot}$  – the angle between the direction of motion of the observer and segment linked the observer and the target

$\theta_o$  – the orientation of the observing robot,

$x_o, y_o$  – estimated coordinates of the observing robot,

$x_t, y_t$  – estimated coordinates of the vehicle which serves as a reference point,

$\omega_o$  – the observation error with the normal distribution and zero mean value.

For a single robot, we can describe a complete observation set as follows:

$$Z_k = [d_{k1} \dots d_{kk-1} \quad d_{kk+1} \dots d_{kn} \quad \varphi_{k1} \dots \varphi_{kk-1} \quad \varphi_{kk+1} \dots \varphi_{kn}]^T, \quad (11)$$

where  $n$  is the number of robots in the formation. Now, for the entire group of robots, observations can be described as a single vector:

$$Z = [Z_1^T \quad \dots \quad Z_n^T]^T. \quad (12)$$

#### 2.4. Extended Kalman filter

The model described above can be characterized by two functions – a function of motion  $f$  and a function of observation  $h$ :

$$\begin{aligned} X_{i+1} &= f(X_i, U_i, v_i) \\ Z_i &= h(X_i, \omega_i) \end{aligned} \quad (13)$$

Our model makes use of the Extended Kalman Filter to estimate the current configuration (position, orientation and values of systematic parameters) of the moving vehicles. The Kalman Filter uses encoder readings  $U$  and observations  $Z$  to estimate the current values of parameters, and does it in two steps – a prediction step and a correction step. The prediction equations, seen below, project the state  $X$  and covariance  $P$  estimates from the time step  $i$  to step  $i + 1$ .

$$\begin{aligned} \hat{X}_{i+1}^- &= f(\hat{X}_i, U_i, 0) \\ P_{i+1}^- &= A_i P_i A_i^T + W_i Q W_i^T, \end{aligned} \quad (14)$$

where  $W$  is the Jacobian matrix of partial derivatives of  $f$  with respect to random error  $v$ ,  $A$  is the Jacobian of partial derivatives of  $f$  with respect to state  $X$ , and  $Q$  is the covariance matrix of  $v$ .

At the correction step, equations update the state and covariance estimates with the measurement:

$$\begin{aligned}
K_i &= P_i^- H_i^T (H_i P_i^- H_i^T + R)^{-1}, \\
\hat{X}_i &= \hat{X}_i^- + K_i (Z_i - h(\hat{X}_i^-, 0)), \\
P_i &= (I - K_i H_i) P_i^-,
\end{aligned} \tag{15}$$

where  $K$  is the Kalman gain matrix,  $H$  is the Jacobian matrix of partial derivatives of  $h$  with respect to  $X$ ,  $R$  is the covariance matrix of  $\omega$ .

### 3. Test rigs based on algorithms implementation

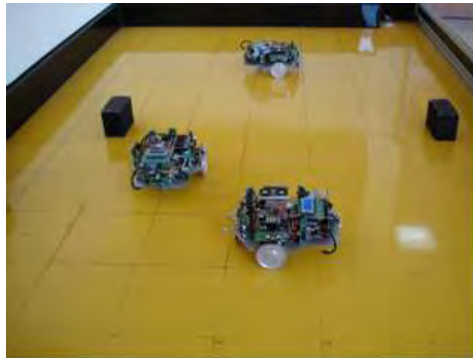
2-wheeled mobile robots for indoor solutions are mechatronic systems treated as anholonomic. This anholonomic condition has been presented below, and is connected with the characteristic point on robot, in this case point A from Fig. 1 has been taken into consideration.

$$\dot{y} = \dot{x} \tan(\Theta). \tag{16}$$

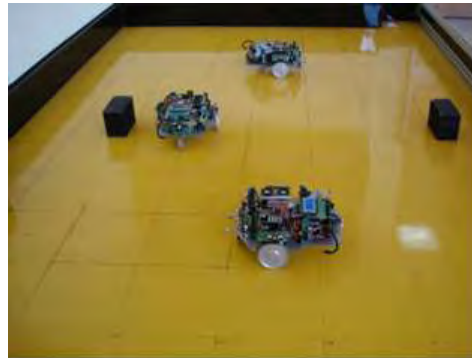
On the basis of our anholonomic condition we are assuming that there is no skid between the wheels and the ground. In order to present the results of the implemented algorithm a test rig based on the interaction between 1 robot in motion and 2 robots motionless has been carried on. The robot navigate on a square trajectory where the side is equal 0.9 [m] (Fig. 2). There are four reference objects necessary to precise navigation. Two passive obstacles (the robot calculating the distance to them Fig. 2b and Fig. 2f) in the environment and two active robots (both robots calculating the distance between each other Fig. 2d and Fig. 2h) with sending and receiving information about the environment. The movable robot calculates the position and orientation on the basis of fusion sensors information. Two infrared sensors (rangars) giving information about the robot present position and additional two ultrasonic sensors giving information about the distance between robots.

The robot navigates on the given square trajectory by moving in the sequence presented in Fig. 14 and moves the distance equal to 8.68 rounds, which means the covered distance is 31.28 [m]. The significant points of the robot traveling based on a real test rig have been presented in Fig. 2. Generally the robots move between clusters, but in order to precisely calculate the deviation from the desired trajectory in the real environment, the robot move on the border of the clusters, it means that characteristic point A (Fig. 1) of the robot is covering the cluster border line.

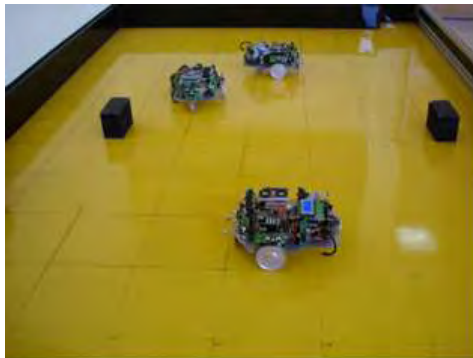
The presented test rig with the use of the Python language and specially prepared libraries in the Matlab environment has been used. The result has been presented in the form of information about the distance covered (Fig. 3), systematic errors in the distance covered by wheels (Fig. 5 and Fig. 6), and the error  $\delta_d$  of wheel axis length with respect to its nominal value (Fig. 4).



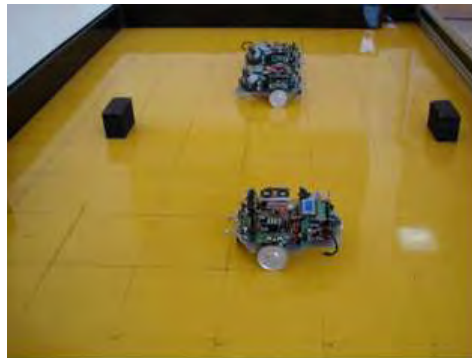
**Fig. 2a.** The robot starting point



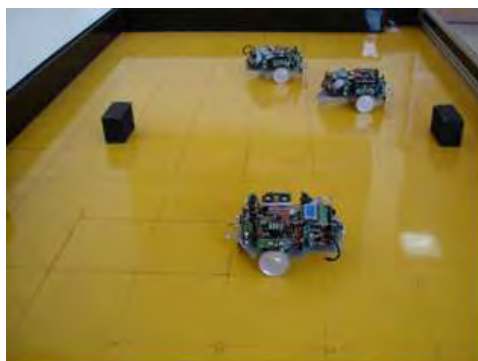
**Fig. 2b.** The first reference point



**Fig. 2c.** The first corner of the square trajectory



**Fig. 2d.** The second active reference point

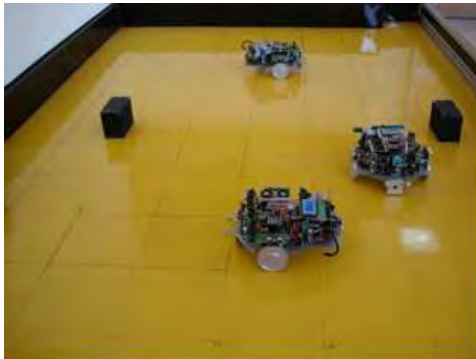


**Fig. 2e.** The second corner of the square trajectory

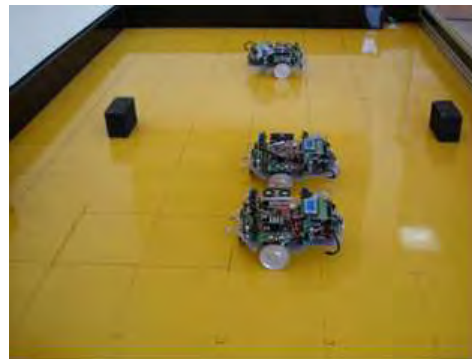


**Fig. 2f.** The third passive reference point

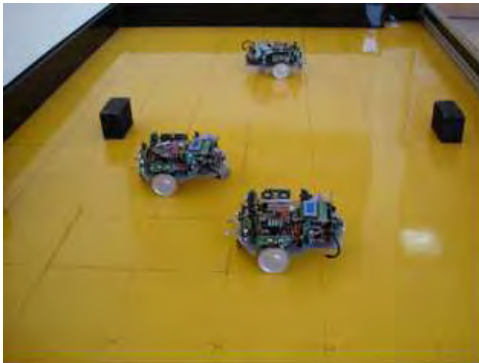




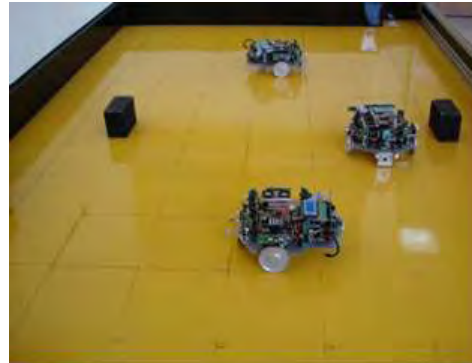
**Fig. 2g.** The third corner of the square trajectory



**Fig. 2h.** The fourth active reference point

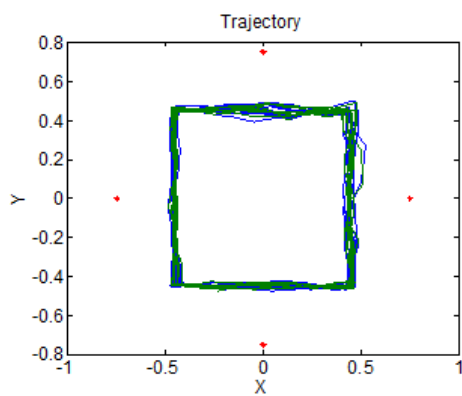


**Fig. 2i.** The fourth corner of the square trajectory

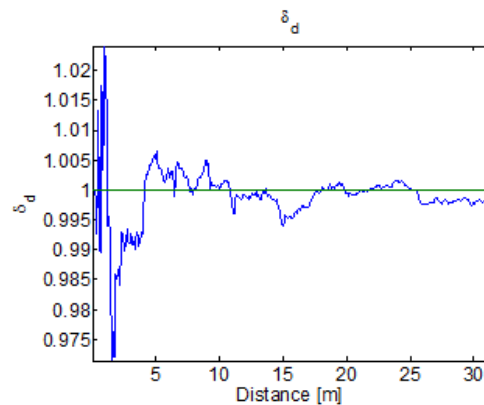


**Fig. 2j.** The robot end point trajectory

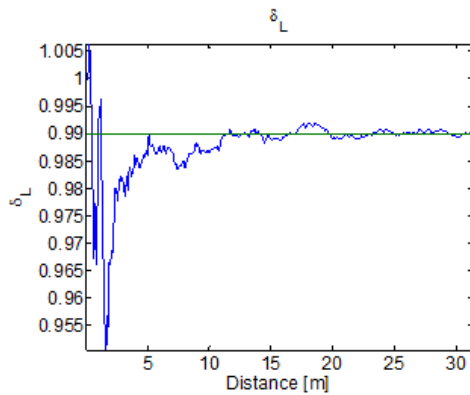
**Fig. 2.** A test rig of the precise position and orientation calculation based on the Extended Kalman filter with the use of one movable robot, two motionless robots and two obstacles, as reference points



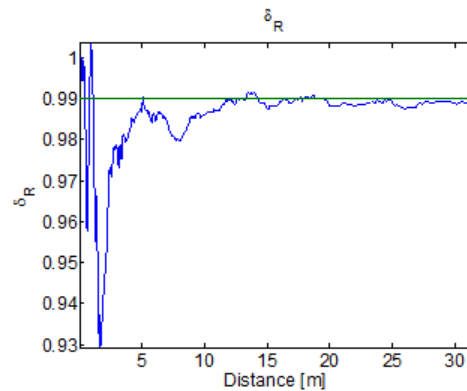
**Fig. 3.** The trajectory received from test rig



**Fig. 4.** The error of the wheel axis



**Fig. 5.** The systematic error in the distance covered by the left wheel



**Fig. 6.** The systematic error in the distance covered by the right wheel

In the presented test rig of the precise position and orientation calculation based on the Extended Kalman filter with the use of one movable robot, two motionless robots and two obstacles in the distance 0.3 [m] from the trajectory line, as reference points we can say that the algorithm positively fulfills expectations. The odometry error has been reduced by the Kalman filter, which means the robot can move on large distances, avoid obstacles, and navigate on the desired trajectory. After the analysis of the results from the test rig we can say also that after the distance of 18 [m] there was a significant reduction of the error connected with the axis measurement and after the distance of 15 [m] systematic errors connected with particular drive wheels also have been reduced. The novel approach presented in the article based on the Kalman filter application in the precise navigation of mobile robots is connected with the multi-robot motion model formulation and observation model description.

#### 4. Conclusions

The presented work is connected with the problem based on the precise navigation. The described test rig proved that robots are able to send and receive information about distance and use it in order to navigate. Additionally, designed algorithms and the software environment do not radically decrease computational abilities, which is very important in order to use vision systems to find a target. Further works based on the multi-robot motion model formulation and an observation description based on the Extended Kalman filter are connected with the application of these algorithms

for a group of robots where all mechatronic machines are in motion in order to avoid obstacles and move on the desired trajectory. Also a different sensors application based on laser technology will be considered. The idea of developing such algorithms is also connected with space research where we have *i.e.* satellites and want to calculate the precise position and orientation of these devices in space or we are trying to control the group of satellites. The presented mathematical description of the algorithms is universal and is tested on anholonomic mobile robots (2D operation) but can be applied also for objects operating in 3D scenario. However, it is easier (from the technological point of view) and cheaper to simulate and carry out tests on earth before the implementation on real space objects.

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